

SX

21MAT21

Second Semester B.E. Degree Examination, June/July 2023 Advanced Calculus and Numerical Methods

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Evaluate $\int_{a}^{c} \int_{b}^{b} \int_{c}^{a} (x+y+z) dx dy dz$

(06 Marks)

b. Evaluate $\int_{0}^{1} \int_{0}^{\sqrt{x}} xy dy dx$ by changing the order of integration.

(07 Marks)

c. Prove that $\pi(\frac{1}{2}) = \sqrt{\pi}$, using definition of Gama function.

(07 Marks)

OR

2 a. Evaluate $\int_{0}^{\infty} \int_{0}^{\infty} e^{-(x^2+y^2)} dxdy$ by changing into polar coordinates.

(06 Marks)

b. Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ by using double integration.

(07 Marks)

c. Show that $\beta(m,n) = \frac{\prod(m)\prod(n)}{\prod(m+n)}$

(07 Marks)

Module-2

- 3 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point (1, -2, -1) along $2\hat{i} \hat{j} 2\hat{k}$. (06 Marks)
 - b. If $\vec{F} = \nabla (xy^3z^2)$, find div \vec{F} and curl \vec{F} at the point (1, -1, 1). (07 Marks)
 - c. If $\vec{F} = (x + y + az)\hat{i} + (bx + 2y z)\hat{j} + (x + cy + 2z)\hat{k}$, find a, b, c such that $\vec{curl F} = 0$.

 (07 Marks)

OR

- 4 a. If $\vec{F} = xy\hat{i} + yz\hat{j} + zx\hat{k}$, evaluate $\int_{C} \vec{F} \cdot d\vec{v}$ where 'c' is the curve represented by x = t, $y = t^{2}$, $z = t^{3}$, $-1 \le t \le 1$. (06 Marks)
 - b. Using Green's theorem, evaluate $\int_C (xy + y^2)dx + x^2dy$, where 'c' is bounded by y = x and $y = x^2$. (07 Marks)
 - c. Apply Stoke's theorem to evaluate $\iint \text{curl } \vec{F} \cdot \hat{n} \, ds$ where $\vec{F} = (x^2 + y^2) \hat{i} 2xy \hat{j}$ taken around the rectangle bounded by the lines $x = \pm a$, y = 0 and y = b. (07 Marks)

Module-3

- 5 a. Form a partial differential equation by eliminating arbitrary function from Z = f(x + at) + g(x at). (06 Marks)
 - b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ when x = 0 and z = 0 when y is an odd

multiple of $\frac{\pi}{2}$.

(07 Marks)

c. Derive one dimensional heat equation.

(07 Marks)

OR

- 6 a. Form a partial differential equation by eliminating arbitrary constant from $Z = (x a)^2 + (y b)^2$. (06 Marks)
 - b. Solve (y-z)p + (z-x)q = x y.

(07 Marks)

c. Solve $\frac{\partial^2 z}{\partial y^2} = z$ given that when y = 0, $z = e^x$ and $\frac{\partial z}{\partial y} = e^x$.

(07 Marks)

Module-4

7 a. The area of a circle (A) corresponding to diameter (D) is given below:

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 105 using an appropriate interpolation formula.

(06 Marks)

(07 Marks)

- b. Find a real root of $x^3 2x 5 = 0$ using Regula-Falsi method correct to 3 decimal places whose root lies between 2 and 2.5. (07 Marks)
- c. Evaluate $\int_{0}^{\pi/2} \sqrt{\cos\theta} \, d\theta$ by taking 7 ordinates by Simpson's $1/3^{rd}$ rule.

OF

8 a. Use Newton's divided difference formula to find f(4) given the data:

X	0	2	3	6
f(x)	-4	2	14	158

(06 Marks)

- b. Use Newton-Raphson method to find a real root of $x \sin x + \cos x = 0$ near $x = \Pi$. Carry out the iterations upto 4 decimal places. (07 Marks)
- c. Use Lagrange's interpolation formula to find y when x = 35 to the following data:

X	25	30	40	60	
f(x)	50	55	70	95	

(07 Marks)

Module-5

- 9 a. Use the Taylor series method to find y(0.2) from $\frac{dy}{dx} = y + \sin x$, y(0) = 1. (06 Marks)
 - b. Use Runge-Kutta method of order 4, find y at x = 0.1, given that $\frac{dy}{dx} = 3e^x + 2y$, y(0) = 0 with h = 0.1.
 - with n = 0.1.

 c. Apply Milne's predictor-corrector method, to find y(1.4) from $\frac{dy}{dx} = x^2 + \frac{y}{2}$ given that y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514. (07 Marks)

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OR RANGALORE - 560 037 10 a. Use modified Euler's method to solve $\frac{dy}{dx} = x^2 + y$ with y(0) = 1, h = 0.05 at x = 0.1.

(06 Marks)

- b. Use Taylor series method to find y(0.1) from $\frac{dy}{dx} = x^2 + y^2$ with y(0) = 1. (07 Marks)
- c. Use Runge-Kutta method of 4th order, find y(0.1) given that $\frac{dy}{dx} = 3x + \frac{y}{2}$, y(0) = 1 with h = 0.1.