CBCS SCHEME

USN | | | | | | | | | | **17MAT41**

Fourth Semester B.E. Degree Examination, June/July 2023
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find the value of y at x = 0.1 and 0.2 from $\frac{dy}{dx} = x^2y - 1$, y(0) = 1 upto third degree term by using Taylor's series method. **(06 Marks)**

b. Using the modified Euler's method, solve the initial value problem $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1 at the point x = 0.1. Take h = 0.1 and carryout two iterations. **(07 Marks)**

c. Solve the differential equation $\frac{dy}{dx} = x - y^2$ at x = 0.8 by using Adam – Bashforth method, given that y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795 and y(0.6) = 0.1762. Apply corrector twice. **(07 Marks)**

OR

2 a. Find the approximate solution of $\frac{dy}{dx} = 2y + 3e^x$, y(0) = 0 at the points x = 0.1 and x = 0.2 by using Taylor's series method. **(06 Marks)**

b. Using Runge – Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) = 1 at x = 0.2 by taking h = 0.2. **(07 Marks)**

c. If $y' = 2e^x - y$, y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040 and y(0.3) = 2.090, find y(0.4) using Milne's predictor – corrector formula. Apply corrector formula twice. **(07 Marks)**

Module-2

3 a. Obtain the solution of the equation : $2\frac{d^2y}{dx^2} = 4x + \frac{dy}{dx}$ by computing the values of the dependent variable corresponding to the value x = 1.4 of the independent variable by applying Milne's method using the following data :

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y′	2	2.3178	2.6725	3.0657

(07 Marks)

b. If $x^3 + 2x^2 - 4x + 5 = aP_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$, find a, b, c, d. **(07 Marks)**

c. If α and β are two distinct roots of $J_n(x) = 0$, then prove that $\int_0^1 x\, J_n(\alpha x) J_n(\beta x)\,dx = 0$ if $\alpha \neq \beta$. **(06 Marks)**

OR

4 a. Using the Runge – Kutta method, find y(0.2) and y′(0.2), given that y satisfies the differential equation $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ and the initial conditions y(0) = 1, y′(0) = 0, h = 0.2. **(07 Marks)**

b. Prove the Rodrigues' formula : $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. **(07 Marks)**

c. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. **(06 Marks)**

Module-3

5 a. Derive Cauchy – Riemann equations in polar form. **(07 Marks)**

b. By using Cauchy's Residue theorem, evaluate the integral $\int_C \frac{z^2}{(z-1)^2(z+2)}dz$ where C is the circle $|z| = 5/2$. **(07 Marks)**

c. Find the bilinear transformation which maps z = –1, i, 1 into w = 1, i, –1, respectively. **(06 Marks)**

OR

6 a. Find the analytic function f(z) = u + i v in terms of z whose imaginary part is $e^x[(x^2-y^2)\cos y - 2xy\sin y]$. **(07 Marks)**

b. State and prove Cauchy's integral formula. **(07 Marks)**

c. Discuss the transformation $w = z^2$. **(06 Marks)**

Module-4

7 a. Derive the expressions for mean and variance of binomial distribution. **(07 Marks)**

b. The mean weight of 500 students at a certain school is 50kgs and the standard deviation is 6kgs. Assuming that the weights are normally distributed, find the expected number of students weighing :
i) between 40 and 50kgs
ii) more than 60kgs, given that A(1.6667) = 0.4525. **(07 Marks)**

c. Alpha particles are emitted by a radioactive source at an average rate of 5 in a 20 minutes interval. Using Poisson distribution, find the probability that there will be :
i) Exactly two emissions
ii) At least two emissions, in a randomly chosen 20 minutes interval. **(06 Marks)**

OR

8 a. The probability density function P(x) of a variate X is given by the following table :

x	−2	−1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	K

Determine the value of K and find the mean, variance and standard deviation. Also find $P(-1 < x \le 2)$. **(07 Marks)**

b. In a certain town the duration of a shower is exponentially distributed with mean equal to 5 minutes. What is the probability that a shower will last for :
i) Less than 10 minutes
ii) 10 minutes or more? **(07 Marks)**

c. The joint probability distribution of two random variables X and Y is given. Find the marginal distribution of X and Y and evaluate cov(x, y) and ρ(x, y).

X \ Y	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

(06 Marks)

Module-5

9 a. Results extracts revealed that in a certain school, over a period of 5 years, 725 students had passed and 615 students had failed. Test whether success and failure are in equal proportion. **(06 Marks)**

b. Two types of batteries are tested for their length of life and the following results are obtained

Battery	n_1	$\overline{x_1}$	σ^2
A	10	560 hrs	100
B	10	500 hrs	121

Test whether there is a significant difference in two means. (Given $t_{0.05} = 2.101$ for 18 df). **(07 Marks)**

c. Find the fixed probability vector of the regular stochastic matrix :

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}.$$

(07 Marks)

OR

10 a. Define :

i) Null hypothesis

ii) Significance level

iii) Type I and Type II errors. **(06 Marks)**

b. The number of accidents per day (x) over a period of 400 days is given below. Test Poisson distribution is a good fit or not. $\left(\chi^2_{0.05} = 9.49 \text{ for } 4\text{d.f}\right)$.

x	0	1	2	3	4	5
f	173	168	37	18	3	1

(07 Marks)

c. A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night. In the long run, how often does he study? **(07 Marks)**

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