CBCS SCHEME

USN

18MATDIP31

Third Semester B.E. Degree Examination, June/July 2023
Additional Mathematics - I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Express the complex number $\frac{(3+i)(1-3i)}{2+i}$ in the form x + iy. Also find its magnitude. **(06 Marks)**

b. Find the cube roots of ℓ - i and represent them in an argand plane. **(07 Marks)**

c. If $\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{b} = 8\hat{i} - 4\hat{j} + \hat{k}$ then show that $\vec{a}$ is perpendicular to $\vec{b}$, also find $|\vec{a} \times \vec{b}|$. **(07 Marks)**

OR

2 a. Find the modulus and amplitude of $1 - \cos\alpha + i\sin\alpha$. **(06 Marks)**

b. If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$; $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} - \hat{k}$, find

i) $\vec{a} \cdot (\vec{b} \times \vec{c})$ ii) $\vec{b} \times (\vec{a} \times \vec{c})$. **(07 Marks)**

c. Prove that $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$. **(07 Marks)**

Module-2

3 a. Using Maclaurin's series, prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - - - -$. **(06 Marks)**

b. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. **(07 Marks)**

c. If $u = 1 - x$, $v = x(1-y)$, $w = xy(1-z)$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. **(07 Marks)**

OR

4 a. Obtain the Maclaurin's expansion of the function $\log(1 + e^x)$. **(06 Marks)**

b. If $u = f(x-y, y-z, z-x)$, Prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(07 Marks)**

c. If $u = x + y + z$, $w = y + z$, $z = uvw$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. **(07 Marks)**

Module-3

5 a. A particle moves along a curve C with parametric equations $x = t - \frac{t^3}{3}$, $y = t^2$ and $z = t + \frac{t^3}{3}$, where t is the time. Find the velocity and acceleration and any time t and also find their magnitudes at t = 3. **(06 Marks)**

b. Find div $\vec{F}$ and Curl $\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

c. Find the directional derivative of $\phi = x^2yz^3$ at (1, 1, 1) in the direction of $\hat{i} + \hat{j} + 2\hat{k}$. **(07 Marks)**

OR

6 a. Show that the vector field $\vec{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$ is solenoidal vector field. **(06 Marks)**

b. If $\vec{F} = (x+y+1)\ \hat{i} + \hat{j} - (x+y)\ \hat{k}$, show that $\vec{F}\ .\ \text{curl}\ \vec{F} = 0$. **(07 Marks)**

c. Find the constants a , b , c such that $\vec{F} = (x+y+az)\ \hat{i} + (x+cy+2z)\ \hat{k} + (bx+2y-z)\ \hat{j}$ is irrotational. **(07 Marks)**

Module-4

7 a. Obtain the Reduction formula for $\int_0^{\pi/2} \cos^n x\, d\,x$. **(06 Marks)**

b. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)\ dy\,dx$. **(07 Marks)**

c. Evaluate $\int_0^1 \int_0^1 \int_0^1 (x+y+z)\, dx\, dy\, dz$. **(07 Marks)**

OR

8 a. Evaluate $\int_1^2 \int_0^{3-y} xy\ dx\ \ dy$. **(06 Marks)**

b. Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\, dx\, dy\, dz$. **(07 Marks)**

c. Obtain the Reduction formula $\int \sin^m x \cos^n x\, dx$. **(07 Marks)**

Module-5

9 a. Solve : $(x^2 + y)\, dx + (y^3 + x)\, dy = 0$. **(06 Marks)**

b. Solve : $x \log x \dfrac{dy}{dx} + y = 2 \log x$. **(07 Marks)**

c. Solve : $\dfrac{dy}{dx} + \dfrac{y}{x} = y^2 x$. **(07 Marks)**

OR

10 a. Solve : $y\, e^y\, dx = (y^3 + 2x\, e^y)\, dy$. **(06 Marks)**

b. Solve : $(x^2 - y^2)\, dx = 2xy\, dy$. **(07 Marks)**

c. Solve : $[1 + (x+y) \tan y] \dfrac{dy}{dx} + 1 = 0$. **(07 Marks)**

* * * * *