CBCS SCHEME

USN | | | | | | | | | | 18EE54

Fifth Semester B.E. Degree Examination, Jan./Feb. 2023
Signals and Systems

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Describe the classifications of signals. **(06 Marks)**

b. Is the signal shown in Fig.Q1(b) in power or energy signal? Given reasons for your answer and further determine its energy or power.

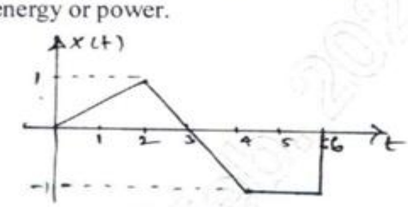

Fig.Q1(b) **(06 Marks)**

c. Determine whether the following signal are periodic, if periodic determine the fundamental period :

i) $x(t) = \cos 2t + \sin 3t$

ii) $x(n) = \cos(\frac{1}{5}\pi n) \sin(\frac{1}{3}\pi n)$. **(08 Marks)**

OR

2 a. Sketch the following signals and determine their even and odd signals
$r(t + 2) - r(t + 1) - r(t - 2) + r(t - 3)$. **(08 Marks)**

b. Given signal x(t) as shown in Fig.Q2(b). Sketch the following : i) $x(-2t + 3)$ ii) $x(t/2 - 2)$.

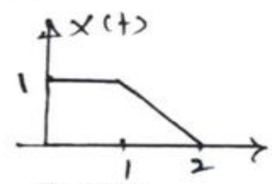

Fig.Q2(b) **(06 Marks)**

c. For each of the system, state whether the system is linear, shift variant, stable, causal and memory. i) $y(n) = \log[x(n)]$ ii) $y(t) = x(t^2)$. **(06 Marks)**

Module-2

3 a. Compute the convolution of two sequences $x_1(n)$ and $x_2(n)$ given below :

$x_1(n) = \left\{\underset{\uparrow}{1}, 2, 3\right\}$ $x_2(n) = \left\{1, 2, \underset{\uparrow}{3}, 4,\right\}$. **(06 Marks)**

b. Convolute the following two signals

$x(t) = 1$; $0 < t < T$ $h(t) = t$; $0 < t < 2T$

0 ; otherwise 0 ; otherwise

Obtain expression for the output y(t). **(08 Marks)**

c. An LTI system represented by the impulse response :

i) $h(t) = e^{t2^t} u(t-1)$ ii) $h(n) = a^n u(n + 2)$

Determine whether its stable, causal and memory. **(06 Marks)**

OR

4 a. Find the forced response for the system described by

$$\frac{d^2y(t)}{dt^2}+\frac{5dy(t)}{dt}+6y(t)=2x(t)+\frac{dx(t)}{dt}$$

with input $x(t)=2e^{-t}u(t)$. **(08 Marks)**

b. Find the natural response of the system described by difference equation :

$y(n)-\frac{1}{4}y(n-1)-\frac{1}{8}y(n-2)=x(n)+x(n-1)$ with $y(-1)=0$ and $y(-2)=1$. **(06 Marks)**

c. Draw the direct form I and II realization for the following system :

$$2\frac{d^3y(t)}{dt^3}+\frac{dy(t)}{dt}+3y(t)=x(t).$$ **(06 Marks)**

Module-3

5 a. What are the properties of continuous time Fourier transform and prove Parsavel's theorem. **(08 Marks)**

b. Obtain the Fourier transform of the signal :

i) $x(t)=e^{-at}u(t)$

ii) $x(t)=e^{-a|t|}$. **(06 Marks)**

c. Using convolution theorem, find the inverse Fourier transform of

$$X(\omega)=\frac{1}{(a+j\omega)^2}.$$ **(06 Marks)**

OR

6 a. Using partial fraction expansion, determine the inverse Fortier transform

$$X(j\omega)=\frac{5j\omega+12}{(j\omega)^2+(5j\omega)+6}$$ **(06 Marks)**

b. Find the Fourier transform of the following signal using appropriate properties.

$x(t)=\sin(\pi t)e^{-2t}u(t)$. **(06 Marks)**

c. Consider the continuous time LTI system described by

$$\frac{dy(t)}{dt}+2y(t)=x(t).$$

Using Fourier transform, find the output y(t) with input signal $x(t)=e^{-t}u(t)$. **(08 Marks)**

Module-4

7 a. Describe the following properties of DTFT

i) Frequency differentiation

ii) Scaling

iii) Modulation. **(06 Marks)**

b. Find the DTFT of the following signals :

i) $x(n)=(0.5)^{n+2}u(n)$

ii) $x(n)=n(0.5)^{2n}u(n)$. **(06 Marks)**

c. Find the inverse DTFT

$$X(\Omega)=\frac{3-\frac{5}{4}e^{-j\Omega}}{\frac{1}{8}e^{-j2\Omega}-\frac{3}{4}e^{-j\Omega}+1}.$$ **(08 Marks)**

OR

8 a. Find the frequency response and the impulse response of discrete time system described by difference equation :

$y(n-2) + 5y(n-1) + 6y(n) = 8x(n-1) + 18x(n)$ **(10 Marks)**

b. Determine the difference equation for the system with following impulse response

$h(n) = \delta(n) + 2(\frac{1}{2})^n u(n) + [-\frac{1}{2}]^n u(n)$. **(10 Marks)**

Module-5

9 a. Explain the properties of ROC. **(06 Marks)**

b. For the signal $x(n) = 7(\frac{1}{3})^n - 6(\frac{1}{2})^n u(n)$, find the Z – transform and ROC. **(06 Marks)**

c. By using suitable properties of Z – transform find the Z – transform and ROC of the following :

i) $x(n) = (\frac{1}{2})^n u(n) - 3^n u(-n-1)$

ii) $x(n) = n\, a^n u(n-3)$. **(08 Marks)**

OR

10 a. Find the inverse Z – transform of the sequence $x(z) = \dfrac{z}{3z^2 - 4z + 1}$, for the following :

i) $|z| > 1$ ii) $|z| < \frac{1}{3}$ iii) $\frac{1}{3} < |z| < 1$. **(06 Marks)**

b. Solve the following linear constant co-efficient difference equation using unilateral Z – transform method.

$y(n) = \frac{3}{2}y(n-1) + \frac{1}{2}y(n-2) = (\frac{1}{4})^n u(n)$, with I.C. $y(-1) = 4$, $y(-2) = 10$. **(08 Marks)**

c. A system has impulse response $h(n) = (\frac{1}{2})^n u(n)$. Determine the input to the system if the output is given by $y(n) = \frac{1}{3}u(n) + \frac{2}{3}(-\frac{1}{2})^n u(n)$. **(06 Marks)**

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Module 1

1a. Signals are classified into the following categories:

- Continuous Time and Discrete Time Signals
- Deterministic and Non-deterministic Signals
- Even and Odd Signals
- Periodic and Aperiodic Signals
- Energy and Power Signals

b.

This is non periodic. It is Energy signal. → 1M

$$x(t) = \begin{cases} t/2 & 0 \le t \le 2 \\ -t+3 & 2 \le t \le 4 \\ -1 & 4 \le t \le 6 \end{cases}$$ 2 Mark

$$E = \int_0^2 (t/2)^2 dt + \int_2^4 (-t+3)^2 dt + \int_4^6 (-1)^2 dt = 10/3$$ → 03

c.

(i) $x(t) = \cos 2t + \sin 3t$

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2} = \pi$, $T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{3}$ — 02M

$\frac{T_1}{T_2} = \frac{\pi}{2\pi/3} = 3/2$ → 1M

which is rational.

∴ periodic → 1m

$T = 2\pi$

$x(t)$

(ii) $x(n) = \cos\left(\frac{\pi n}{5}\right) \sin\left(\frac{\pi n}{3}\right)$

$= \frac{1}{2}\left\{\sin\left(\frac{\pi n}{3} - \frac{\pi n}{5}\right) + \sin\left(\frac{\pi n}{3} + \frac{\pi n}{5}\right)\right\}$

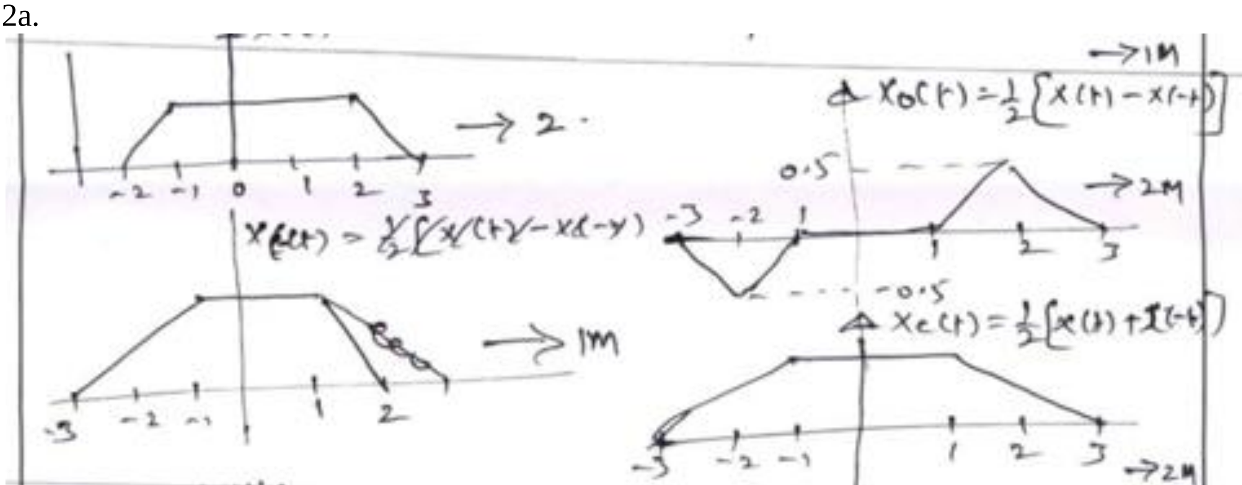
$= \frac{1}{2}\left\{\sin \frac{2\pi n}{15} + \sin \frac{8}{15}\pi n\right\}$ 02M

$N_1 = \frac{2\pi}{\Omega_1} = \frac{2\pi}{2\pi/15} = 15$

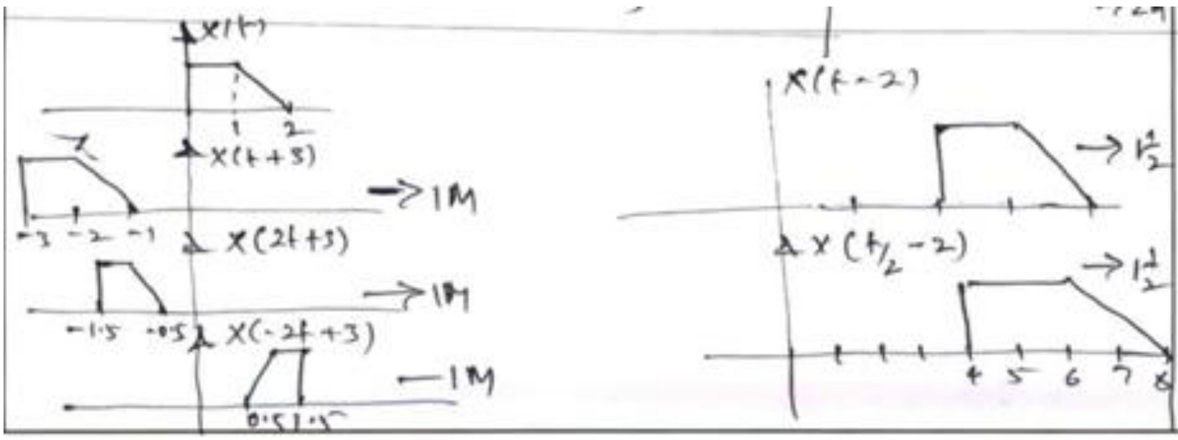
$N_2 = \frac{2\pi}{\Omega_2} = \frac{2\pi}{8/15\,\pi} = \frac{4}{15}$ 2M

periodic: $N = 15$

2a.



b.



c.

1. Stability: A system is stable if its output is bounded for all bounded inputs. In this case, if the input signal x(n) is bounded, then the output signal y(n) will also be bounded. However, if the input signal x(n) approaches zero, then the output signal y(n) will approach negative infinity, which means the system is not bounded. Therefore, the system is not stable.
2. Linearity: A system is linear if it satisfies the superposition principle, which means that the response to a sum of inputs is the sum of the responses to each individual input. In this case, if we have two input signals x1(n) and x2(n), then the output signals y1(n) and y2(n) are given by:

   y1(n) = log(x1(n)) y2(n) = log(x2(n))

   The output signal for the sum of these two input signals x1(n) + x2(n) is:

   y(n) = log(x1(n) + x2(n))

   However, we cannot write y(n) as y1(n) + y2(n), which means the system is not linear.

3. Shift invariance: A system is shift-invariant if a shift in the input signal results in a corresponding shift in the output signal. In this case, if we shift the input signal x(n) by k samples, then the output signal y(n) will also be shifted by k samples. Therefore, the system is shift-invariant.
4. Causality: A system is causal if its output depends only on past and present input values, and not on future input values. In this case, the output signal y(n) depends only on the current and past input values x(n), x(n-1), x(n-2), ... and not on future input values. Therefore, the system is causal.
5. Memory: A system has memory if its output depends on past and/or present input values. In this case, the output signal y(n) depends only on the current input value x(n) and not on past input values, which means the system has no memory.

In summary, the system y(n) = log(x(n)) is not stable and not linear, but it is shift-invariant, causal, and has no memory.

1. Stability: A system is stable if its output is bounded for all bounded inputs. In this case, if the input signal x(n) is bounded, then the output signal y(n) = x(n^2) may or may not be bounded depending on the input signal. For example, if the input signal is bounded between -1 and 1, the output signal will be unbounded. Hence, the system is not stable.
2. Linearity: A system is linear if it satisfies the superposition principle, which means that the response to a sum of inputs is the sum of the responses to each individual input. In

this case, if we have two input signals x1(n) and x2(n), then the output signals y1(n) and y2(n) are given by:

y1(n) = x1(n^2) y2(n) = x2(n^2)

The output signal for the sum of these two input signals x1(n) + x2(n) is:

y(n) = (x1(n^2) + x2(n^2))

We can write y(n) as y1(n) + y2(n), which means the system is linear.

3. Shift invariance: A system is shift-invariant if a shift in the input signal results in a corresponding shift in the output signal. In this case, if we shift the input signal x(n) by k samples, then the output signal y(n) will be:

y(n) = x((n-k)^2)

However, this is not the same as y(n-k) = x((n-k)^2), which means the system is not shift-invariant.

4. Causality: A system is causal if its output depends only on past and present input values, and not on future input values. In this case, the output signal y(n) depends only on the present and past input values x(n^2), x((n-1)^2), x((n-2)^2), ... and not on future input values. Therefore, the system is causal.
5. Memory: A system has memory if its output depends on past and/or present input values. In this case, the output signal y(n) depends on the present and past input values x(n^2), x((n-1)^2), x((n-2)^2), ... which means the system has memory.

In summary, the system y(n) = x(n^2) is not stable, linear, and shift-invariant, but it is causal and has memory.

Module 2

3a.

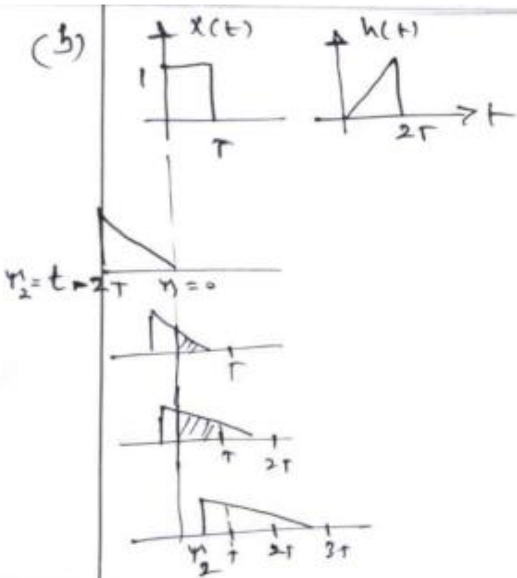
$y(n) = x_1(n) * x_2(n)$

$(\delta(n) + 2\delta(n-1) + 3\delta(n-2)) * [\delta(n+2) + 2\delta(n+1) + 3\delta(n) + 4(\delta(n-1)] \rightarrow 2m$

$\therefore y(n) = 1, 4, \underset{\Uparrow}{10}, 16, 17, 12 \rightarrow 4m$

b.

(3)



$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$t = 0 \quad \int_{-\infty} x(\tau)\, h(-\tau)\, d\tau.$

$\tau_1 < 0 \quad y(t) = 0 \longrightarrow 2M$

$\tau_1 > 0 \quad \tau_1 < T$

$y(t) = \int_0^{\tau_1 = t} (t-\tau)\, d\tau = \frac{t^2}{2} \longrightarrow 2M$

$\tau_1 > T \quad \tau_1 < 2T$

$y(t) = \int_0^{T} (t-\tau)\, d\tau = Tt - \frac{T^2}{2} \longrightarrow 2M$

$\tau_1 > 2T, \tau_1 < 3T$

$y(t) = \int_{t-2T}^{T} (t-\tau)\, d\tau = tT - \frac{t^2}{2} + \frac{3}{2}T^2 \longrightarrow 2M$

(c) (i) $h(t) = e^{2t} u(t-1)$

c.

$s^2 + 5s + 6 = 0$

$y^h(t) = c_1 e^{-3t} + c_2 e^{-2t} \longrightarrow (1) \longrightarrow 2M$

$y^p(t) = K e^{-t}, \; y^p(t) = e^{-t} \longrightarrow 1M$

$\boxed{K = 1}$

$y^f(t) = c_1 e^{-3t} + c_2 e^{-2t} + e^{-t} \longrightarrow 1M$

Assum I.C = 0

$c_1 + c_2 = -1 \longrightarrow 1M$

diff eqn

$-3c_1 - 2c_2 = 1 \longrightarrow 1M$

$c_1 = 1, \; c_2 = -2 \longrightarrow 1M$

$\therefore y^f(t) = e^{-3t} - 2e^{-2t} + e^{-t} \longrightarrow 1M$

b.

$1 - \frac{1}{4}r^{-1} - \frac{1}{8}r^{-2} = 0$

$r_1 = \frac{1}{2}, \; r_2 = -\frac{1}{4} \longrightarrow 1M$

$y^h(n) = c_1 (\frac{1}{2})^n + c_2 (-\frac{1}{4})^n \longrightarrow 1M$

$n = 0 \quad y(-1) = 0$

$c_1 + c_2 = \frac{1}{8} \longrightarrow (1) \longrightarrow 1M$

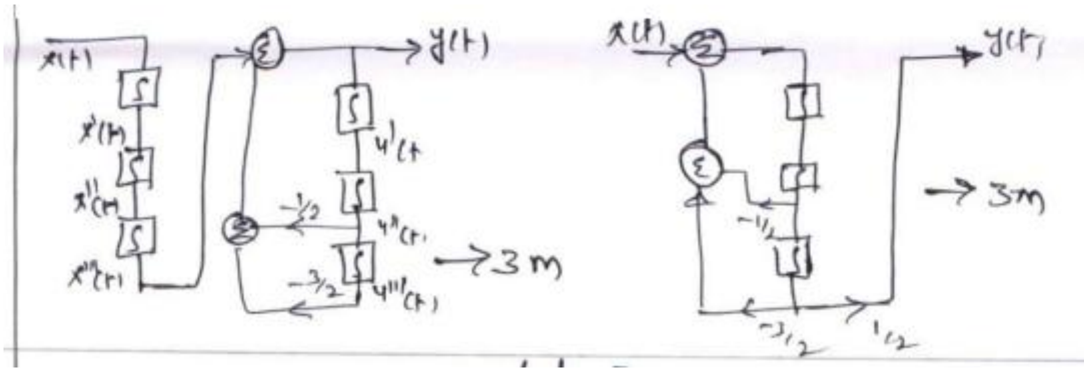
$n = 0 \quad y(-2) = -\frac{1}{4}$

$\frac{1}{2}c_1 - \frac{1}{4}c_2 = \frac{1}{32} \longrightarrow 1M$

$c_1 = \frac{1}{12} \quad c_2 = \frac{1}{24} \longrightarrow 1M$

$y^h(n) = \frac{1}{12}(\frac{1}{2})^n + \frac{1}{24}(-\frac{1}{4})^n \longrightarrow 1M$

c.



Module 3

5a

1. Linearity: The CTFT is a linear operator. This means that if a signal f(t) can be expressed as the sum of two other signals g(t) and h(t), then the CTFT of f(t) is equal to the sum of the CTFTs of g(t) and h(t).
2. Time-Shifting: The CTFT of a time-shifted signal f(t - t0) is equal to the original CTFT of f(t) multiplied by e^(-jωt0), where ω is the angular frequency.
3. Frequency-Shifting: The CTFT of a frequency-shifted signal f(t)exp(jω0t) is equal to the original CTFT of f(t) shifted by ω0.
4. Time Reversal: The CTFT of a time-reversed signal f(-t) is equal to the complex conjugate of the CTFT of f(t).
5. Duality: If the CTFT of a signal f(t) is F(ω), then the CTFT of F(t) is 2πf(-ω).
6. Convolution: The CTFT of the convolution of two signals f(t) and g(t) is equal to the product of their respective CTFTs.
7. Multiplication: The CTFT of the product of two signals f(t) and g(t) is equal to the convolution of their respective CTFTs.
8. Parseval's Theorem: The integral of the square of the magnitude of a signal f(t) in the time domain is equal to the integral of the square of the magnitude of its CTFT F(ω) in the frequency domain.

If $x(t) \xleftrightarrow{FT} X(\omega)$, then

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega = \int_{-\infty}^{\infty} |X(f)|^2\, df \quad \ldots (5.1$$

Meaning :

Energy of the signal can be obtained by interchanging its energy spectrum.

Proof :

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} x(t)x^*(t)\, dt \quad \ldots (5.1$$

Inverse Fourier transform states that,

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

Taking conjugate of both the sides,

$$x^*(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)\, e^{-j\omega t}\, d\omega$$

Putting above expression for $x^*(t)$ in equation 5.1.16,

$$E = \int_{-\infty}^{\infty} x(t)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)\, e^{-j\omega t}\right] d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)\cdot X(\omega)\, d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$$

b.

(i) $x(t) = e^{-at}u(t)$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-at} u(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt = \frac{1}{a+j\omega}$$

$\longrightarrow$ 3M

(ii) $x(t) = e^{-a|t|}$

$x(t) = e^{-at}$ $t>0$; 1 $t=0$; e^{at} $t<0$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt + \int 1\, e^{-j\omega t}\, dt$$

$$= \frac{2a}{a^2+\omega^2} \rightarrow 03$$

c.

$$X(\omega) = \frac{1}{(a+j\omega)^2} = \frac{1}{(a+j\omega)(a+j\omega)} = X_1(j\omega)\,X_2(j\omega) \longrightarrow 1M$$

$$X_1 = \frac{1}{a+j\omega} \qquad X_2(j\omega) = \frac{1}{a+j\omega}$$

$$x_1(t) = e^{-at}u(t) \qquad x_2(t) = e^{-at}u(t) \longrightarrow 2M$$

$$x(t) = x_1(t) * x_2(t) \longrightarrow 1M$$

$$= \int_{-\infty}^{\infty} e^{-at}\; e^{-a(t-\tau)}\, d\tau.$$

$$= te^{-at}u(t) \longrightarrow 2M$$

6a.

$$X(j\omega) = \frac{5j\omega+12}{(j\omega+3)(j\omega+2)} = \frac{A}{j\omega+3} + \frac{B}{j\omega+2} \longrightarrow 2M$$

$$A = 3 \quad B = 2 \longrightarrow 2M$$

$$X(j\omega) = \frac{3}{j\omega+3} + \frac{2}{j\omega+2} \longrightarrow$$

$$x(t) = [3e^{-3t} + 2e^{-2t}]u(t) \longrightarrow 2M$$

b.

$$x(t) = \sin\pi t\; e^{-2t}u(t)$$

$$= \left[\frac{e^{+j\pi t} - e^{-j\pi t}}{2j}\right] e^{-2t}u(t)$$

$$= e^{-2t}\frac{e^{j\pi t}}{2j}u(t) - e^{-2t}\frac{e^{-j\pi t}}{2j}u(t)$$ 2M.

using frequency shifting property

$$e^{-j\pi t}e^{-2t}u(t) \longrightarrow \frac{1}{2-j(\omega-\pi)}$$

$$\frac{1}{2j}e^{j\pi t}e^{-2t} \longrightarrow \frac{1}{2j-2+j(\omega-\pi)}$$ 2M

$$x(t) = \frac{1}{j2}\left[\frac{1}{2+j(\omega-\pi)} - \frac{1}{2+j(\omega+\pi)}\right] \longrightarrow 2M$$

c.

$$\frac{dy(t)}{dt} + 2y(t) = x(t) \qquad x(t) = e^{-t}u(t)$$

$$jw\, Y(jw) + 2Y(jw) = X(w) \rightarrow 2m \qquad X(jw) = \frac{1}{1+jw} \rightarrow 1m$$

$$\frac{Y(w)}{X(w)} = \frac{1}{jw+2} \rightarrow 2m \qquad Y(w) = \frac{1}{1+jw} - \frac{1}{jw+2}$$

$$y(t) = [e^{-t} - e^{-2t}]u(t) \rightarrow 3m$$

Module 4

7a.i

Proof : By definition of DTFT,

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

Differentiating both the sides with respect to Ω, we get,

$$\frac{d}{d\Omega} X(\Omega) = \frac{d}{d\Omega} \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

Changing the order of summation and differentiation,

$$\frac{d}{d\Omega} X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) \frac{d}{d\Omega}[e^{-j\Omega n}] = \sum_{n=-\infty}^{\infty} x(n)(-jn)\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} [-jn\, x(n)]\, e^{-j\Omega n}$$

Comparing above equation with the definition of DTFT, we find that $-jnx(n)$ has DTFT of $\frac{d}{d\Omega} X(\Omega)$. i.e.,

$$-jnx(n) \overset{DTFT}{\longleftrightarrow} \frac{d}{d\Omega} X(\Omega)$$

ii.

Proof : By definition of DTFT,

$$Y(\Omega) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} x(p\,n)\, e^{-j\Omega n}$$

Here put $p\,n = m$. Since n has the range of $-\infty$ to ∞, m will also have the same range. Then above equation becomes,

$$Y(\Omega) = \sum_{m=-\infty}^{\infty} x(m)\, e^{-j\Omega m/p} = \sum_{m=-\infty}^{\infty} x(m)\, e^{-j\left(\frac{\Omega}{p}\right)m}$$

$$= X\left(\frac{\Omega}{p}\right)$$

iii.

This property states that

If $\quad x(n) \xleftrightarrow{DTFT} X(\Omega)$

and $\quad y(n) \xleftrightarrow{DTFT} Y(\Omega)$

then $$z(n) = x(n)\, y(n) \xleftrightarrow{DTFT} Z(\Omega) = \frac{1}{2\pi}\,[X(\Omega) * Y(\Omega)] \qquad \text{... (5.2.14)}$$

Proof : By definition of DTFT,

$$Z(\Omega) = \sum_{n=-\infty}^{\infty} z(n)\, e^{-j\Omega n}$$

Putting for $z(n) = x(n)\, y(n)$ in above equation,

$$Z(\Omega) = \sum_{n=-\infty}^{\infty} x(n)\, y(n)\, e^{-j\Omega n} \qquad \text{... (5.2.15)}$$

From the inverse DTFT of equation 5.2.2 we know that,

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\lambda)\, e^{j\lambda n}\, d\lambda$$

Here we have used separate frequency variable λ. Putting the above expression of $x(n)$ in equation 5.2.15,

$$Z(\Omega) = \sum_{n=-\infty}^{\infty} \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\lambda) e^{j\lambda n}\, d\lambda \cdot y(n)\, e^{-j\Omega n}$$

Interchanging the order of summation and integration,

$$Z(\Omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\lambda) \sum_{n=-\infty}^{\infty} y(n) e^{j\lambda n} e^{-j\Omega n}\, d\lambda$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\lambda) \left[\sum_{n=-\infty}^{\infty} y(n)\, e^{-j(\Omega-\lambda)n}\right] d\lambda$$

The term in square brackets is $Y(\Omega - \lambda)$, hence above equation becomes,

$$Z(\Omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\lambda)\, Y(\Omega-\lambda)\, d\lambda$$

b.

(b)

(i) $x(n) = (0.5)^{n+2} u(n)$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}.$$

$$\left.\begin{aligned} &= \sum_{n=0}^{\infty} (0.5)^{n+2} e^{-j\Omega n} \\ &= \frac{1/4}{1 - \frac{1}{2} e^{j\Omega}} \end{aligned}\right\} 3m$$

(ii) $x(n) = n\,(0.5)^{2n} u(n)$

$$\left.\begin{aligned} &\sum_{n=0}^{\infty} n \cdot \left(\tfrac{1}{2}\right)^{2n} e^{-j\Omega n} \\ &= \frac{\frac{1}{4} e^{-j\Omega}}{\left(1 - \frac{1}{4} e^{-j\Omega}\right)^2} \end{aligned}\right\} 3m$$

c.

(c) $X(\Omega) = \dfrac{3 - \frac{5}{4}v}{\frac{1}{8}v^2 - \frac{3}{4}v + 1} = \dfrac{A}{\frac{v}{4} - 1} + \dfrac{B}{\frac{v}{2} - 1}$ $\rightarrow$ 2M

$A = -2 \quad B = -1$ $\rightarrow$ 2M

$X(\Omega) = \dfrac{}{\frac{v}{4} - 1} - \dfrac{}{\frac{v}{2} - 1}$ $\rightarrow$ 1M (08)

$X(\Omega) = \dfrac{2}{1 - \frac{1}{4}e^{-j\Omega}} + \dfrac{1}{1 - \frac{1}{2}e^{-j\Omega}}$ $\rightarrow$ 1M

$x(n) = 2\left(\frac{1}{4}\right)^n u(n) + \left(\frac{1}{2}\right)^n u(n)$ $\rightarrow$ 2M

8a.

$e^{-j2\Omega}Y(\Omega) + 5e^{-j\Omega}Y(\Omega) + 6Y(\Omega) = 8e^{-j\Omega}X(\Omega) + 18X(\Omega)$

$\dfrac{Y(\Omega)}{X(\Omega)} = \dfrac{8e^{-j\Omega} + 18}{(e^{-j\Omega})^2 + 5e^{-j\Omega} + 16} = \dfrac{8v + 18}{v^2 + 5v + 16}$ $\quad v = e^{-j\Omega}$ $\rightarrow$ 2M

$H(\Omega) = \dfrac{Y(\Omega)}{X(\Omega)} = \dfrac{8v + 18}{v^2 + 5v + 16} = \dfrac{A}{(v+1)} + \dfrac{B}{(v+4)}$ $\rightarrow$ 2M

$\alpha^n u(n) \Leftrightarrow \dfrac{1}{1 - \alpha e^{-j\Omega}}$ $\quad A = 10/3 \quad B = -14/3$ $\rightarrow$ 2M

$h(n) =$ $\rightarrow$ (4)

b.

$H(\Omega) = 1 + 2 \cdot \dfrac{1}{1 - \frac{1}{2}e^{-j\Omega}} + \dfrac{1}{1 + \frac{1}{2}e^{-j\Omega}}$ $\rightarrow$ 2M

$= 1 + \dfrac{2}{1 - \frac{1}{2}e^{-j\Omega}} + \dfrac{1}{1 + \frac{1}{2}e^{-j\Omega}}$ $\rightarrow$ 2M

$H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{4 + \frac{1}{2}e^{-j\Omega} - \frac{1}{4}e^{-j2\Omega}}{1 - \frac{1}{4}e^{-j2\Omega}}$ $\rightarrow$ 2M

$Y(\Omega) - \frac{1}{4}e^{-j2\Omega}Y(\Omega) = 4X(\Omega) + \frac{1}{2}e^{-j\Omega}X(\Omega) - \frac{1}{4}e^{-j2\Omega}X(\Omega)$ $\rightarrow$ 2M

taking inverse DTFT

$y(n) - \frac{1}{4}y(n-2) = 4x(n) + \frac{1}{2}x(n-1) - \frac{1}{4}x(n-2)$ $\rightarrow$ 2M

Module 5

9a.

- ROC has a ring form or a disc form
- The Fourier transform of x(n) has Fourier transform if and only if that its z-transform's ROC includes unit circle
- ROC cannot contain any pole
- If the sequence x(n) has finite length then ROC contains all z-plane (excluding z=0 or z=∞)
- If x(n) is right-sided, then ROC is located outside of the largest pole.
- If x(n) is left sided then ROC is located inside of the smallest pole.
- If the sequence x(n) is both-sided then the ROC has ring shape which is limited to inside and outside poles and there is no pole in ROC.
- ROC must be a connected area.

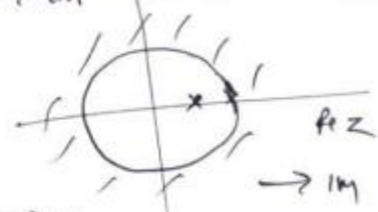
b.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=-\infty}^{\infty} \{7(1/3)^n u(n) - 6(1/2)^n u(z)\} z^{-n} \rightarrow 2m$$

$$= 7 \sum_{n=-0}^{\infty} (1/3)^n z^{-n} - 6 \sum_{n=0}^{\infty} (1/2)^n z^{-n} \rightarrow 1m$$

$$= 7 \cdot \frac{1}{1-\frac{z^{-1}}{3}} - 6 \times \frac{1}{1-\frac{z^{-1}}{2}} \rightarrow 1m$$

$$= \frac{z(z-3/2)}{(z-1/3)(z-1/2)} \qquad |z| > 1/2 \rightarrow m$$



$\rightarrow 1m$

c.

(i) $x(n) = \left(\frac{1}{2}\right)^n u(n) - 3^n u(-n-1)$

$$X(z) = \frac{z}{z-1/2} - \left[\frac{-z}{z-3}\right]$$

$$= \frac{z}{z-1/2} + \frac{z}{z-3}$$

ROC $|z| > 1/2$ & $|z| < 3$

ROC $1/2 < |z| < 3$ $\Big\}$ 02m

(ii) $x(n) = n\, a^n u(n-3)$

$x_1(n) = u(n)$

$X_1(z) = \frac{z}{z-1}$

$x_2(n) = u(n-3)$ $\Big\}$ 02

$X_2(z) = z^{-3} X_1(z)$

$x_3(n) = a^n u(n-3)$

$= a^n x_2(n)$

$X_3(z) = X_2(z/a)$ $\Big\}$ 02 $\rightarrow 0$

$x(n) = n\, x_3(n)$

$= -z \frac{d}{dz} X_3(z)$

$= \frac{a^3}{z(z-a)^2}$ $\Big\}$ 02

10a.

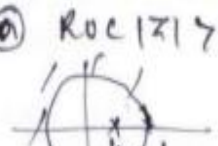
$$\frac{X(z)}{z} = \frac{1}{3(z-1/3)(z-1)} = \frac{K_1}{z-1/3} + \frac{K_2}{z-1}$$

$K_1 = -1/2$, $K_2 = 1/2 \rightarrow 1m$

$$\frac{X(z)}{z} = \frac{-1/2}{z-1/2} + \frac{1/2}{z-1}$$

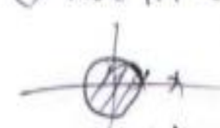
$$X(z) = -\frac{1}{2}\left(\frac{z}{z-1/3}\right) + \frac{1}{2}\left(\frac{z}{z-1}\right) \rightarrow 2m$$

(a) ROC $|z| > 1$



$x(n) = -\frac{1}{2}(1/3)^n u(n) + \frac{1}{2} u(n)$

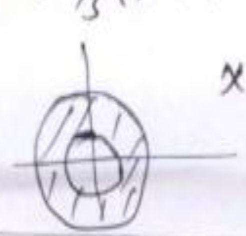
(b) ROC $|z| < 1/3 \rightarrow m$



$x(n) = \frac{1}{2}(1/3)^n u(-n-1) - \frac{1}{2} u(-n-1)$

$\rightarrow 1m$

(c) $1/3 < |z| < 1$



$x(n) = -\frac{1}{2}(1/3)^n u(n) - \frac{1}{2} u(-n-1)$

$\rightarrow 1m$

b.

$$y(n) - \frac{3}{2}y(n-1) + \frac{1}{2}y(n-2) = (1/4)^n u(n)$$

taking UZT

$$Y(z) - \frac{3}{2}[y(-1) + z^{-1}Y(z)] + \frac{1}{2}[y(-2) + y(-1)z^{-1} + z^{-2}Y(z)] = \frac{1}{1 - 1/4 z^{-1}}$$

$\longrightarrow$ 2M

$$Y(z) = \frac{2 - \frac{9}{4}z^{-1} + \frac{1}{2}z^{-2}}{(1 - 1/4 z^{-1})(1 - 1/2 z^{-1})(1 - z^{-1})} = \frac{A}{1 - \frac{1}{4}z^{-1}} + \frac{B}{1 - \frac{1}{2}z^{-1}} + \frac{C}{1 - z^{-1}}$$

$A = 1/3 \qquad B = 1 \qquad C = 2/3$ $\longrightarrow$ 4

$$Y(z) \cdot \frac{-1/3}{1 - 1/4 z^{-1}} + \frac{1}{1 - 1/2 z^{-1}} + \frac{2/3}{1 - z^{-1}}$$

$$y(n) = \frac{1}{3}\left(\frac{1}{4}\right)^n u(n) + (1/2)^n u(n) + \frac{2}{3}u(n)$$ $\longrightarrow$ 2M

c.

$$H(z) = \frac{1}{1 - 1/2 z^{-1}}$$ $\longrightarrow$ 1M

$$Y(z) = \frac{1/3}{1 - z^{-1}} + \frac{2/3}{1 + 1/2 z^{-1}}$$

$$= \frac{1 - 1/2 z^{-1}}{(1 - z^{-1})(1 + 1/2 z^{-1})}$$ $\longrightarrow$ 1M

$$X(z) = \frac{Y(z)}{H(z)} = \frac{(1 - 1/2 z^{-1})^2}{(1 - z^{-1})(1 + 1/2 z^{-1})}$$

$$= \frac{1 - z^{-1} + 1/4 z^{-2}}{1 - 1/2 z^{-1} - 1/2 z^{-2}}$$ $\longrightarrow$ 2M

$$X(z) = \frac{-1}{2} + \frac{-5/4 z^{-1} + 3/2}{(1 - z^{-1})(1 + 1/2 z^{-1})}$$

$$X(z) = \frac{-1}{2} + \frac{A}{(1 - z^{-1})} + \frac{B}{1 + 1/2 z^{-1}}$$

$$= \frac{-1}{2} + \frac{1/6}{1 - z^{-1}} + \frac{4/3}{1 + 1/2 z^{-1}}$$

$$x(n) = -\frac{1}{2}\delta(n) + \frac{1}{6}u(n) + \frac{4}{3}(-1/2)^n u(n)$$