

18MATDIP41

Fourth Semester B.E. Degree Examination, June/July 2023 Additional Mathematics – II

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

a. Find the rank of the matrix by applying elementary row operations:

$$A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 8 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$$
 (06 Marks)

b. Test for consistency and solve the system

$$x + y + z = 6$$

 $x - y + 2z = 5$
 $3x + y + z = 8$.

(07 Marks)

c. Find the eigen value and the corresponding eigen vectors of the matrix:

$$\mathbf{A} = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

(07 Marks)

OR

2 a. Reduce the matrix A to the echelon firm, where

$$A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}.$$

(06 Marks)

b. Find the values of λ and μ such that the system

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

may have

- i) unique solution
- ii) infinite solution
- iii) no solution.

(07 Marks)

c. Solve:

$$2x + y + 4z = 12$$

$$4x + 11y - z = 33$$

 $8x - 3y + 2z = 20$

(07 Marks)

(06 Marks)

Module-2

The area of a circle (A) corresponding to diameter (D) is given in the following table:

Ι)	80	85	90	95	100
A	1	5026	5674	6362	7088	7854

Find the area when D = 105 using an appropriate interpolation formula. (06 Marks)

- b. Find the real root of the equation $\cos x = 3x 1$ correct to three decimal places using Regula – Falsi method.
- c. Evaluate $\int_{0}^{1} \frac{x dx}{1+x^2}$ using Weddle's rule. Take seven ordinates. (07 Marks)

- Find $u_{0.5}$ from the data $u_0 = 225$, $u_1 = 238$, $u_2 = 320$, $u_3 = 340$ by using an appropriate interpolation formula.
 - Use Newton Raphson method to find a real root of the equation $x^3 + 5x 11 = 0$ correct to (07 Marks) the three decimal places.
 - Using Simpson's $1/3^{rd}$ rule, evaluate $\int_0^1 \frac{dx}{1+x^2}$ by dividing the interval [0, 1] into six equal parts. Hence deduce the value of \log_e^2 (07 Marks)

5 a. Solve
$$(D^3 - 6D^2 + 11D - 6) y = 0$$
.

b. Solve
$$(D^2 - 4)y = \cos h (2x - 1) + 3^x$$
. (07 Marks)

c. Solve
$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4\cos^2 x$$
. (07 Marks)

6 a. Solve
$$\frac{d^3y}{dx^3} + y = 0$$
. (06 Marks)

b. Solve
$$y'' + 9y = \cos 2x \cdot \cos x$$
 (07 Marks)

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$$y'' + 9y = \cos 2x \cdot \cos x$$
 (07 Marks)
c. Solve $y'' - (a + b)y' + aby = e^{ax} + e^{bx}$. (07 Marks)

- Form a partial differential equation by eliminating the arbitrary constants in $ax^2 + by^2 + z^2 = 1$ (06 Marks)
 - b. Form the partial differential equation by eliminating the arbitrary function from $\ell_x + my + nz = \phi(x^2 + y^2 + z^2).$ (07 Marks)
 - c. Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$, given that when x = 0, z = 0 and $\frac{\partial z}{\partial x} = a \sin y$. (07 Marks) 2 of 3

OR

Form a partial differential equation by eliminating the arbitrary constructs from:

$$z = xy + y\sqrt{x^2 - a^2} + b.$$

(06 Marks)

b. Solve $\frac{\partial^2 z}{\partial y^2} = x + y$ by direct integration.

(07 Marks)

c. Solve
$$\frac{\partial^2 z}{\partial y^2} = z$$
, given that $z = 0$, $\frac{\partial z}{\partial y} = \sin x$, when $y = 0$.

(07 Marks)

Module-5

Define:

i) Sample space

ii) Mutually exclusive events

(06 Marks)

iii) Mutually independent events. b. A box contains 4 black, 5 white and 6 red balls. If 2 balls are drown at random, what is the probability that:

i) both are red

ii) one black and one white.

(07 Marks)

State and prove Baye's theorem.

(07 Marks)

If A and B are events with $P(A \cup B) = \frac{7}{8}$, $P(A \cap B) = \frac{1}{4}$ and $P(A \cap \overline{B}) = \frac{1}{3}$. 10

Find:

- i) P(A)
- ii) P(B)
- iii) $P(A \cap B)$.

(06 Marks)

- b. A problem is given to four students A, B, C, D whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ respectively. Find the probability that the problem is solved. (07 Marks)
- c. Three machines A, B and C produce 50%, 30% and 20% of the items in a factory. The percentage of defective outputs of these machines are 3, 4, and 5 respectively. If an item is selected at random, what is the probability that it is defective? If a selected item is defective, what is the probability that it is from machine A? (07 Marks)