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Internal Assessment Test II – August 2023

Sub:	Mathematics – II for CSE Stream					Sub Code:	BMATS201
Date:	08/08/2023	Duration:	90 mins	Max Marks:	50	Sem / Sec:	**II / I,J,K,L**

Answer any FIVE questions. Keep accuracy up to four decimal places for all questions.

	Question	MARKS	CO	RBT
1.	Using Taylor's series method Solve $\frac{dy}{dx} = x^2 - y$, y(0)=1 to get y(0.1) considering up to fourth degree term..	[10]	CO5	L3
2.	Use Runge-Kutta method to find y(0.2) for $\frac{dy}{dx} = \frac{y-x}{y+x}$,y(0) =1 taking h=0.2	[10]	CO5	L3
3.	Use Modified Euler's method to compute y(0.1) given $\frac{dy}{dx} = x + y$,y(0) =1 taking h=0.1	[10]	CO5	L3
4.	Use Milne's Predictor-corrector method to compute y(1.4) if given $\frac{dy}{dx} = x^2 + \frac{y}{2}$, from the data	[10]	CO5	L3

x	1	1.1	1.2	1.3
y	2	2.2156	2.4549	2.7514

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5. Evaluate $I = \int_1^4 e^{\left(\frac{1}{x}\right)} dx$, taking four ordinates using [5+5] CO4 L3
(1) Simpson's(3/8)th rule (2) Trapezoidal rule

6. Find an approximate root of the equation $x^3 - 3x + 4 = 0$ using the method of false position, correct to three decimal places which lie between -3 and -2. [10] CO4 L3

7. **Use Newton's divided difference formulae to fit the polynomial to the data** [10] CO4 L3

x	4	7	9	12
y=$f(x)$	-43	83	327	1053

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6. Find an approximate root of the equation $x^3 - 3x + 4 = 0$ using the method of false position, correct to three decimal places which lie between -3 and -2. [07] CO4 L3

7. **Use Newton's appropriate formulae to find f (24) and f (42) given that** [10] CO4 L3

x	20	25	30	35	40
y=$f(x)$	0.3420	0.4226	0.5000	0.5736	0.6428

Mathematics - II for CSE Stream - BMATS201

IAT 2

1\. Given $y' = x^2 - y$ with $x_0 = 0$, $y_0 = 1$

7

$$y' = x^2 - y \Rightarrow y'(x_0) = 0 - 1 = -1$$
$$y'' = 2x - y' \Rightarrow y''(x_0) = 2(0) + 1 = 1$$
$$y''' = 2 - y'' \Rightarrow y'''(x_0) = 2 - 1 = 1$$
$$y^{IV} = -y''' \Rightarrow y^{IV}(x_0) = -1$$

Taylor's series expansion is:

$$y(x) = y(x_0) + (x - x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \frac{(x-x_0)^4}{4!}y^{IV}(x_0)$$

$$y(x) = 1 - x + \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \cdots$$

$$y(0.1) = 1 + (0.1 - 0)(-1) + \frac{(0.1)^2}{2!}(1) + \frac{(0.1)^3}{3!}(1) + \frac{(0.1)^4}{4!}(-1)$$

$$= 0.9051$$

2\. By data $f(x,y) = \frac{y-x}{y+x}$, $x_0 = 0$, $y_0 = 1$, $h = 0.2$

$$k_1 = hf(x_0, y_0) = 0.2\left[\frac{1-0}{1+0}\right] = 0.2$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2\left[\frac{1.1 - 0.1}{1.1 + 0.1}\right] = 0.1667$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = 0.1662$$

$$k_4 = hf(x_0 + h, y_0 + k_3) = 0.1414$$

$$y_1 = y(x_1) = y_0 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$= 1.1676$$

3\. $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$f(x,y) = x + y$; $f(x_0, y_0) = x_0 + y_0 = 0 + 1 = 1$

$x_1 = x_0 + h = 0.1$, $y(x_1) = y_1 = ?$

From Euler's formula,

$$y_1^{(0)} = y_0 + h f(x_0, y_0) = 1 + 0.1\,(1) = 1.1$$

Modified Euler's formula $\Rightarrow$

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$
$$= 1.11$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$
$$= 1.1105$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 1 + 0.05\left[1 + 0.1 + 1.1105\right]$$
$$= 1.1105$$

$$\therefore y_1 = y(0.1) = 1.1105$$

4.

x	y	$y' = x^2 + \frac{y}{2}$
$x_0 = 1$	$y_0 = 2$	$y_0' = 2$
$x_1 = 1.1$	$y_1 = 2.2156$	$y_1' = 2.3178$
$x_2 = 1.2$	$y_2 = 2.4549$	$y_2' = 2.67245$
$x_3 = 1.3$	$y_3 = 2.7514$	$y_3' = 3.0657$
$x_4 = 1.4$	$y_4 = ?$	

$$y_4^{(P)} = y_0 + \frac{4h}{3}\left(2y_1' - y_2' + 2y_3'\right) = 3.0793$$

$$y_4' = x_4^2 + \frac{y_4^{(P)}}{2} = 3.49965$$

$$y_4^{(C)} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$
$$= 3.0794$$

$$y_4' = 3.4997$$

$$y_4^{(C)} = 3.0794$$

$$\therefore y_4 = y(1.4) = 3.0794$$

5. $n = 3$, $a = 1$, $b = 4$

$$h = \frac{b-a}{n} = \frac{4-1}{3} = 1 \quad \text{(i)} \quad y = e^{1/x}$$

x	1	2	3	4
y	2.7183	1.6487	1.3956	1.2840
	y_0	y_1	y_2	y_3

~~Substituting these fns~~

(1) Simpson's $3/8^{th}$ Rule:

$$I = \frac{3h}{8}\left[(y_0 + y_3) + 3(y_1 + y_2)\right]$$

$$= 4.9257$$

(2) Trapezoidal Rule:

$$I = \frac{h}{2}\left[(y_0 + y_3) + 2(y_1 + y_2)\right]$$

$$= 5.0455$$

6) $f(-3) = -14$, $f(-2) = 2$ Root lies in $(-2.2, -2.1)$

$f(-2.2) = -0.048 < 0$

$f(-2.1) = 1.039 > 0$

I Iteration: $a = -2.2$, $b = -2.1$, $f(a) = -0.048$, $f(b) = 1.039$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = -2.1956 \; ; \quad f(-2.1956) = 0.0026$$

II Iteration: $a = -2.2$, $b = -2.1956$

$f(a) = -0.048$, $f(b) = 0.0026$

$$x_2 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = -2.1957$$

$f(-2.1957) = 0.0014$

III iterⁿ: $a = -2.2$, $f(a) = -0.048$

$b = -2.1957$, $f(b) = 0.0014$

$x_3 = -2.1958$ $\quad f(x_3) = 0.0003$

IV itrⁿ $a = -2.2$, $f(a) = -0.048$

$b = -2.1958$, $f(b) = 0.0003$

$x_4 = -2.1958$

7)

x	y	I DD	II DD	III DD
4	-43			
		42		
7	83		16	
		122		1
9	327		24	
		242		
12	1053			

Above is the Divided Difference table for the given data.

Newton's Divided Difference Formula is given by

$$y(x) = y(x_0) + (x-x_0)\, f(x_0,x_1) + (x-x_0)(x-x_1)\, f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2)\, f(x_0,x_1,x_2,x_3)$$

$$= -43 + (x-4)42 + (x-4)(x-7)16 + (x-4)(x-7)(x-9)1$$

$$= x^3 - 4x^2 - 7x - 15$$

This is the required polynomial.