CMR
INSTITUTE OF
TECHNOLOGY

USN										

Internal Assessment Test - III

Sub:	**Control Systems**							Code:	**18EE61**
Date:	05.07.2023 8.30 – 10AM	Duration:	90 mins	Max Marks:	50	Sem:	VI	Branch:	EEE

Answer Any FIVE FULL Questions

		Marks	OBE CO	OBE RBT
1	For the closed loop transfer function when the input is unit step $\frac{C(s)}{R(s)} = \frac{16}{s^2+4s+16}$. Determine undamped natural frequency, damping ratio, Maximum overshoot, Peak time and settling time. Solution: $s^2+2\delta\omega_n s+\omega_n^2 \Leftrightarrow s^2+4s+16$ $\omega_n^2=16$ $\quad$ $2\delta\omega_n=4$ $\omega_n=4$ rad/sec $\quad$ $\delta=0.5$ ∴ undamped natural freq, $\omega_n=4$ rad/sec Damping ratio, $\delta=0.5$ Maximum overshoot, $M_p = e^{-\delta\pi/\sqrt{1-\delta^2}} \times 100$ $=16.3\%$	10	CO3	L3
2	A unity feedback system is characterized by open-loop transfer function $G(s)=\frac{10}{s^2(1+0.4s)(1+0.3s)}$. Determine steady state errors for unit step, unit ramp and unit parabola Solution: Unit step: $K_P = \lim_{s\to 0} G(s) = \infty$, $e_{ss} = \frac{1}{1+K_P} = 0$ Unit Ramp $K_v = \lim_{s\to 0} sG(s) = \infty$, $e_{ss} = \frac{1}{K_v} = 0$	10	CO3	L3

	Unit Parabola $K_a = \lim_{s\to 0} s^2 G(s) = 10, \; e_{ss} = \frac{1}{K_a} = \frac{1}{10}$			
3	Determine the range of K for stability of unity feedback system whose open loop transfer function is $G(s) = \frac{K}{s(s+1)(s+2)}$ using Routh Hurwitz criteria Solution: Solution closed loop T.F $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$ characteristic equation is, $1+G(s)H(s) = 1 + \frac{K}{s(s+1)(s+2)} = 0.$ $s(s+1)(s+2) + K = 0.$ $s^3 + 3s^2 + 2s + K = 0.$ Routh table, s^3 | 1 2 s^2 | 3 K s^1 | $\frac{6-K}{3}$ s^0 | K For stable system, s^0 row $\Rightarrow K > 0.$ s^1 row $\Rightarrow \frac{6-K}{3} > 0 \Rightarrow K < 6$ $\therefore$ Range of K for stable system is $0 < K < 6.$	10	CO3	L3
4	Sketch the root locus of the system whose open loop transfer function is $G(s) = \frac{K}{s(s+2)(s+4)}$ Solution:	10	CO4	L3

Step:-1

Starting points are $s=0, s=-2, s=-4$.

Ending points are $s=\infty, \infty, \infty$.

Step:-2.

Number of branches $= 3$.

Step:-3 Symmetrical.

Step:-4.

Angle of Asymptotes $\phi_a = \frac{(2q+1)180}{n-m}$.

$n=3, m=0, n-m-1 = 3-0-1 = 2$.

If $q=0$, $\phi_a = 60°$.

$q=1$, $\phi_a = 180°$.

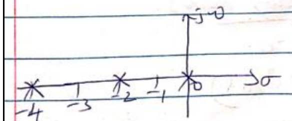
$q=2$, $\phi_a = -60°$.

Intersection of Asymptote

$\sigma_a = \frac{(0-2-4)-0}{3-0}$

$= -\frac{6}{3} = -2$.

Step:-5. root locus of real axis.



$-\infty$ to $-4 \Rightarrow$ odd $\Rightarrow$ Yes.

-4 to $-2 \Rightarrow$ Even $\Rightarrow$ NO.

-2 to $0 \Rightarrow$ odd $\Rightarrow$ Yes.

0 to $\infty \Rightarrow$ Even $\Rightarrow$ NO.

Step:-6. No complex pole & complex zero.

$\therefore$ No Angle of Departure/Arrival.

Step:-7

$$\frac{C(s)}{R(s)} = \frac{K}{s(s+2)(s+4)+K}$$

$s(s+2)(s+4) + K = 0$.

$s^3 + 6s^2 + 8s + K = 0$.

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	
s^0	K	

$s^0 \Rightarrow K > 0$.

$s^1 \Rightarrow K < 48$.

For stable,

$0 < K < 48$.

$K = 48$, $6s^2 + K = 0$.

$6s^2 + 48 = 0$.

$s^2 + 8 = 0$.

$s^2 = -8$.

$s = \pm 2.8j$

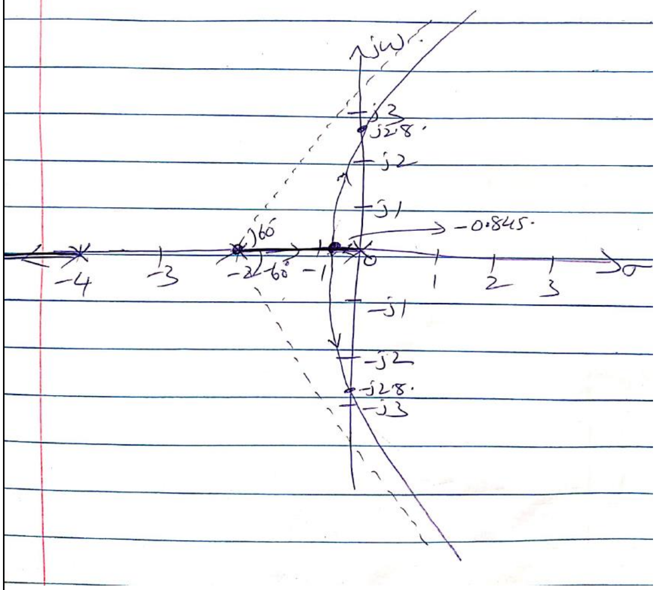
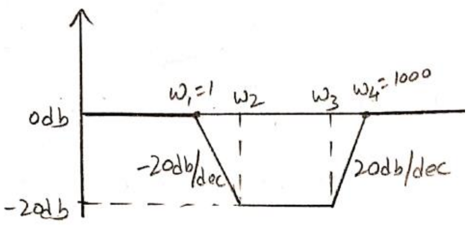
Step:-8. Breakaway points.

$s^3 + 6s^2 + 8s + K = 0$.

$K = -(s^3 + 6s^2 + 8s)$

$\frac{dK}{ds} = -(3s^2 + 12s + 8) = 0$.

$s = -0.845, -3.154$.

	$S = -0.845$ is valid breakaway point. $S = -3.154$ is invalid breakaway point. sketch root locus.  			
5	Determine the transfer function from the given bode plot.  Solution: corner frequencies, $\omega_{c1} = 1 \Rightarrow \frac{1}{T_1} = 1 \Rightarrow T_1 = 1.$ $\omega_{c2} = 10 \Rightarrow \frac{1}{T_2} = 10 \Rightarrow T_2 = 0.1$ $\omega_{c3} = 100 \Rightarrow \frac{1}{T_3} = 100 \Rightarrow T_3 = 0.01$ $\omega_{c4} = 1000 \Rightarrow \frac{1}{T_4} = 1000 \Rightarrow T_4 = 0.001.$ $G(s) = \frac{K(1+0.1s)(1+0.01s)}{(1+s)(1+0.001s)}$ $20\log K = 0.$ $K = 1.$ $\therefore$ Transfer function, $G(s) = \frac{(1+0.1s)(1+0.01s)}{(1+s)(1+0.001s)}$	10	CO4	L3
6	Determine the frequency domain specifications for the unity feedback system $G(s) = \frac{225}{s(s+6)}$ Solution:	10	CO4	L3

$$\therefore \frac{C(s)}{R(s)} = \frac{225}{s^2+6s+225} \longrightarrow ①$$

* By Comparing Equation ① with $\frac{W_n^2}{s^2+2\xi W_n s+W_n^2}$, we have

$$W_n^2 = 225$$
$$W_n = \sqrt{225}$$
$$\boxed{W_n = 15 \text{ rad/sec}}$$

$$2\xi W_n = 6$$
$$\xi = \frac{6}{2W_n} = \frac{6}{2 \times 15}$$
$$\boxed{\xi = 0.2}$$

→ Resonant Peak.

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$
$$= \frac{1}{2 * 0.2 * \sqrt{1-(0.2)^2}}$$
$$\boxed{M_r = 2.60}$$

→ Resonant frequency

$$W_r = W_n\sqrt{1-2\xi^2}$$
$$= 15 * \sqrt{1-2*(0.2)^2}$$
$$\boxed{W_r = 14.39 \text{ rad/sec}}$$

→ Bandwidth

$$\omega_b = W_n\sqrt{(1-2\xi^2)+\sqrt{2-4\xi^2+4\xi^4}}$$
$$= 15\sqrt{(1-2(0.2)^2)+\sqrt{2-4(0.2)^2+4(0.2)^4}}$$
$$= 15\sqrt{(1-2(0.2)^2)+1.3588}$$
$$\boxed{\omega_b = 22.656 \text{ rad/sec.}}$$