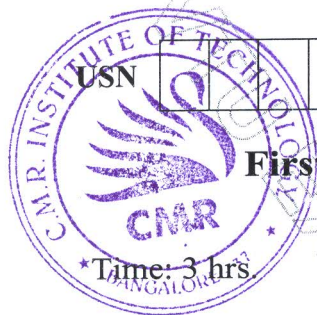


CBCS SCHEME



First Semester B.E. Degree Examination, June/July 2023 Calculus and Linear Algebra

18MAT11

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. With usual notations, prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\theta} \right)^2$. (06 Marks)
- b. Find the radius of curvature of the curve $y^2 = \frac{a^2(a-x)}{x}$ at the point (a, 0). (06 Marks)
- c. For the curve $\theta = \frac{1}{a} \sqrt{r^2 - a^2} - \cos^{-1} \left(\frac{a}{r} \right)$, prove that $p^2 = r^2 - a^2$. (08 Marks)

OR

- 2 a. Find the angle between the curves $r = a(1 - \cos \theta)$ and $r = 2a \cos \theta$. (06 Marks)
- b. Find the radius of curvature of the curve $r = a \sin n\theta$ at the pole (0, 0). (06 Marks)
- c. Find evolutes curve $y^2 = 4ax$ as $27ay^2 = 4(x+a)^3$. (08 Marks)

Module-2

- 3 a. Obtain Maclaurin's expansion of $e^{\tan^{-1} x}$ upto the term containing x^4 . (06 Marks)
- b. Evaluate $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{\frac{1}{x}}$. (06 Marks)
- c. Find the extreme values of $f(x, y) = x^3 y^2 (1 - x - y)$. (08 Marks)

OR

- 4 a. If $U = f(x-y, y-z, z-x)$, prove that $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = 0$. (06 Marks)
- b. If $u = x + 3y^2 - z^2$, $v = x^2 yz$, $w = 2z^2 - xy$, find the Jacobian $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at (1, -1, 0). (06 Marks)
- c. Find the stationary values of $x^2 + y^2 + z^2$ subject to the condition $xy + yz + zx = 3a^2$. (08 Marks)

Module-3

- 5 a. Evaluate $\int_0^1 \int_{y^2}^{1-y} \int_0^{1-x} x \, dz \, dx \, dy$. (07 Marks)
- b. Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, above x-axis. (07 Marks)
- c. With usual notations, prove that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$. (06 Marks)

OR

- 6 a. Evaluate $\int_0^a \int_y^a \frac{x}{x^2 + y^2} dx dy$ by changing the order of integration. (07 Marks)
- b. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing into polar coordinates. (07 Marks)
- c. Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin \theta}} = \pi$. (06 Marks)

Module-4

- 7 a. Solve $p(p+y) = x(x+y)$. (07 Marks)
- b. Find the orthogonal trajectories to the family of curve, $y^2 = 4ax$. (07 Marks)
- c. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$. (06 Marks)

OR

- 8 a. Solve $(x^2 + y^2 + x)dx + xydy = 0$. (07 Marks)
- b. A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C . What will be the temperature of the body after 40 minutes from the original? (07 Marks)
- c. Find the general solution and singular solution of the equation $\sin px \cos y = \cos px \sin y + p$. (06 Marks)

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- 9 a. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ by reducing to row-reduced echelon form. (06 Marks)
- b. Apply Gauss-elimination method to solve the $x+4y-z=-5$, $x+y-6z=-12$, $3x-y-z=4$. (07 Marks)
- c. Find numerically largest eigen value and corresponding eigen vector of $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by Rayleigh's power method. Take initial eigen vector $[1,0,0]^T$. Carry out five iterations. (07 Marks)

OR

- 10 a. Test for consistency and solve the system of equations, $x+y+z=6$, $x-y+2z=5$, $3x+y+z=8$. (06 Marks)
- b. Solve the system of equations by Gauss-Seidel method $x+y+54z=110$, $27x+6y-z=85$, $6x+15y+2z=72$. Carryout three iterations. (07 Marks)
- c. Diagonalize the matrix $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. (07 Marks)
