CBCS SCHEME

USN

18EC54

Fifth Semester B.E. Degree Examination, June/July 2023
Information Theory and Coding

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. A DMS emits symbols from the source alphabet $S = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7\}$ with $P = \{0.25, 0.25, 0.125, 0.125, 0.125, 0.0625, 0.0625\}$. Compute :
i) H(s) ii) $H(s)_{max}$ iii) Information Rate R if $r_s = 5$ symbols/sec. **(06 Marks)**

b. The state diagram of the Markov source is shown below Q1(b)

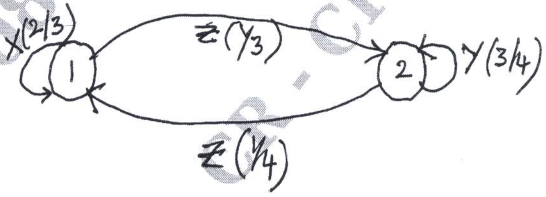

Fig Q1(b)

i) Find the Entropy of the source
ii) Find the Message Entropy G_1, G_2
iii) Verify $G_1 \geq G_2 \geq H(s)$ **(10 Marks)**

c. A zero memory has a source alphabet $S = \{S_1, S_2, S_3\}$ with $P = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right\}$. Construct the second order source and compute its entropy. **(04 Marks)**

OR

2 a. Prove that the Entropy is maximum when the symbols are equiprobable. **(06 Marks)**

b. Design a system to report heading of collection of 400 cars. The heading levels heading straight, turning left and turning right. The information is transmitted every second.
i) On an average during a reporting interval 200 cars were heading straight, 100 were turning left and remaining were turning right.
ii) Out of 200 cars that reported heading straight, 100 were going straight during next reporting interval, 50 turning left and remaining were turning right in the next reporting interval.
iii) Out of 100 cars reported turning during signaling period, 50 continued turning and the remaining headed straight during the next reporting interval.
iv) The dynamics of the car did not allow them to turn left to right and vice versa

Find entropy of the state and source. Also, find Rate of informations. **(10 Marks)**

c. Prove that entropy of the second order Binary source is $S^2 = 2H(s)$ bits/sy m **(04 Marks)**

Module-2

3 a. Construct a Shannon Fano code for the following symbols :
$S = \{S_1, S_2, S_3, S_4, S_5, S_6\}$
$P = \{0.2, 0.4, 0.15, 0.15, 0.06, 0.04\}$ **(10 Marks)**
With code alphabet $X = \{0, 1\}$ and $X = \{0, 1, 2\}$. Find the efficiency of the code.

b. A discrete memory less source has an alphabet of six symbols with probability statistics as given below :

Symbols	:	A	B	C	D	E	F
P	:	0.3	0.25	0.20	0.12	0.08	0.05

i) Construct the Huffman code by moving combined symbols as high as possible. Compute efficiency and variance

ii) Construct the Huffman trainary code by moving symbols combined as high as possible. **(10 Marks)**

OR

4 a. Test whether the following code is a prefix code :

A	1
B	0 1
C	0 0 1
D	0 0 0 1

(04 Marks)

b. Encode the symbols using Shannon encoding algorithm and compute the coding efficiency and variance for the following symbol set :

$X = \{x_1, x_2, x_3, x_4, x_5\}$

$$P = \left\{\frac{5}{16}, \frac{1}{4}, \frac{3}{16}, \frac{1}{8}, \frac{1}{8}\right\}$$ **(10 Marks)**

c. A DMS has an alphabet

$S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$

$$P = \left\{\frac{1}{12}, \frac{1}{8}, \frac{1}{12}, \frac{1}{8}, \frac{1}{3}, \frac{1}{4}\right\}$$

Construct Huffman code for the code alphabet X = {0, 1, 2}. Compute coding efficiency. **(06 Marks)**

Module-3

5 a. Compute Entropy function H(x), H(y) H(xy), H(x/y), H(y/x), Data transmission rate and channel capacity, given $\tau = 0.1$ sec/sym and $P(xy) = \begin{bmatrix} 0.15 & 0 & 0 & 0.15 \\ 0 & 0.2 & 0.15 & 0 \\ 0 & 0 & 0.1 & 0.05 \\ 0.1 & 0.1 & 0 & 0 \end{bmatrix}$ **(07 Marks)**

b. Compute the channel capacity for the channel given below :

$P(y/x) = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$. Given $r_s = 1000$ sym/sec. **(05 Marks)**

c. Derive an expression for the channel capacity of a Binary Erasure channel. **(08 Marks)**

OR

6 a. Prove that Mutual information is always positive. **(06 Marks)**

b. Compute the channel capacity for the channel with $r_s = 1000$ sym/sec and

$P(y/x) = \begin{bmatrix} 0.6 & 0.4 \\ 0.7 & 0.3 \end{bmatrix}$ **(06 Marks)**

c. A Binary channel has the following characteristics :

$$P(y/x)=\begin{bmatrix}\frac{2}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{2}{3}\end{bmatrix} \quad P(x)=\begin{bmatrix}\frac{3}{4} & \frac{1}{4}\end{bmatrix}$$

Compute :
i) Mutual information
ii) Channel capacity if $r_s = 100$ sym/sec. **(08 Marks)**

Module-4

7 a. For a (6, 3) Linear Block code, the check bits are related to the message bits as per the equations given below :
$C_4 = d_1 + d_2$; $C_5 = d_1 + d_2 + d_3$; $C_6 = d_2 + d_3$
i) Obtain the Generator Matrix G.
ii) Find all possible code words.
iii) Find H and H^T.
iv) Computer syndrome if there is an error in the 3rd bit of a transmitted codeword [110 001] and show how it can be corrected. **(10 Marks)**

b. For a (6, 3) cyclic code find the following :
i) g(x) ii) G in systematic form iii) find all possible code words. **(06 Marks)**

c. For a (7, 3) Hamming code with $g(x) = 1 + x + x^2 + x^4$, design a suitable encoder to generate systematic cyclic codes. **(04 Marks)**

OR

8 a. Prove that $C.\,H^T = 0$ there by show that $S = E.\,H^T$ **(06 Marks)**

b. A (7, 4) cyclic code has the generator polynomial $g(x) = 1 + x + x^4$. Design a syndrome computation circuit and verify the circuit for the message polynomial $d(x) = 1 + x^3$. **(07 Marks)**

c. For a (7, 4) Linear Block code the syndrome is given by
$S_1 = r_1 + r_2 + r_3 + r_5$
$S_2 = r_1 + r_2 + r_4 + r_6$
$S_3 = r_1 + r_3 + r_4 + r_7$
i) Find G and H matrix
ii) Draw the Encoder and syndrome computation circuit. **(07 Marks)**

Module-5

9 a. Consider (3, 1, 2) convolutional encoder with g(1) = (110), g(2) = (101), g(3) = (111)
i) Write the Encoder circuit.
ii) Write the state transition table.
iii) Write the state diagram.
iv) Write the code tree. **(10 Marks)**

b. For a (2, 1, 3) convolutional encoder with $g^1 = (1101)$, $g^2 = (1011)$
i) Find the constraint length.
ii) Find the rate efficiency.
iii) Find the codeword for the message sequence (11101) using matrix and frequency domain approach. **(10 Marks)**

OR

10 a. Explain Viterbi Decoding algorithm with an example. **(08 Marks)**

b. For the State show below with $S_0 = 00$, $S_1 = 10$, $S_2 = 01$, $S_3 = 11$, draw the trellis diagram. For the input sequence m = {1 0 1} trace the output.

Fig Q10(b) **(06 Marks)**

c. Define the following distance properties of convolution codes
   i) Minimum free distance
   ii) Column distance function
   iii) Minimum distance **(06 Marks)**

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