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Internal Assessment Test I July 2023

Sub:	**Mathematical Foundations for Computing, Proability and Statistics**	Code:	21MATCS41

Date:	0 /07/2023	Duration:	90 mins	Max Marks:	50	Sem:	III	Branch:	

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE CO	OBE RBT
1	The following table gives the stopping distance y in meters of a motor bike moving at a speed of x Kms/hour. Find the correlation coefficient and the regression lines. Find the maximum speed at which the motor bike could be driven if the stopping distance is not to exceed 5 meters.	[8]	CO4	L3
2	Using the method of least squares, fit a linear relation of the form P = a+bW to the following data. Estimate P when W is 150.	[7]	CO4	L3
3	In a laboratory record of correlation data, the following results are obtained. Variance of x is 9. The regression equations are $4x-5y+33=0, 20x-9y=107$. Find the mean values of x and y, standard deviation of y and correlation coefficient.	[7]	CO4	L3

Table for Question 1:

x	16	24	32	40	48	56
y	0.39	0.75	1.23	1.91	2.77	3.81

Table for Question 2:

W	50	70	100	120
P	12	15	21	25

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1	The following table gives the stopping distance y in meters of a motor bike moving at a speed of x Kms/hour when the breaks are applied. Find the correlation coefficient and the regression lines. Estimate the maximum speed at which the motor bike could be driven is the stopping distance is not to exceed 5 meters.	[8]	CO4	L3
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No.	Question	Marks	CO	Level
4	Find the rank correlation coefficient between two sets of marks.	[7]	CO4	L3

Subject 1	80	72	45	55	56	58	69	65	76	85
Subject 2	84	70	56	50	48	60	64	65	82	81

No.	Question	Marks	CO	Level
5	Fit a parabola of the form $y = a + bx + cx^2$ to the following data	[7]	CO4	L3

X	0	`1	2	3	4	5
P(X)	1	3	7	13	21	31

No.	Question	Marks	CO	Level
6	Fit a curve of the form $y = ax^b$ to the following data. Use natural logarithm	[7]	CO4	L3

x	1	2	3	4	5
Y	0.5	2	4.5	8	12.5

No.	Question	Marks	CO	Level
7	A fair coin is tossed 3 times. Let X denote the number of heads showing up. Find the distribution of X. Find the mean, variance and standard deviation.	[7]	CO3	L3
8	The pdf of a variate X is given by the following table. For what value of K, does this represent a valid probability distribution? Find i)P(X < 4) ii)P(X ≥5) iii) P(3<X≤6)	[7]	CO3	L3

X	0	`1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13K

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P(X)	K	3K	5K	7K	9K	11K	13K

Q1 $n = 6$ $\quad \bar{x} = \frac{1}{n} \sum x_i = \frac{216}{6} = 36$

$\bar{y} = \frac{1}{n} \sum y_i = \frac{10.86}{6} = 1.81$

x	$X = x - \bar{x}$	X^2	y	$Y = y - \bar{y}$	Y^2	XY
16	-20	400	0.39	-1.42	2.0164	28.4
24	-12	144	0.75	-1.06	1.1236	12.72
32	-4	16	1.23	-0.58	0.3364	2.32
40	4	16	1.91	0.1	0.001	0.4
48	12	144	2.77	0.96	0.9216	11.5
56	20	400	3.81	2.0	4	40
		1120			8.399	95.3

$$r = \frac{\sum XY}{\sqrt{\sum X^2 \sum Y^2}} = \frac{95.36}{\sqrt{(1120)(8.399)}} = \frac{95.36}{96.989} = 0.983$$

$$\sigma_x^2 = \frac{1}{n} \sum X^2 = \frac{1120}{6} = 186.667$$

$$\sigma_y^2 = \frac{1}{n} \sum Y^2 = \frac{8.399}{6} = 1.3998$$

$\sigma_x = 13.663 \qquad \sigma_y = 1.1831$

$$b_{yx} = r \frac{\sigma_y}{\sigma_x} = 0.983 \frac{1.1831}{13.663} = 0.0851$$

$$b_{xy} = r\frac{\sigma_x}{\sigma_y} = 0.983\left(\frac{13.663}{1.1831}\right) = 11.352$$

Regression line of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 1.81 = 0.0851(x - 36)$$

$$\boxed{y = 0.0851x - 1.2536}$$

Regression line of x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 36 = 11.352(y - 1.81)$$

$$x = 11.352y + 15.453$$

For $y = 5$ $\quad x = 11.352(5) + 15.453 = 72.213$

Speed should not exceed 72 kms/hr

Q2 $n = 4$

x	y	x^2	xy
50	12	2500	600
70	15	4900	1050
100	21	10000	2100
120	25	14400	3000
340	73	31800	6750

NE's

$$na + b\sum x = \sum y$$

$$a\sum x + b\sum x^2 = \sum xy$$

$$4a + 340b = 73$$

$$340a + 31800b = 6750$$

$a \approx 2.2785 \qquad b \approx 0.1879$

$P = a + bW \quad = 2.2785 + 0.1879\,W$

at $W = 150$, $\quad P = 2.2785 + (0.1879)(150)$

$= 30.5$

Q3 Regression lines pass thro' $(\bar{x}\ \bar{y})$

$4\bar{x} - 5\bar{y} = -33$

$20\bar{x} - 9\bar{y} = 107$

Solving $\quad \bar{x} = 18 \quad \bar{y} = 17$

Regression eqns $\quad 5y = 4x + 33 \ ; \quad 20x = 9y + 107$

$y = \frac{4}{5}x + \frac{33}{5} \ ; \quad x = \frac{9}{20}y + \frac{107}{20}$

$b_{xy} = \frac{9}{20}$

$b_{yx} = \frac{4}{5}$

$r = \sqrt{b_{yx}\,b_{xy}} = \sqrt{\left(\frac{4}{5}\right)\left(\frac{9}{20}\right)} = \frac{3}{5} = 0.6$

$b_{yx} = r\frac{\sigma_y}{\sigma_x} \quad \Rightarrow \quad \frac{4}{5} = 0.6\,\frac{\sigma_y}{3}$

$5(0.6)\sigma_y = 12 \qquad 3\sigma_y = 12 \Rightarrow \sigma_y = 4$

Q4

x	80	72	45	55	56	58	69	65	76	85
rank$_x$	2	4	10	9	8	7	5	6	3	1
y	84	70	56	50	48	60	64	65	82	81
rank$_y$	1	4	8	9	10	7	6	5	2	3

r	2	4	10	9	8	7	5	6	3	1
s	1	4	8	9	10	7	6	5	2	3
$r-s$	1	0	2	0	-2	0	-1	1	1	-2
$(r-s)^2$	1	0	4	0	4	0	1	1	1	4

$$\Sigma (r-s)^2 = 16$$

$$\rho = 1 - \frac{6\,\Sigma d^2}{n(n^2-1)} = 1 - \frac{6(16)}{10(10^2-1)} \approx 0.903$$

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	3	1	1	1	3	3
2	7	4	8	16	14	28
3	13	9	27	81	39	117
4	21	16	64	256	84	336
5	31	25	125	625	155	775
15	76	55	225	979	295	1259

NE's

$$na + b\Sigma x + c\Sigma x^2 = \Sigma y$$

$$a\Sigma x + b\Sigma x^2 + c\Sigma x^3 = \Sigma xy$$

$$a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 = \Sigma x^2 y$$

$$6a + 15b + 55c = 76$$

$$15a + 55b + 225c = 295$$

$$55a + 225b + 979c = 1259$$

$a = \cancel{24.568}$ 1 $\quad b = \cancel{28.034}$ 1 $\quad c = \cancel{6.2\ldots}$ 1

Parabola $\quad y = a + bx + cx^2 = 1 + x + x^2$

$\cancel{= 24.568 - 28.034x + 6.2\ldots x^2}$

Q6

x	y	$X = \log x$	$Y = \log y$	X^2	XY
1	0.5	0	-0.6931	0	0
2	2	0.6931	0.6931	0.4804	0.4804
3	4.5	1.0986	1.5041	1.2069	1.6524
4	8	1.3863	2.0794	1.9232	2.8827
5	12.5	1.6094	2.5254	2.5903	4.0649
		4.7874	6.1092	6.2008	9.0803

NE's

$$nA + b\Sigma X = \Sigma Y$$

$$A\Sigma X + b\Sigma X^2 = \Sigma XY$$

$$5A + 4.7874\,b = 6.1092$$

$$4.7874A + 6.2008\,b = 9.0803$$

Solving $\quad A \sim \bar{0}.6913 \quad b \sim 1.998$

$a = e^A \sim 0.501 \qquad y = ax^b$

$$y = 0.501\,x^{1.998}$$

Q7 $\quad S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Let X denote the no. of heads showing up.

$P(X=0) = \frac{1}{8}$ $\quad P(X=1) = \frac{3}{8}$ $\quad P(X=2) = \frac{3}{8}$

$P(X=3) = \frac{1}{8}$

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean $\mu = \sum x P(x) = (0 \times \frac{1}{8}) + (1 \times \frac{3}{8}) + (2 \times \frac{3}{8}) + (3 \times \frac{1}{8}) = \underline{1.5}$

Variance $\sigma^2 = \sum (x-\mu)^2 P(x)$

$= (0-1.5)^2 \frac{1}{8} + (1-1.5)^2 \frac{3}{8} + (2-1.5)^2 \frac{3}{8} + (3-1.5)^2 \frac{1}{8}$

$= 0.75$

$\sigma = 0.866$

8) We K.T $\sum P(x) = 1$

$K + 3K + 5K + 7K + 9K + 11K + 13K = 1$

$49K = 1 \Rightarrow K = 1/49$

$P(X<4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= K + 3K + 5K + 7K = 16K = \frac{16}{49}$

$P(X \geq 5) = P(X=5) + P(X=6) = 11K + 13K = \frac{24}{49}$

$P(3 < X \leq 6) = P(4) + P(5) + P(6)$

$= 9K + 11K + 13K = 33K = \frac{33}{49}$