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Internal Assessment Test III – Sept 2023

Sub:	Mathematics – II for CSE Stream				Sub Code:	BMATS201		
Date:	04/09/2023	Duration:	90 mins	Max Marks:	50	Sem / Sec:	II / I, J, K, L	OBE

Answer any FIVE questions.

		MARKS	CO	RBT
1.	Find the directional derivative of $\varphi = x^2yz + 4xz^2$ at (1, -2,-1) in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$.	[10]	CO2	L3
2.	If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$, $find\ div\ \vec{F}\ and\ curl\ \vec{F}$. Is $\vec{F}$ solenoidal or irrotational?	[10]	CO2	L3
3.	Find the angle between the surfaces $x \log z = y^2 - 1$ and $x^2y + z = 2$ at the point $(1,1,1)$.	[10]	CO2	L3
4.	Show that the cylindrical coordinate system is orthogonal.	[10]	CO2	L2

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4.	Show that the cylindrical coordinate system is orthogonal.	[10]	CO2	L2

5.	Prove that the subset $W = \{(x, y, z)\ /\ x - 3y + 4z = 0\}$ of the vector space R^3 is a subspace of R^3.	[10]	CO3	L3
6.	Determine whether the matrix $\begin{pmatrix} -1 & 7 \\ 8 & -1 \end{pmatrix}$ is a linear combination of $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 2 & -3 \\ 0 & 2 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$ in the vector space $M_{22}\ of\ 2 \times 2$ matrices.	[10]	CO3	L3
7.	Prove that the transformation $T: R^2 \rightarrow R^2$ defined by $T(x, y) = (3x, x + y)$is linear. Find the images of the vectors $(1,3)\ and\ (-1,2)$ under this transformation.	[10]	CO3	L3

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DD of $\phi = x^2yz + 4xz^2$ at $(1,-2,-1)$ in $2\hat{i} - \hat{j} - 2\hat{k}$

$$\nabla\phi = \frac{\partial(x^2yz+4xz^2)}{\partial x}\hat{i} + \frac{\partial(x^2yz+4xz^2)}{\partial y}\hat{j} + \frac{\partial(x^2yz+4xz^2)}{\partial z}\hat{k}$$

$$\nabla\phi = (2xyz + 4z^2)\hat{i} + (x^2z)\hat{j} + (x^2y + 8xz)\hat{k}$$

$$\nabla\phi_{(1,-2,-1)} = \left[(2)(1)(-2)(-1) + 4(-1)^2\ \hat{i}\right] + \left[(1^2)(-1)\hat{j}\right] + \left[-2 - 8\right]\hat{k}$$

$$= (4+4)\hat{i} - \hat{j} - 10\hat{k}$$

$$= 8\hat{i} - \hat{j} - 10\hat{k}$$

$$DD = \nabla\phi \cdot \hat{d}$$

$$= \nabla\phi \cdot \frac{\vec{d}}{|\vec{d}|}$$

$$= \frac{(8\hat{i} - \hat{j} - 10\hat{k})(2\hat{i} - \hat{j} - 2\hat{k})}{\sqrt{4+1+4}}$$

$$= \frac{16+1+20}{\sqrt{9}} = \frac{37}{\sqrt{9}} = \frac{37}{3}$$

2) $\vec{F} = \nabla(x^3+y^3+z^3-3xyz)$

div $\vec{F}$? curl $\vec{F}$

$\text{div}\,\vec{F} = \nabla\phi \cdot \vec{F}$

$\nabla\phi = (3x^2-3xyz)\,\hat{i} + (3y^2-3xz)\,\hat{j} + (3z^2-3xy)\,\hat{k}$

$\text{div}\,\vec{F} = \frac{\partial(3x^2-3yz)}{\partial x} + \frac{\partial(3y^2-3xz)}{\partial y} + \frac{\partial(3z^2-3xy)}{\partial z}$

$\text{div}\,\vec{F} = 6x + 6y + 6z$

$\because$ div $\vec{F} \neq \vec{0}$

$\therefore$ $\vec{F}$ is not solenoidal.

$\text{curl}\,\vec{F} = \nabla\phi \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2-3yz & 3y^2-3xz & 3z^2-3xy \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(3z^2-3xy) - \frac{\partial}{\partial z}(3y^2-3xz)\right] - \hat{j}\left[\frac{\partial}{\partial x}(3z^2-3xy) - \frac{\partial}{\partial z}(3x^2-3yz)\right]$$

$$+ \hat{k}\left[\frac{\partial}{\partial x}(3y^2-3xz) - \frac{\partial}{\partial y}(3x^2-3yz)\right]$$

$= \hat{i}\,[-3x+3x] - \hat{j}\,[-3y+3y] + \hat{k}\,[-3z+3z]$

$= 0\hat{i} + 0\hat{j} + 0\hat{k}$

$\therefore$ curl $\vec{F} = \vec{0}$

$\therefore$ $\vec{F}$ is Irrotational

3) Find angle b/w $x \log z = y^2 - 1$

$x^2 y + z = 2$ at $(1,1,1)$

$\phi_1 = x \log z - y^2 + 1$

$\nabla\phi_1 = (\log z)\hat{i} - 2y\,\hat{j} + \frac{x}{z}\hat{k}$

$\nabla\phi_{1(1,1,1)} = 0\hat{i} - 2\hat{j} + 1\hat{k}$

$\phi_2 = x^2 y + z - 2$

$\nabla\phi_2 = 2xy\,\hat{i} + x^2\hat{j} + 1\hat{k}$

$\nabla\phi_2 = 2\hat{i} + 1\hat{j} + 1\hat{k}$

$\cos\theta = \dfrac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1| \cdot |\nabla\phi_2|}$

$\nabla\phi_1 \cdot \nabla\phi_2 = (0\hat{i} - 2\hat{j} + 1\hat{k})(2\hat{i} + 1\hat{j} + 1\hat{k})$

$= 0 - 2 + 1 = -1$

$|\nabla\phi_1| |\nabla\phi_2| = (\sqrt{4+1})(\sqrt{4+1+1}$

$= \sqrt{30}$

$\cos\theta = \dfrac{-1}{\sqrt{30}}$

$\theta = \cos^{-1}\left(\dfrac{-1}{\sqrt{30}}\right)$

CMR

4) cylindrical coordinates is orthogonal.

sol) Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

In cylindrical system,

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$z = z$

and scale factors are $h_1 = 1$, $h_2 = \rho$, $h_3 = 1$

$\therefore \vec{r} = \rho\cos\phi\,\hat{i} + \rho\sin\phi\,\hat{j} + z\hat{k}$

$\hat{e}_\rho = \frac{1}{h_1}(\cos\phi\,\hat{i} + \sin\phi\,\hat{j}) = \frac{1}{1}(\cos\phi\,\hat{i} + \sin\phi\,\hat{j} + 0\hat{k}) \rightarrow (1)$

$\hat{e}_\phi = \frac{\partial(\vec{r})}{\partial\phi} = \frac{1}{\rho}(-\rho\sin\phi\,\hat{i} + \rho\cos\phi\,\hat{j})$

$= \frac{\rho}{\rho}(-\sin\phi\,\hat{i} + \cos\phi\,\hat{j} + 0\hat{k})$

$\hat{e}_\phi = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j} \rightarrow (2)$

$\hat{e}_z = \frac{\partial\vec{r}}{\partial z} = 0\hat{i} + 0\hat{j} + 1\hat{k} \rightarrow (3)$

from (1), (2), (3),

$\hat{e}_\rho \cdot \hat{e}_\phi = -\cos\phi\sin\phi + \sin\phi\cos\phi + 0 = 0$

$\hat{e}_\rho \cdot \hat{e}_\phi = 0$

$\hat{e}_\phi \cdot \hat{e}_z = 0 + 0 + 0 = 0$

$\hat{e}_z \cdot \hat{e}_\rho = 0 + 0 + 0 = 0$

since, $\hat{e}_\rho \cdot \hat{e}_\phi = 0$

$\hat{e}_\phi \cdot \hat{e}_z = 0$

$\hat{e}_z \cdot \hat{e}_\rho = 0$

The cylindrical coordinate system is orthogonal

5) $W = \{(x, y, z) / x - 3y + 4z = 0\}$

Let u, v be a point in W, such that.

$\alpha, \beta \in W$

$\therefore u = (x_1, y_1) = (x_1, -3y_1 + 4z_1)$

$v = (x_2, y_2) = (x_2, -3y_2 + 4z_2)$

$x - 3y + 4z = 0 \rightarrow (1).$

$T(\alpha u + \beta v)$

$= \{(\alpha x_1 - 3\alpha y_1 + \alpha 4z_1) + (\beta x_2 - 3\beta y_2 + 4\beta z_2)\}$

$= \{\alpha(x_1 - 3y_1 + 4z_1) + \beta(x_2 - 3y_2 + 4z_2)\}$

From (1)

$= \alpha(0) + \beta(0)$

$= 0 \in R$

$\therefore$ subset $W = \{(x, y, z) / x - 3y + 4z = 0\}$ of vector space R^3 is a subspace of R^3

6) Let $A = \begin{bmatrix} -1 & 7 \\ 8 & -1 \end{bmatrix}$ $B = a\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$C = b\begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}$ $D = c\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$

~~multipy a to A~~
~~b to B~~
~~c to C~~
~~d to D~~

$A = B + C + D$

$\therefore \begin{bmatrix} -1 & 7 \\ 8 & -1 \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + \left(b\begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}\right) + c\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$

$\begin{bmatrix} -1 & 7 \\ 8 & -1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 2a & a \end{bmatrix} + \begin{bmatrix} 2b & -3b \\ 0 & 2b \end{bmatrix} + \begin{bmatrix} 0 & c \\ 2c & 0 \end{bmatrix}$

by comparing LHS & RHS

$-1 = a + 2b \rightarrow 1$

$7 = -3b + c \rightarrow 2$

$8 = 2a + 2c \rightarrow 3$

$-1 = a + 2b \rightarrow 4$

$3/2 \Rightarrow 4 = a + c \rightarrow 5$

$a = -1 - 2b$

$c = 7 + 3b$

subs, a, c in 5

$4 = -1 - 2b + 7 + 3b$

$4 = 6 - b \Rightarrow \boxed{b = -2}$

~~b = 4 = -6~~

~~$\boxed{b = -2}$~~

$-1 = a + 2b$

$-1 = a + (-4)$

$\boxed{a = 3}$

$4 = a + c$

$4 = 3 + c$

$\boxed{c = 1}$

$\therefore$ The matrix $\begin{bmatrix} -1 & 7 \\ 8 & -1 \end{bmatrix}$ is a linear combn of B, C, D in vector space M_{22} of 2×2 matri

7) $T(x,y) = (3x, x+y)$ is linear.

Let, u, v be two points and $\alpha, \beta \in R$

$u = x_1, y_1$

$v = x_2, y_2$

~~T(αu + αβ) =~~

$T(u+v) = \{(x_1, y_1) + (x_2, y_2)\}$

$= \{(3x_1, x_1+y_1) + (3x_2, x_2+y_2)\}$

$T(\alpha u + \beta v) = \{(3\alpha x_1, \alpha x_1 + \alpha y_1) + (3\beta x_2, \beta x_2 + \beta y_2)\}$

$= \{\alpha(3x_1, x_1+y_1) + \beta(3x_2, x_2+y_2)\}$

$= T(u) + T(v)$

$\therefore$ $T: R^2 \to R^2$ defined by $T(x,y) = (3x, x+y)$ is linear.

Images of vectors:-

$(1,3) \Rightarrow T(1,3) = (3(1), (1+3))$

$= (3, 4)$

$(-1,2) \Rightarrow T(-1,2) = (3(-1), (-1+2))$

$= (-3, 1)$