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Third Semester B.E. Degree Examination, June/July 2023
Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Prove that $\left[\frac{\cos \theta + i \sin \theta}{\sin \theta + i \cos \theta} \right]^4 = \cos 8\theta + i \sin 8\theta$. (06 Marks)
- b. Express $\frac{1}{(2+i)^2} - \frac{1}{(2-i)^2}$ in the form of $a + ib$. (06 Marks)
- c. If $\vec{a} = 4i + 3j + k$, $\vec{b} = 2i - j + 2k$. Find a unit vector \vec{N} perpendicular to vectors \vec{a} and \vec{b} such that \vec{a} , \vec{b} , \vec{N} form a right handed system. Also find the angle between the vectors \vec{a} and \vec{b} . (08 Marks)

OR

- 2 a. Show that $(1 + \cos \theta + i \sin \theta)^n + (1 + \cos \theta - i \sin \theta)^n = 2^{n+1} \cos^n \left(\frac{\theta}{2} \right) \cos \left(\frac{n\theta}{2} \right)$. (06 Marks)
- b. Find the value of λ so that the vectors $\vec{a} = i - 2j + 3k$, $\vec{b} = 7i + j + k$ and $\vec{c} = 3i + 4j - k$ are coplanar. (06 Marks)
- c. Define scalar and vector products of two vectors. If $\vec{a} = 2i - 3j - k$ and $\vec{b} = i + 4j - 2k$. Find $\vec{a} \cdot \vec{b}$ and $\vec{a} \times \vec{b}$. (08 Marks)

Module-2

- 3 a. If $y = \tan^{-1} x$, prove that $(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. (06 Marks)
- b. If $u = x^2y + y^2z + z^2x$, prove that $u_x + u_y + u_z = (x + y + z)^2$. (06 Marks)
- c. Show that the pair of curves $r = a(1 + \cos \theta)$, $r = b(1 - \cos \theta)$ intersect orthogonally. (08 Marks)

OR

- 4 a. Find the pedal equation to the curve $r^m = a^m \cos m\theta$. (06 Marks)
- b. If $u = e^{\left(\frac{x^2 y^2}{x+y} \right)}$, prove that $xu_x + yu_y = 3u \log u$. (06 Marks)
- c. Obtain the MaClaurin's series expansion of $y = \sin x + \cos x$ upto the term containing x^4 . (08 Marks)

Module-3

- 5 a. Obtain a reduction formula for $\int \sin^n x dx$, $n > 0$. (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

b. Evaluate $\int_0^a \frac{x^4}{\sqrt{a^2 - x^2}} dx$. (06 Marks)

c. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xy \, dz \, dy \, dx$. (08 Marks)

OR

6 a. Evaluate $\int_0^1 \frac{x^9}{\sqrt{1-x^2}} dx$. (06 Marks)

b. Evaluate $\int_0^1 \int_1^{x^2} (x^3 + y^2x) dx \, dy$. (06 Marks)

c. Evaluate $\int_{-1}^1 \int_0^{x+z} \int_{x-z}^x (x+y+z) dy \, dx \, dz$. (08 Marks)

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7 a. If $\vec{F} = (x+y+1)\mathbf{i} + \mathbf{j} - (x+y)\mathbf{k}$, show that $\vec{F} \cdot \text{curl} \vec{F} = 0$. (06 Marks)

b. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$. (06 Marks)

c. A particle moves along a curve $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time variable. Determine its velocity and acceleration and also magnitudes of velocity and acceleration at $t = 0$. (08 Marks)

OR

8 a. Find the values of the constants a, b, c such that $\vec{F} = (x+y+az)\mathbf{i} + (bx+2y-z)\mathbf{j} + (x+cy+2z)\mathbf{k}$ is irrotational. (06 Marks)

b. Show that the vector field $\vec{F} = \frac{x\mathbf{i} + y\mathbf{j}}{x^2 + y^2}$ is solenoidal. (06 Marks)

c. Find $\text{div} \vec{F}$ and $\text{curl} \vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. (08 Marks)

Module-5

9 a. Solve $(x^2 + y)dx + (y^2 + x)dy = 0$. (06 Marks)

b. Solve $\frac{dy}{dx} + y \tan x = \sec x$. (06 Marks)

c. Solve $x^2ydx - (x^3 + y^3)dy = 0$. (08 Marks)

OR

10 a. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2x$. (06 Marks)

b. Solve $\frac{dy}{dx} = \frac{y}{x} + \cos^2\left(\frac{y}{x}\right)$. (06 Marks)

c. Solve $3x(x+y^2)dy + (x^3 - 3xy - 2y^3)dx = 0$. (08 Marks)
