USN

15MATDIP31

Third Semester B.E. Degree Examination, June/July 2023
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Express the complex number $\frac{1}{(2+i)^2}$ in the form a + ib. **(05 Marks)**

b. Simplify $\frac{(\cos\theta - i\sin\theta)^2(\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9(\cos\theta + i\sin\theta)^5}$. **(05 Marks)**

c. Find the value of λ such that the vectors (1, -2, 3), (-2, 3, -4) and (1, -3, λ) are coplanar. **(06 Marks)**

OR

2 a. Find the modulus and amplitude of the complex number $1 + i\sqrt{3}$. **(05 Marks)**

b. If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} - \hat{k}$ find

i) $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$

ii) $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$. **(05 Marks)**

c. If $\vec{a} = (1, -1, 2)$, $\vec{b} = (1, 2, 3)$ and $\vec{c} = (3, -2, 4)$. Evaluate $\vec{a} \times (\vec{b} \times \vec{c})$ and $(\vec{a} \times \vec{b}) \times \vec{c}$. Are they equal? **(06 Marks)**

Module-2

3 a. If y = A cos (logx) + Bsin (logx), show that $x^2 y_{n+2} + (2n+1)xy_{x+1} + (x^2+1)y = 0$. **(05 Marks)**

b. Expand log(1 + x) in a Macluarin's series upto the term involving x^4. **(05 Marks)**

c. If $u = \sin^{-1}\left[\frac{x^4 + y^4}{x + y}\right]$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$. **(06 Marks)**

OR

4 a. Write the n^{th} derivatives of

i) $\frac{1}{2x+3}$ ii) log (3x + 2) iii) $e^{2x}\cos 3x$. **(05 Marks)**

b. Find the pedal equation of $r^n = a^n \cos n\theta$. **(05 Marks)**

c. If $x = r\cos\theta$, $y = r\sin\theta$, $z = z$ then find $\frac{\partial(x,y,z)}{\partial(r,\theta,z)}$. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-3

5 a. Evaluate

i) $\int_{0}^{\pi/2} \sin^6 x\, dx$

ii) $\int_{0}^{\pi/2} \sin^4 x \cos^5 x dx$. (05 Marks)

b. Evaluate $\int_{0}^{2a}\int_{0}^{x^2/4a} xy\, dy\, dx$. (05 Marks)

c. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z} (x+y+z) dydxdz$. (06 Marks)

OR

6 a. Evaluate $\int_{0}^{\pi/6} \cos^4 3\theta \sin^3 6\theta d\theta$. (05 Marks)

b. Evaluate $\int_{1}^{2}\int_{1}^{3} xy^2\, dx\, dy$. (05 Marks)

c. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a} (x^2+y^2+z^2)\, dx\, dy\, dz$. (06 Marks)

Module-4

7 a. A particle moves along a curve $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time variable. Determine the magnitudes of velocity and acceleration at $t = 0$. (05 Marks)

b. If $\phi = \log(x^2+y^2+z^2)$, find the magnitude of the grad ϕ at (1, 2, 3). (05 Marks)

c. If $\vec{R} = x\hat{i} + y\hat{i} + z\hat{k}$ find $\nabla \cdot \vec{R}$ and $\nabla \times \vec{R}$. (06 Marks)

OR

8 a. Find a unit vector normal to the surface $xy^3z^2 = 4$ at (-1, -1, 2). (05 Marks)

b. Find the value of 'a' if the vector $(ax^2y+yz)\hat{i} + (xy^2 - xz^2)\hat{j} + (2xyz - 2x^2y^2)\hat{k}$ is solenoidal. (05 Marks)

c. Show that grad $(x^3 + y^3 + z^3 - 3xyz)$ is irrotational. (06 Marks)

Module-5

9 a. Solve $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$. (05 Marks)

b. Solve $\frac{dy}{dx} - \frac{y}{x} = (x+1)$. (05 Marks)

c. Solve $(x^2 + y^2 + 1)\, dx + 2xydy = 0$. (06 Marks)

OR

10 a. Solve $(3x^2 + 6xy^2)dx + (6x^2y + 4y^3)dy = 0$. (05 Marks)

b. Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$. (05 Marks)

c. Solve $\frac{dy}{dx} = y\tan x - y^2 \sec x$ (06 Marks)

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