



CBCS SCHEME

21MAT31

Third Semester B.E. Degree Examination, June/July 2023 Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the Laplace transform $2^t + \frac{\cos 2t + \cos 3t}{t}$ (06 Marks)
- b. Find the Laplace transform of the triangular wave of period $2C$ given by $f(t) = \begin{cases} t & 0 < t < c \\ 2c - t & c < t < 2c \end{cases}$ (07 Marks)
- c. Using convolution theorem find the inverse Laplace transform of $\frac{s}{(s^2 + a^2)^2}$ (07 Marks)

OR

- 2 a. Express the function $f(t)$ in terms of unit step function and hence find the Laplace transform of $f(t) = \begin{cases} \sin t & 0 < t < \pi \\ \sin 2t & \pi < t < 2\pi \\ \sin 3t & t \geq 2\pi \end{cases}$ (06 Marks)
- b. Find the inverse Laplace transform $\frac{2s^2 - 6s + 5}{(s-1)(s-2)(s-3)}$ (07 Marks)
- c. Solve the using Laplace transform method $y''(t) + 4y'(t) + 4y = e^{-t}$ $y(0) = 0$ $y'(0) = 0$ (07 Marks)

Module-2

- 3 a. Obtain the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$. Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$ (06 Marks)
- b. Obtain the half range cosine series for the function $f(x) = 2x - 1$ in $0 < x < 1$ (07 Marks)
- c. Obtain the Fourier series of y upto the first harmonic for the following values:

x°	45	90	135	180	225	270	315	360
y	4.0	3.8	2.4	2.0	-1.5	0	2.6	3.4

(07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

- 4 a. Obtain the Fourier series of $f(x) = x \cos x$ in the interval $-\pi \leq x \leq \pi$. (06 Marks)
 b. Obtain the sine half range Fourier series, for the function,

$$f(x) = \begin{cases} \frac{2Kx}{\ell} & \text{in } 0 \leq x \leq \frac{\ell}{2} \\ \frac{2K}{\ell}(\ell - x) & \text{in } \frac{\ell}{2} \leq x \leq \ell \end{cases} \quad (07 \text{ Marks})$$

- c. Obtain the constant term and the first three coefficients in the Fourier cosine series of y in the following data :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(07 Marks)

Module-3

- 5 a. Find the complex Fourier transform of the function,

$$f(x) = \begin{cases} a^2 - x^2 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$$

Hence evaluate $\int_0^{\infty} \left(\frac{\sin s - s \cos s}{s^3} \right) ds = \frac{\pi}{2}$. (06 Marks)

- b. Find the Fourier sine transform of e^{-ax} . (07 Marks)
 c. Find the z-transform of $\cos n\theta$ and $\sin n\theta$. (07 Marks)

OR

- 6 a. Find the Fourier cosine transform of the function, $f(x) = \begin{cases} 4x & 0 < x < 1 \\ 4 - x & 1 < x < 4 \\ 0 & x > 4 \end{cases}$. (06 Marks)

b. Find the inverse z-transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. (07 Marks)

- c. Solve by using z-transform $y_{n+2} - 4y_n = 0$ given that $y_0 = 0$ and $y_1 = 2$. (07 Marks)

Module-4

- 7 a. Classify the following partial differential equation

i) $\frac{\partial^2 u}{\partial x^2} + 4 \frac{\partial^2 u}{\partial x \partial y} + 4 \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0$

ii) $x^2 \frac{\partial^2 u}{\partial x^2} + (1 - y^2) \frac{\partial^2 u}{\partial y^2} = 0$ $-\infty < x < \infty, -1 < y < 1$

iii) $(1 + x^2) \frac{\partial^2 u}{\partial x^2} + (5 + 2x^2) \frac{\partial^2 u}{\partial x \partial t} + (4 + x^2) \frac{\partial^2 u}{\partial t^2} = 0$

iv) $(x + 1) \frac{\partial^2 u}{\partial x^2} - 2(x + 2) \frac{\partial^2 u}{\partial x \partial y} + (x + 3) \frac{\partial^2 u}{\partial y^2} = 0$ (10 Marks)

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- b. Find the values of $u(x, t)$ satisfying the parabolic equation $\frac{\partial^2 u}{\partial x^2} = 2 \frac{\partial u}{\partial t}$ and its boundary conditions $u(0, t) = 0 = u(4, t)$ and $u(x, 0) = x(4 - x)$ by taking $h = 1$ find the value up to $t = 5$. (10 Marks)

OR

- 8 a. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in $0 < x < 5, t \geq 0$ given that $u(x, 0) = 20$ $u(0, t) = 0$ $u(5, t) = 100$ compute U for the time step $h = 1$ by Crank Nicholson method. (10 Marks)
- b. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$ subject to the condition $u(0, t) = 0$ $u(4, t) = 0$ $u_t(x, 0) = 0$ and $u(x, 0) = x(4 - x)$ by taking $h = 1, K = 0.5$ up to four steps. (10 Marks)

Module-5

- 9 a. Given $\frac{d^2 y}{dx^2} - x^2 \frac{dy}{dx} - 2xy = 1, y(0) = 1, y'(0) = 0$ evaluate $y(0.1)$ using Runge-Kutta method of order 4. (06 Marks)
- b. Derive the Euler's equation of the form $\frac{\partial t}{\partial y} - \frac{d}{dx} \left(\frac{\partial t}{\partial y_1} \right) = 0$. (07 Marks)
- c. Find the extremal of the functional $I = \int_0^{\pi/2} (y^2 - y'^2 - 2y \sin x) dx$ under the conditions $y(0) = y(\pi/2) = 0$. (07 Marks)

OR

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- 10 a. Apply Milne's predictor corrector method to solve $\frac{d^2 y}{dx^2} = 1 - 2y \frac{dy}{dx}$ at 0.8 given that $y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762,$
 $y'(0) = 0, y'(0.2) = 0.1996, y'(0.4) = 0.3937, y'(0.6) = 0.5689$. (06 Marks)
- b. Show that the geodesics on a plane are straight line. (07 Marks)
- c. Which curve the functional $\int_0^{\pi/2} (y'^2 - y^2 + 2xy) dx, y(0) = 0, y(\pi/2) = 0$ be extremized. (07 Marks)
