

CBCS SCHEME



18MAT21

Second Semester B.E. Degree Examination, June/July 2023 Advanced Calculus and Numerical Methods

Max. Marks: 100

Time: 3 hrs.

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, 1)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. (06 Marks)
- b. If $\vec{A} = x^2y\hat{i} + y^2z\hat{j} + z^2x\hat{k}$, find
 - i) $\text{curl}(\text{curl}\vec{A})$ (07 Marks)
 - ii) $\text{div}(\text{curl}\vec{A})$ (07 Marks)
- c. Show that $\vec{F} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational. (07 Marks)

OR

- 2 a. Find the workdone in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the straight line from $(0, 0, 0)$ to $(2, 1, 3)$. (06 Marks)
- b. Using Green's theorem, evaluate $\oint_C (x^2 + xy)dx + (x^2 + y^2)dy$, where C is the square formed by the lines $x = \pm 1, y = \pm 1$. (07 Marks)
- c. Using Gauss divergence theorem, evaluate $\oint_S \vec{F} \cdot \hat{n} ds$, where $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$ over the region $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$ (07 Marks)

Module-2

- 3 a. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x}$ (06 Marks)
- b. Solve the variation of parameters methods $\frac{d^2y}{dx^2} + y = \sec x$ (07 Marks)
- c. A body weighing 4.9kg is hung from a spring. A pull of 10kg will stretch the spring to 5 cm. The body is pulled down to 6cm below the static equilibrium position and then released. Find the displacement of the body from its equilibrium position at time t seconds. (07 Marks)

OR

- 4 a. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \sin 2x$. (06 Marks)
- b. Solve $(D^2 + 3D + 2)y = 1 + 3x + x^2$ (07 Marks)
- c. Solve $x^2y'' - 3xy' + 4y = (1 + x)^2$ (07 Marks)

Module-3

- 5 a. Form a partial differential equation from the relation $xyz = f(x + y + z)$ (06 Marks)
- b. Solve the Lagrange's partial differential equation $x(y^2 - z^2)p + y(z^2 - x^2)q - z(x^2 - y^2)r = 0$ (07 Marks)
- c. With suitable assumptions derive one dimensional wave equation. (07 Marks)

OR

- 6 a. Using the method of direct integration solve

$$\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0$$

(06 Marks)

- b. Solve
- $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial z}{\partial x} - 4z = 0$
- , given that when
- $x = 0$
- ,
- $z = 1$
- and
- $\frac{\partial z}{\partial x} = y$
- .

(07 Marks)

- c. Find all possible solutions of the one dimensional heat equation

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$$

(07 Marks)

Module-4

- 7 a. Test for convergence the series

$$1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \dots \text{to } \infty$$

(06 Marks)

- b. If
- α
- and
- β
- are the roots of Bessel equation
- $J_n(x) = 0$
- , prove that

$$\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0, \text{ for } \alpha \neq \beta.$$

(07 Marks)

- c. Express
- $f(x) = 3x^3 - x^2 + 5x - 2$
- in terms of Legendre polynomial.

(07 Marks)

OR

- 8 a. Test for convergence the series

$$\left[\frac{2^2}{1^2} - \frac{2}{1} \right]^{-1} + \left[\frac{3^3}{2^3} - \frac{3}{2} \right]^{-2} + \left[\frac{4^4}{3^4} - \frac{4}{3} \right]^{-3} + \dots \infty$$

(06 Marks)

- b. Show that
- $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$

(07 Marks)

- c. Show that
- $P_4(\cos \theta) = \frac{1}{64} [35 \cos 4\theta + 20 \cos 2\theta + 9]$

(07 Marks)

Module-5

- 9 a. Using Newton's forward interpolation find
- y
- when
- $x = 38$
- from the following data:

x	40	50	60	70	80	90
y	184	204	226	250	276	304

(06 Marks)

- b. Using Newton - Raphson method find the root of the equation
- $x \sin x + \cos x = 0$
- near
- $x = \pi$
- correct to four decimal places.

(07 Marks)

- c. Using Simpson's
- $\frac{3^{\text{th}}}{8}$
- rule, evaluate
- $\int_0^{0.3} \sqrt{1-8x^3} dx$
- by taking seven ordinates.

(07 Marks)

OR

- 10 a. Obtain Newton's divided difference interpolation polynomial and hence find
- $f(2)$
- from

x	3	7	9	10
f(x)	168	120	72	63

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(06 Marks)

- b. Find a real root of
- $x \log_{10} x = 1.2$
- by Regula - Falsi method in three iterations, given that root lies in the interval (2, 3).

(07 Marks)

- c. Evaluate
- $\int_0^1 \frac{x}{1+x^2} dx$
- taking six equal sub-intervals by using Weddle's rule.

(07 Marks)
