USN | | | | | | | | | | |

18EC44

Fourth Semester B.E. Degree Examination, June/July 2023
Engineering Statistics and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Discuss the CDF and PDF of a random variable. List the properties of PDF. (08 Marks)

b. Given the data in the following table:

k	1	2	3	4	5
y_k	2.1	3.2	4.8	5.4	6.9
$P\{y_k\}$	0.2	0.21	0.19	0.14	0.26

i) Plot the PDF and CDF of the discrete random variable Y.

ii) Write expressions for PDF and CDF using unit delta and unit-step functions. (08 Marks)

c. A continuous random variable X has a PDF, $f_x(x) = 3x^2 \quad 0 \le x \le 1$. Find 'a' such that $P\{x > a\} = 0.05$. (04 Marks)

OR

2 a. Define an exponential random variable. Obtain the characteristic function of an exponential random variable and using the characteristic function derive its mean and variance. (10 Marks)

b. Given the data in the following table:

k	1	2	3	4	5
y_k	2.1	3.2	4.8	5.4	6.9
$P(y_k)$	0.2	0.21	0.19	0.14	0.26

i) What are the mean and variance of Y.

ii) If $W = y^2 + 1$, what are mean and variance of W. (10 Marks)

Module-2

3 a. Define correlation coefficient of random variables x and y. Show that it is bounded by limits ±1. (05 Marks)

b. The joint PDF $f_{xy}(x, y) = C$, a constant when $0 < x < 3$ and $0 < y < 3$ and is '0' otherwise.

i) What is the value of the constant 'C'?

ii) What are the PDFs for X and Y?

iii) What $F_{xy}(x, y)$ when $0 < x < 3$ and $0 < y < 3$?

iv) What are $F_{xy}(x, \infty)$ and $F_{xy}(\infty, y)$?

v) Are x and y independent? (10 Marks)

c. Prove that $COV(ax, by) = ab\ cov(xy)$. (05 Marks)

OR

4 a. Define central limit theorem and show that the sum of two independent Gaussian random variables is also Gaussian. (06 Marks)

b. For a bivariate random variable CDF is given by $c(x+1)^2(y+1)^2$ for $\begin{Bmatrix} -2 < x < 4, \\ -1 < y < 2 \end{Bmatrix}$ and "0" outside. Find:
   i) The value of 'c'
   ii) Bivariate PDF
   iii) $F_x(x)$ and $F_y(y)$
   iv) Evaluate $P\{(x \le 2) \cap (y \le 1)\}$
   v) Are there variables independent? **(10 Marks)**

c. Explain briefly the following random variables:
   i) Chi-square random variable
   ii) Student-t random variable. **(04 Marks)**

Module-3

5 a. Define random process, with help of examples discuss different types of random processes. **(08 Marks)**

b. Explain strict-sense-stationary and wide-sense-stationary random process. **(04 Marks)**

c. A random process is defined by $x(t) = A \sin(w_c t + \Theta)$ where A, w_c are constants and Θ is a uniformly distributed random variable, distributed between $-\pi$ and π. Check whether x(t) is WSS. If yes list its mean and ACF. **(08 Marks)**

OR

6 a. Define Auto Correlation Function (ACF) of a random process and discuss its properties. **(10 Marks)**

b. The random process x(t) and y(t) are jointly wide-sense stationary and independent. Given that W(t) = x(t) + y(t) and

$$R_x(\tau) = 10 e^{-\frac{|\tau|}{3}}$$

$$R_y(\tau) = 10^{\left(\frac{3-|\tau|}{3}\right)} \quad -3 \le \tau \le 3$$

$= 0$ (otherwise).

For W(t), find i) ACF ii) Total power iii) ac power iv) dc power v) check whether W(t) is W.S.S. **(10 Marks)**

Module-4

7 a. Define vector space and explain four fundamental subspaces with example. **(08 Marks)**

b. Determine the column space and null space of the matrix $B = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix}$. **(06 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ to the Echelon (u) form and find the rank of the matrix. **(06 Marks)**

OR

8 a. What is basis for a vector space? Explain. **(06 Marks)**

b. Given the vectors (1, -3, 2), (2, 1, -3) and (-3, 2, 1). Identify the basis. Verify they are independent or not. **(08 Marks)**

c. Determine orthonormal vectors for $u = \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix}$. (06 Marks)

Module-5

9 a. By applying row operations to produce upper triangular matrix u, compute $|A|$ (det A).

$$A = \begin{bmatrix} 3 & 1 & 4 & 2 \\ 1 & 5 & 2 & 6 \\ 2 & 3 & 7 & 1 \\ 4 & 1 & 2 & 3 \end{bmatrix}.$$ (08 Marks)

b. For the given upper triangular matrix, determine i) $|u|$ ii) $|u^T|$ iii) $|u^{-1}|$.

$$u = \begin{bmatrix} 4 & 4 & 2 & 8 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$ (06 Marks)

c. What is cofactor? Explain with an example. (06 Marks)

OR

10 a. Find x, y and z using CRAMER's rule for the system of equations,

$x + 4y - z = 1$
$x + y + z = 0$
$2x + 3z = 0.$ (06 Marks)

b. Determine the eigen values of matrix $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$. (04 Marks)

c. i) List the properties of Singular Value Decomposition (SVD).
ii) Prove that Identity matrix is positive definite using all required tests. (10 Marks)

* * * * *