CBCS SCHEME

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15MATDIP41

Fourth Semester B.E. Degree Examination, June/July 2023
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the rank of the matrix $\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by applying elementary row transformations. **(06 Marks)**

b. Solve by Gauss-elimination method:
$x - 2y + 3z = 2, \quad 3x - y + 4z = 4, \quad 2x + y - 2z = 5$ **(05 Marks)**

c. Find the inverse of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ by Caylay-Hamilton theorem. **(05 Marks)**

OR

2 a. Find the rank of the matrix $\begin{bmatrix} 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \\ 0 & 1 & 2 & -2 \end{bmatrix}$ by reducing to echelon form. **(06 Marks)**

b. Find all the eigen values and the eigen vector corresponding to smallest eigen value of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. **(05 Marks)**

c. Solve by Gauss-elimination method:
$2x + y + 4z = 12, \quad 4x + 11y - z = 33, \quad 8x - 3y + 2z = 20$ **(05 Marks)**

Module-2

3 a. Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$. **(05 Marks)**

b. Solve $y'' - 6y' + 9y = 5e^{-2x}$. **(05 Marks)**

c. Solve $y'' - 2y' - 3y = e^{2x}$ by the method of undetermined coefficients. **(06 Marks)**

OR

4 a. Solve $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$. Given $y(0) = 0$, $\frac{dy}{dx}(0) = 15$. **(05 Marks)**

b. Solve $y'' + 4y' - 12y = 3\sin 2x$ **(05 Marks)**

c. Solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by the method of variation of parameters. **(06 Marks)**

Module-3

5 a. Find the Laplace transforms of (i) $1+2t^3-4e^{3t}+5e^{-t}$ (ii) $\cos^2 2t + 2t$ **(05 Marks)**

b. Find Laplace transforms of (i) $e^{-t}\sin 4t + t\cos 2t$ (ii) $\frac{e^{-2t}-e^{-3t}}{t}$ **(05 Marks)**

c. For the periodic function f(t) of period 4 defined by $f(t)=\begin{cases}3t, & 0<t<2\\ 6, & 2<t<4\end{cases}$, find $L\{f(t)\}$. **(06 Marks)**

OR

6 a. Evaluate : (i) $L\{\sin 2t \sin 3t\}$ (ii) $L\{e^t(t+1)^2\}$ **(05 Marks)**

b. Evaluate $L\left\{e^{-4t}\int_0^t \frac{\sin 3t}{t}\,dt\right\}$. **(05 Marks)**

c. Express f(t) interms of unit step function, hence find $L\{f(t)\}$, where $f(t)=\begin{cases}\cos t; & 0<t<\pi\\ \sin t; & t>\pi\end{cases}$. **(06 Marks)**

Module-4

7 a. Evaluate: (i) $L^{-1}\left\{\frac{3(s^2-1)^2}{2s^5}\right\}$ (ii) $L^{-1}\left\{\frac{3s-4}{s^2-16}\right\}$ **(05 Marks)**

b. Find inverse Laplace transform of $\frac{s+5}{s^2-6s+13}$. **(05 Marks)**

c. Solve $\frac{dy}{dt}+y=\sin t$, $y(0)=0$ by using Laplace transforms. **(06 Marks)**

OR

8 a. Find (i) $L^{-1}\left\{\tan^{-1}\left(\frac{a}{s}\right)\right\}$ (ii) $L^{-1}\left\{\log\frac{(s+a)}{(s+b)}\right\}$. **(08 Marks)**

b. Solve the simultaneous differential equations $\frac{dx}{dt}+y=\sin t$, $\frac{dy}{dt}+x=\cos t$, x = 2 and y = 0 for t = 0 by using Laplace transforms. **(08 Marks)**

Module-5

9 a. If A, B, C are any three events, prove that
$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$ **(06 Marks)**

b. There are 10 students of which three are graduates. If a committee of five is to be formed, what is the probability that atleast 2 graduates are there in a committee. **(05 Marks)**

c. Three machines A, B and C produce respectively 60%, 30%, 10% of the total number of items of a factory. The percentages of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine C. **(05 Marks)**

OR

10 a. State and prove Baye's theorem. **(06 Marks)**

b. A problem in mathematics is given to three students A, B and C, whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved? **(05 Marks)**

c. If A and B are events with $P(A)=\frac{1}{2}$, $P(A\cup B)=\frac{3}{4}$ and $P(\overline{B})=\frac{5}{8}$, find $P(A\cap B)$ and $P(\overline{A}\cap\overline{B})$. **(05 Marks)**