CBCS SCHEME

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15MAT11

First Semester B.E. Degree Examination, June/July 2023
Engineering Mathematics - I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find n^{th} derivation of $\frac{x^3}{x^2-1}$. **(05 Marks)**

b. Find Pedal equation for $\frac{2a}{r} = 1 - \cos\theta$. **(05 Marks)**

c. Find radius of curvature for $y^2 = \frac{a^2(a-x)}{x}$, where the curve cuts x – axis. **(06 Marks)**

OR

2 a. If $y = \sin(m \cos^{-1} x)$, Show that $(1-x^2)\, Y_{n+2} - (2n+1)x\, Y_{n+1} + (n^2 - m^2)\, Y_n = 0$. **(06 Marks)**

b. ST curves $r^n = a^n \sec n\theta$, $r^n \sin n\theta = b^n$ cuts orthogonally. **(05 Marks)**

c. If $\sqrt{r}\,\cos\left(\frac{\theta}{2}\right) = a$, Show that $\rho = \frac{2}{a} r^{3/2}$. **(05 Marks)**

Module-2

3 a. If $u = f\left(\frac{y-x}{xy}, \frac{x-z}{xz}\right)$, show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$. **(06 Marks)**

b. Evaluate $\underset{x\to 0}{\ell t}\left[\frac{\tan x}{x}\right]^{1/x^2}$. **(05 Marks)**

c. Obtain the expansion of tan x in terms of powers of $\left(x - \frac{\pi}{4}\right)$ upto 3^{rd} degree. **(05 Marks)**

OR

4 a. For $y = \log_e(1+x)$, find Maclaurin's series expansion upto 4^{th} degree, hence evaluate $\log_e^2$. **(06 Marks)**

b. If $u = \sin^{-1}\left[\frac{x^2+y^2}{\sqrt{x}-\sqrt{y}}\right]$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}$ tones. **(05 Marks)**

c. If $x+y+z = u$, $y+z = v$, $z = uvw$.
ST $J\left[\frac{x, y, z}{u, v, w}\right] = uv$. **(05 Marks)**

Module-3

5 a. Find the components of velocity and acceleration of partical whose position vector is $\vec{r} = xi + yi + zk$, where $x = 2\sin 3t$, $y = 2\cos 3t$, $z = 8t$ at $t = \pi/2$ along the vector $i + j + k$. **(06 Marks)**

b. Find div $\vec{F}$, Curl $\vec{F}$ for $\vec{F} = x^2yzi + y^2zxj + z^2xyk$ at (1, 1, 1). **(05 Marks)**

c. If ϕ and ψ are any two scalar function. Show that grad $(\phi\,\psi) = \phi$ grad $\psi + \psi$ grad ϕ. **(05 Marks)**

OR

6 a. If $\vec{F} = (\sin y + z)\, i + (x \cos y - z) j + (x - y) k$. Show that $\vec{F}$ is irrotational. Hence find scalar potential ϕ such that $\vec{F} = \nabla \phi$. **(06 Marks)**

b. Find Directional Derivative of scalar function $\phi = xy^2 + yz^3$ at (2, -1, 1) along $i + 2j + 2k$. **(05 Marks)**

c. If $\vec{f} = r^n \vec{r}$, where $\vec{r} = xi + xj + zk$ and $r = |\vec{r}|$, find div $\vec{f}$. Hence find n for which $\vec{f}$ is Solenoidal. **(05 Marks)**

Module-4

7 a. Find Reduction formula for $\int_0^{\pi/2} \sin^n x\, dx$. **(06 Marks)**

b. Find Orthogonal trajectory of $r^n = a^n \sin n\theta$. **(05 Marks)**

c. Solve $(y \log x - 2)\, y dx = x\, dy$. **(05 Marks)**

OR

8 a. Evaluate $\int_0^{\pi} x \sin^5 x \cos^6 x\, dx$. **(06 Marks)**

b. Solve $y[2x - y + 1]dx + x[3x - 4y + j]dy = 0$. **(05 Marks)**

c. A body in air at 25°C cools from 100°C to 75°C on 1 minute. Find temperature of body at end of 3 minutes. **(05 Marks)**

Module-5

9 a. Solve by Gauss elimination

$$x + y + z = 6$$
$$x - y + 2z = 5$$
$$3x + y + z = 8.$$

(06 Marks)

b. Find Spectral (Diagonal) and Modal matrix for

$$\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}.$$

(05 Marks)

c. Show that Linear Transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular. Hence find Inverse transformation. **(05 Marks)**

OR

10 a. Using Gauss – Seidel, solve

$$10x + y + z = 12$$
$$x + 10y + z = 12$$
$$x + y + 10z = 12.$$

Taking (0, 0, 0) as Initial Vector. **(06 Marks)**

b. Using Rayleigh's Power method, find Numerically largest Eigen value and corresponding Eigen vector.

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ taking $x_0 = (1, 1, 1)'$, carry out 5 iteration. **(05 Marks)**

c. Reduce quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ onto Canonical form. **(05 Marks)**

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