



Second Semester B.E./B.Tech. Degree Examination, June/July 2023
Mathematics – II for CSE Stream

Time: 3 hrs.

Max. Marks: 100

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
 2. VTU Formula Hand Book is permitted.
 3. M : Marks , L: Bloom's level, C: Course outcomes.*

Module – 1			M	L	C
Q.1	a.	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dz \, dy \, dx$.	7	L2	CO1
	b.	Evaluate by changing the order of integration $\int_0^a \int_y^a \frac{x}{x^2 + y^2} \, dx \, dy$.	7	L3	CO1
	c.	Show that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.	6	L2	CO1
OR					
Q.2	a.	Evaluate $\int_{-2}^2 \int_0^{\sqrt{4-x^2}} (2-x) \, dy \, dx$ by changing into polar coordinates.	7	L3	CO1
	b.	A pyramid is bounded by three coordinate planes and the plane $x + 2y + 3z = 6$. Compute the volume by double integration.	7	L3	CO1
	c.	Using Mathematical tools, write the code to find the area of the cardioids $r = a(1 + \cos \theta)$ by double integration.	6	L3	CO5
Module – 2					
Q.3	a.	Show that the two surfaces $xz + y + z^2 = 9$ and $z = 4 - 4xy$ at $(1, -1, 2)$ are orthogonal.	7	L3	CO2
	b.	If $F = \text{grad}(xy^3z^2)$, find $\text{div} F$ and $\text{curl} F$ at the point $(1, -1, 1)$.	7	L2	CO2
	c.	Prove that the cylindrical coordinate system is orthogonal.	6	L3	CO2
OR					
Q.4	a.	Find the directional derivative of $\phi = x \log z - y^2 + 4$ at $(-1, 2, 1)$ in the direction of the vector $2i - j - 2k$.	7	L2	CO2
	b.	Find the constants a, b and c such that $F = (axy - z^3)i + (bx^2 + z)i + (bxz^2 + cy)k$ is irrotational.	7	L2	CO2
	c.	Using the Mathematical tools, write the codes to find the gradient of $\phi = xy^2z^3$.	6	L3	CO5

Module – 3

Q.5	a.	Let $W = \{(x, y, z) \mid lx + my + nz = 0\}$, then prove that W is a subspace of \mathbb{R}^3 .	7	L2	CO3
	b.	Find the basis and the dimension of the subspace spanned by the vectors $\{(2, 4, 2), (1, -1, 0), (1, 2, 1), (0, 3, 1)\}$ in $V_3(\mathbb{R})$.	7	L2	CO3
	c.	Prove that $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be defined by $T(x, y, z) = (2x - 3y, x + 4, 5z)$ is not a linear transformation.	6	L3	CO3

OR

Q.6	a.	Show that the matrix $E = \begin{bmatrix} -1 & 7 \\ 8 & -1 \end{bmatrix}$ lies in the sub space span $\{A, B, C\}$ of vector space M_{22} of 2×2 matrices, where $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$.	7	L2	CO3
	b.	Verify the Rank-nullity theorem for the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.	7	L3	CO3
	c.	Define an Inner product space. Consider $f(t) = 4t + 3$, $g(t) = t^2$, the inner product $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$. Find $\langle f, g \rangle$ and $\ g\ $.	6	L2	CO3

Module – 4

Q.7	a.	Find the real root of the equation $x \log_{10} x - 1.2$ by the Regula-Falsi method between 2 and 3. (Carryout three iterations).	7	L2	CO4												
	b.	From the following table, estimate the number of students who have obtained the marks between 40 and 45.	7	L2	CO4												
		<table border="1"> <tr> <td>Marks</td> <td>30 – 40</td> <td>40 – 50</td> <td>50 – 60</td> <td>60 – 70</td> <td>70 – 80</td> </tr> <tr> <td>Number of students</td> <td>31</td> <td>42</td> <td>51</td> <td>35</td> <td>31</td> </tr> </table>	Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	Number of students	31	42	51	35	31			
Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80												
Number of students	31	42	51	35	31												
	c.	Compute the value of $\int_{0.2}^{1.4} (\sin x - \log x + e^x) dx$ using Simpson's $\frac{3}{8}$ rule taking six parts.	6	L3	CO4												

OR

Q.8	a.	Using Newton-Raphson method compute the real root of the equation $x \sin x + \cos x = 0$ near $x = \pi$, correct to four decimal places.	7	L2	CO4
	b.	If $y(0) = -12$, $y(1) = 0$, $y(3) = 6$ and $y(4) = 12$, find the Lagrange's interpolation polynomial and estimate $y(2)$.	7	L2	CO4
	c.	Evaluate $\int_0^3 \frac{dx}{4x + 5}$ using Trapezoidal rule by taking 7 ordinates.	6	L3	CO4

Module – 5

Q.9	a.	Employ Taylor's series method to obtain $y(0.1)$ for $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$ considering upto 4 th degree terms.	7	L2	CO4
	b.	Using Runge-Kutta method of fourth order, solve $y' = \log_{10} \left[\frac{y}{1-x} \right]$ given $y(0) = 1$ at $x = 0.2$	7	L3	CO4

c.	Solve $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$, $y(0.3) = 2.090$, find $y(0.4)$ using Milne's method.	6	L2	CO4
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OR

Q.10 a.	Given $\frac{dy}{dx} = x + \sqrt{y}$, $y(0) = 1$. Compute $y(0.4)$ with $h = 0.2$ using Euler's modified method. Perform two modifications in each stage.	7	L2	CO4
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b.	Apply Milne's predictor-corrector formulae to compute $y(4.5)$, given that $5x \frac{dy}{dx} = 2 - y^2$ and	7	L2	CO4										
	<table border="1"> <tr> <td>x</td> <td>4.1</td> <td>4.2</td> <td>4.3</td> <td>4.4</td> </tr> <tr> <td>y</td> <td>1.0049</td> <td>1.0097</td> <td>1.0143</td> <td>1.0187</td> </tr> </table>	x	4.1	4.2	4.3	4.4	y	1.0049	1.0097	1.0143	1.0187			
x	4.1	4.2	4.3	4.4										
y	1.0049	1.0097	1.0143	1.0187										

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c.	Using modern mathematical tools, write the code to find the solution of $\frac{dy}{dx} = x - y^2$ at $y(0.2)$. Given that $y(0) = 1$ by Runge-Kutta 4 th order method. (Take $h = 0.2$)	6	L3	CO5
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