CBCS SCHEME

USN

21MAT41

Fourth Semester B.E. Degree Examination, June/July 2023
Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Use of Statistical table is permitted.

Module-1

1 a. Derive Cauchy – Riemann equations in Cartesian form. (06 Marks)

b. Show that f(z) = sin z is analytic and hence find its derivative. (07 Marks)

c. Evaluate $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3y - x)dy$, along the parabola $x = 2t, y = t^2 + 3$ (07 Marks)

OR

2 a. Determine the analytic function $f(z) = u + iv$, whose imaginary part is $(x^2 - y^2) + \frac{x}{x^2 + y^2}$ by Milne – Thompson method. (06 Marks)

b. State and prove Cauchy's integral theorm. (07 Marks)

c. Evaluate $\int_c \frac{dz}{z^2 - 4}$ over $c : |z| = 1$ (07 Marks)

Module-2

3 a. Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ (06 Marks)

b. If α and β are the two roots of $J_n(x) = 0$ then prove that $\int_0^1 x\, J_n(\alpha x) J_n(\beta x) dx = 0$ if $\alpha \neq \beta$. (07 Marks)

c. Express $f(x) = 2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. (07 Marks)

OR

4 a. Obtain the series solution of Bessel's differential equation $x^2y'' + xy' + (x^2 + n^2)y = 0$ leading to $J_n(x)$. (06 Marks)

b. Show that $J_{+1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ (07 Marks)

c. Prove that, $x^3 + 2x^2 - 4x + 5 = \frac{2}{5}P_3(x) + \frac{4}{3}P_2(x) - \frac{17}{5}P_1(x) + \frac{17}{5}P_0(x)$ (07 Marks)

Module-3

5 a. Find the Karl Pearson's coefficient correlation for the following two groups.

A	92	89	87	86	83	77	71	63	53	50
B	86	83	91	77	68	85	52	82	37	57

(06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

b. Fit a straight line of the form $y = ax + b$ for the data by the least squares method.

x	0	1	2	3	4	5
y	9	8	24	28	26	20

(07 Marks)

c. Using the method of least squares fit a curve $y = ax^b$ for the data

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

(07 Marks)

OR

6 a. Ten students got the percentage of marks in two subjects x and y. Compute their rank correlation coefficient.

Marks in x	78	36	98	25	75	82	90	62	65	39
Marks in y	84	51	91	60	68	62	86	58	53	37

(07 Marks)

b. Compute the means $\bar{x}$, $\bar{y}$ and the coefficient of correlation r from the given regression lines $2x + 3y + 1 = 0$, $x + 6y - 4 = 0$. (07 Marks)

c. Fit a second degree parabola $y = ax^2 + bx + c$ in the least square sense for the following data and hence estimate y at $x = 6$.

x	1	2	3	4	5
y	10	12	13	16	19

(06 Marks)

Module-4

7 a. A random variable X has the following probability function :

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find k and evaluate $P(X \geq 6)$, $P(3 < X \leq 6)$. (06 Marks)

b. Find the mean and standard deviation of Poisson distribution. (07 Marks)

c. The probability that a person aged 60 years will live upto 70 is 0.65. What is the probability that out of 10 persons aged 60 atleast 7 of them will live upto 70? (07 Marks)

OR

8 a. Find a constant K such that

$$f(x) = \begin{cases} kx^2, & 0 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$ is a pdf.

Also, compute : (i) $P(1 < x < 2)$ (ii) $P(x \leq 1)$ (iii) $P(x > 1)$ (06 Marks)

b. Find the mean and standard deviation of Binomial distribution. (07 Marks)

c. In a test of electric bulbs it was found that the lifetime of bulbs of a particular brand was normally distributed with an average life of 2000 hours and standard deviation of 60 hours. If a firm purchases 2500 bulbs, find the number of bulbs that are likely to last for

(i) More than 2100 hours

(ii) Less than 1950 hours

(iii) Between 1900 and 2100 hours

Given that, $\phi(1.67) = 0.4525$; $\phi(0.83) = 0.2967$ (07 Marks)

Module-5

9 a. The joint probability distribution of the random variables X and Y are given as follows:

X \ Y	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

Find (i) E(X) (ii) E(Y) (iii) E(XY) (iv) Cov(X, Y)
(v) Marginal distribution of X and Y **(06 Marks)**

b. Define (i) Null hypothesis (ii) Type-I and Type-II error (iii) Level of Significance **(07 Marks)**

c. A sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 40,650 kms with a standard deviation of 3260. Can it be considered as a true random sample from a population with mean life of 40,000 kms (use 0.05 level of significance). (Given $z_{0.05} = 1.96$, $z_{0.01} = 2.58$) **(07 Marks)**

OR

10 a. The joint probability distribution of two random variables X and Y are as follows:

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine : (i) Marginal distribution of X and Y (ii) Find E(X), E(Y) and E(XY)
(iii) Covariance of X and Y **(06 Marks)**

b. In the experiment of pea breeding the following frequencies of seeds were obtained.

Round and Yellow	Wrinkled and Yellow	Rounded Green	Wrinkled and Green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportions 9:3:3:1. Examine the correspondence between theory and experiment. (Given $\chi^2_{0.05} = 7.815$ for 3df). **(07 Marks)**

c. A group of 10 boys fed on a diet A and another group of 8 boys fed on a different diet B for a period of 6 months recorded the following increase in weight (lbs).

Diet A :	5	6	8	1	12	4	3	9	6	10
Diet B :	2	3	6	8	10	1	2	8	5	5

Test whether diets A and B differ significantly regarding their effect on increase in weight. (Given $t_{0.05}$ for 16 df = 2.12) **(07 Marks)**

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