

## Fifth Semester B.E. Degree Examination, June/July 2023

### Information Theory and Coding

Max. Marks: 80

**Note:** Answer any FIVE full questions, choosing ONE full question from each module.

#### Module-1

1. a. Derive an expression for average information content of symbols in long independent sequence. (03 Marks)
- b. For the Markov source shown below, find i) The stationary distribution ii) State entropies iii) Source entropy iv)  $G_1 G_2$  and show that  $G_1 \geq G_2 \geq H(s)$ . (Refer Fig.Q1(b)). (10 Marks)
- c. Define Self Information, Entropy and Information rate. (03 Marks)

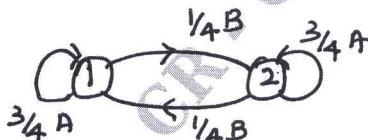


Fig.Q1(b)

2. a. Mention different properties of entropy and prove external property. (07 Marks)
- b. A source emits one of the four symbols  $S_1 S_2 S_3$  and  $S_4$  with probabilities of  $\frac{7}{16}, \frac{5}{16}, \frac{1}{8}$  &  $\frac{1}{8}$ . Show that  $H(S^2) = 2H(S)$ . (04 Marks)
- c. In a facsimile transmission of a picture, there are about  $2.25 \times 10^6$  pixels/frame. For a good reproduction at the receiver 12 brightness levels are necessary. Assume all these levels are equally likely to occur. Find the rate of information if one picture is to be transmitted every 3 min. Also compute the source efficiency. (05 Marks)

#### Module-2

3. a. Apply Shannon's binary encoding algorithm to the following set of symbols given in table below. Also obtain code efficiency. (08 Marks)

Symbols	A	B	C	D	E
P	1/8	1/16	3/16	1/4	3/8

- b. Consider a source  $S = \{s_1, s_2\}$  with probabilities  $3/4$  and  $1/4$  respectively. Obtain Shannon-Fano code for source  $S$  and its 2<sup>nd</sup> extension. Calculate efficiencies for each case. Comment on the result. (08 Marks)

#### OR

4. a. Consider a source with 8 alphabets A to H with respective probabilities of 0.22, 0.20, 0.18, 0.15, 0.10, 0.08, 0.05 and 0.02. Construct Huffman's code and determine its efficiency. (10 Marks)
- b. With an illustrative example, explain arithmetic coding technique. (06 Marks)

#### Module-3

5. a. Find the capacity of the discrete channel whose noise matrix is

$$P(y/x) = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.2 & 0.8 \end{bmatrix} \quad (04 \text{ Marks})$$

- b. Define Mutual Information. List the properties of Mutual information and prove that  $I(x; y) = H(x) + H(y) - H(xy)$  bits/system. (06 Marks)

- c. A channel has the following characteristics :

$P\left(\frac{y}{x}\right) = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$  &  $P(x_1) = p(x_2) = \frac{1}{2}$ . Find  $H(x)$ ,  $H(y)$ ,  $H(x, y)$  and Channel capacity if  $r = 1000$  symbols/sec. (06 Marks)

**OR**

- 6 a. A binary symmetric channel has the following noise matrix with source probabilities of

$$P(x_1) = \frac{2}{3} \text{ and } P(x_2) = \frac{1}{3} \text{ and } P\left(\frac{y}{x}\right) = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

- i) Determine  $H(x)$ ,  $H(y)$ ,  $H(x, y)$ ,  $H(y/x)$ ,  $H(x/y)$  and  $I(x, y)$ .  
ii) Find channel capacity  $C$ . iii) Find channel efficiency and redundancy. (08 Marks)

- b. Derive an expression for channel efficiency for a Binary Erasure channel. (05 Marks)  
c. Write a note on Differential Entropy. (03 Marks)

#### Module-4

- 7 a. Distinguish between "block codes" and "convolution codes". (02 Marks)

- b. For a systematic  $(6, 3)$  linear block code, the parity matrix is  $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ . Find all possible code vectors.  
c. The parity check bits of a  $(8, 4)$  block code are generated by  $c_5 = d_1 + d_2 + d_4$ ,  $c_6 = d_1 + d_2 + d_3$ ,  $c_7 = d_1 + d_3 + d_4$  and  $c_8 = d_2 + d_3 + d_4$  where  $d_1, d_2, d_3$  and  $d_4$  are message bits. Find the generator matrix and parity check matrix for this code. (06 Marks)

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(08 Marks)

- 8 a. A  $(7, 4)$  cyclic code has the generator polynomial  $g(x) = 1 + x + x^3$ . Find the code vectors both in systematic and nonsystematic form for the message bits  $(1001)$  and  $(1101)$ . (12 Marks)  
b. Consider a  $(15, 11)$  cyclic code generated by  $g(x) = 1 + x + x^4$ . Device a feedback shift registers encoder circuit. (04 Marks)

#### Module-5

- 9 a. Design a  $(15, 7)$  binary BCH code with  $r = 2$ . (06 Marks)  
b. Consider the  $(3, 1, 2)$  convolution code with  $g^{(1)} = (1 \ 1 \ 0)$ ,  $g^{(2)} = (1 \ 0 \ 1)$ ,  $g^{(3)} = (1 \ 1 \ 1)$ .  
i) Find the constraint length ii) Find the rate iii) Draw the encoder block diagram  
iv) Find the generator matrix v) Find the code word for the message sequence  $(1 \ 1 \ 1 \ 0 \ 1)$  using time – domain and transfer – domain approach. (10 Marks)

**OR**

- 10 a. Explain why  $(23, 12)$  Golay code is called as perfect code. (04 Marks)  
b. Consider the convolution encoder shown in fig. Q10(b).  
i) Write the impulse response of the encoder.  
ii) Find the output for the message  $(1 \ 0 \ 0 \ 1 \ 1)$  using time – domain approach.  
iii) Find the output for the message  $(1 \ 0 \ 0 \ 1 \ 1)$  using transfer – domain approach.

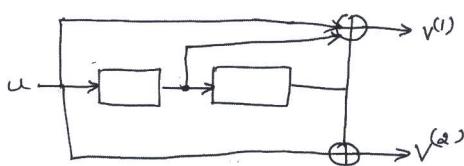


Fig Q10(b)

(12 Marks)

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