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INTERNAL ASSESSMENT TEST – I

Sub:	DIGITAL SIGNAL PROCESSING							Code:	21EC42
Date:	04 / 11 / 2022	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	ECE

Answer any 5 full questions

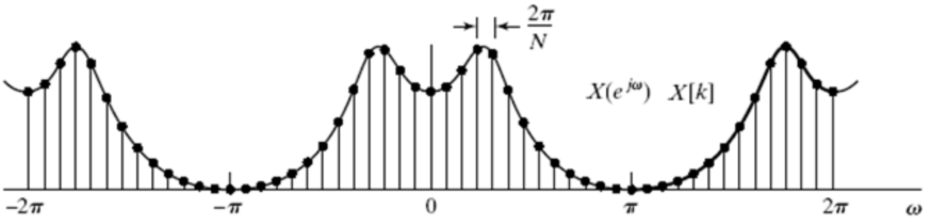
		Marks	**CO**	**RBT**
1	Why is it necessary to perform frequency domain sampling? With a neat diagram, show that sampling of DTFT results in N-point DFT.	[10]	CO2	L3
2	Compute the 4-point DFT of the sequence $x[n] = [1,1,1,1]$. Plot the magnitude spectrum and the phase spectrum.	[10]	CO2	L3
3(a)	Compute the 4-point DFT of the sequence $x[n] = [4,3,2,1]$. Plot the magnitude spectrum and the phase spectrum.	[06]	CO2	L3
3(b)	Compute the IDFT of $X[k] = [7.5, 1.5 - j0.8666, 1.5 + j0.866]$ using matrix method.	[04]	CO2	L3
4(a)	The first 5 samples of 8-point DFT of a real 6-point sequence are as follows $X[k] = [0.25, 0.125 - j0.3018, 0,0.125 - j0.518, 0]$. Determine the remaining samples of $X[k]$. Estimate the value of $x[0]$.	[6]	CO2	L3
4(b)	Compute the DFT of the sequence $x(n) = e^{j\frac{2\pi}{N}ln}, 0 \leq n \leq N-1$	[4]	CO2	L3
5	Compute the DFT of the sequence $x(n) = a^n, 0 \leq n \leq N-1$, hence evaluate the DFT of $x(n) = 0.4^n, 0 \leq n \leq 3$	[10]	CO2	L3
6	Two length-4 sequences are defined as below $x(n) = \cos\left(\frac{\pi n}{2}\right), n = 0,1,2,3\ , h(n) = 2n\ for\ n = 0,1,2,3$ Compute the circular convolution using any two methods	[10]	CO1	L3
7	Sate and prove that the multiplication of two DFTs is equivalent to circular convolution in time domain.	[10]	CO1	L1

Solution.

Q1. Frequency analysis of discrete-time signals is usually and most conveniently performed on a digital signal processor which may be a general-purpose digital computer or specially designed digital hardware. To perform frequency analysis on a discrete-time signal *x*(n) we convert that time-domain sequence into an equivalent frequency-domain representation. We know that such a representation is given by the Fourier transform *X(ω)* of the sequence *x*(n). However *X(ω)* is a continuous function of frequency and therefore it is not a computationally convenient representation of the sequence *x*(n)**.** So we consider the representation oi a sequence *x*(n) by samples of its spectrum *X(ω).*

The FT of DT aperiodic signal is represented by $X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$

- Suppose that we sample X(ω) periodically in frequency domain at a spacing of δω between two successive samples .



- Since X(ω) is periodic with period 2π , only samples in period 0 to 2π are necessary.Replacing ω by $\omega_k = \frac{2\pi k}{N}$
- $X(\omega_k) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\frac{2\pi k}{N}n}$
- $X\left(\frac{2\pi k}{N}\right) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\frac{2\pi k}{N}n}$
- The summation can be divided in to infinite number of summations with N terms in each summation
- $X\left(\frac{2\pi k}{N}\right) = \cdots .. + \sum_{n=-N}^{-1} x(n)\, e^{-j\frac{2\pi k}{N}n} + \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi k}{N}n} + \sum_{n=N}^{2N-1} x(n)\, e^{-j\frac{2\pi k}{N}n}$+..........
- $X\left(\frac{2\pi k}{N}\right) = \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n)\, e^{-j\frac{2\pi k}{N}n}$ Replace *n* by *n-lN*
- $X\left(\frac{2\pi k}{N}\right) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN)\, e^{-j\frac{2\pi k}{N}(n-lN)}$
- $X\left(\frac{2\pi k}{N}\right) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN)\, e^{-j\frac{2\pi k}{N}n} \quad \because e^{j\frac{2\pi k}{N}lN} = 1$
- Interchanging the order of summation
- $\because X\left(\frac{2\pi k}{N}\right) = X(k) = \sum_{n=0}^{N-1} \sum_{l=-\infty}^{\infty} x(n-lN)\, e^{-j\frac{2\pi k}{N}n}$
- Let us define $x_p(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$
- $\therefore X(k) = \sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi k}{N}n}$
- Where $x_p(n)$ is periodic extension of *x*(n) for every N samples for finite duration length sequence *x*(n) with length L<N.
- Then $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi k}{N}n}$ computes the DFT of sequence *x*(n). The expression obtained by sampling the X(ω) is called as discrete Fourier transform (DFT)
- $x_p(n)$ can be expanded using FS as $x_p(n) = \sum_{k=0}^{N-1} c_k\, e^{j\frac{2\pi k}{N}n}$
- With FS coefficients $c_k = \frac{1}{N}\sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi k}{N}n} = \frac{1}{N} X\left(\frac{2\pi k}{N}\right)$
- Now substituting for c_k in expression of $x_p(n)$. $x_p(n) = \sum_{k=0}^{N-1} \frac{1}{N} X\left(\frac{2\pi k}{N}\right) e^{j\frac{2\pi k}{N}n}$
- $x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} X\left(\frac{2\pi k}{N}\right) e^{j\frac{2\pi k}{N}n}$ Since $x_p(n)$ is periodic extension of *x*(n). *x*(n) can be recovered from $x_p(n)$ obtained by above expression if there is no aliasing (L<N) in the time domain.
- Where $x(n)=x_p(n)$ for 0≤n≤N-1
- $x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi k}{N}n}$ This equation computes IDFT of *X*(k).

Q2. Given x(n)=[1,1,1,1], Note to compute 8 point DFT four zeros are padded at the end of given the sequence so x(n)=[4,3,2,1,0,0,0,0]

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk} \text{ where } W_N = e^{-j\frac{2\pi}{N}}$$

For k=0, $X(k) = \sum_{n=0}^{8-1} x(n)\, W_8^0 = \sum_{n=0}^{7} x(n)$= x(0)+x(1)+x(2)+x(3) =4+3+2+1 =10 as samples values from x(4) to x(7) are 0. $X(0) =$ 4.

For k=1, $X(1) = \sum_{n=0}^{7} x(n)\, W_8^{n}$= x(0) W_8^0+x(1) W_8^1+x(2) W_8^2+x(3) W_8^3 samples values from x(4) to x(7) are 0

$W_8^1 = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$, $W_8^2 = -j$, $W_8^3 = \frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$, $W_8^4 = -1$, $W_8^5 = W_8^{*3}$, $W_8^6 = W_8^{*2}$, $W_8^7 = W_8^{*1}$

$\therefore X(1) = 1+1\,(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}})+1(-j)+1\,(\frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}})$ =1 - j2.4142

For k=2, $X(2) = \sum_{n=0}^{7} x(n)\, W_8^{2n}$= x(0) W_8^0+x(1) W_8^2+x(2) W_8^4+x(3) W_8^6 samples values from x(4) to x(7) are 0

$\therefore X(2) = 1+1\,(-j)+1(-1)+1\,(j)$ = 0

For k=3, $X(3) = \sum_{n=0}^{7} x(n)\, W_8^{3n}$= x(0) W_8^0+x(1) W_8^3+x(2) W_8^6+x(3) W_8^9 samples values from x(4) to x(7) are 0, $W_8^9 = W_8^1$

$\therefore X(3) = 1+1\,(\frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}})+1(j)+1\,(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}})$ 1 - j0.4142

For k=4, $X(4) = \sum_{n=0}^{7} x(n)\, W_8^{4n}$= x(0) W_8^0+x(1) W_8^4+x(2) W_8^8+x(3) W_8^{12} samples values from x(4) to x(7) are 0, $W_8^8 = W_8^0$, $W_8^{12} = W_8^4$

$\therefore X(4) = 1+1\,(-1)+1(1)+1\,(-1)$ = 0

For k=5, $X(5) = \sum_{n=0}^{7} x(n)\, W_8^{5n}$= x(0) W_8^0+x(1) W_8^5+x(2) W_8^{10}+x(3) W_8^{15} samples values from x(4) to x(7) are 0, $W_8^{10} = W_8^2$, $W_8^{15} = W_8^7$

$\therefore X(5) = 1+1\,(\frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}})+1(j)+1\,(\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}})$ = 1 + j0.4142

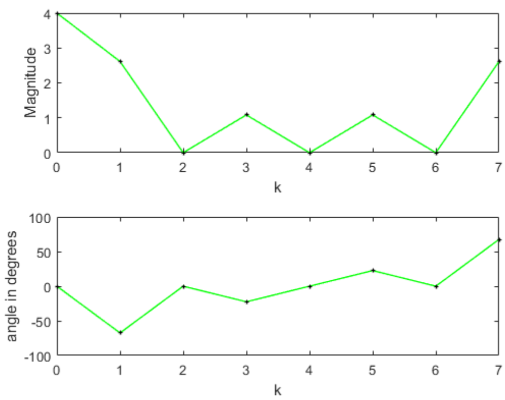
For k=6, $X(2) = \sum_{n=0}^{7} x(n)\, W_8^{6n}$= x(0) W_8^0+x(1) W_8^6+x(2) W_8^{12}+x(3) W_8^{18} samples values from x(4) to x(7) are 0, $W_8^{12} = W_8^4$, $W_8^{18} = W_8^2$

$\therefore X(6) = 1+1\,(j)+1(-1)+1\,(-j)$ = 0

For k=7, $X(7) = \sum_{n=0}^{7} x(n)\, W_8^{7n}$= x(0) W_8^0+x(1) W_8^7+x(2) W_8^{14}+x(3) W_8^{21} samples values from x(4) to x(7) are 0, $W_8^{14} = W_8^6$, $W_8^{21} = W_8^5$

$\therefore X(7) = 1+1\,(\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}})+1(j)+1\,(\frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}})$ = 1 + j 2.4142

$X(k)$ = {4∠0 , 2.613∠-67.5,0∠0, 1.0824∠ − 22.5, 0∠0,1.0824∠22.5, 0∠0, 2.613∠67.5}



Q3a. To compute the 4-point DFT of $x[n] = [4,3,2,1]$ using matrix method $X = W_{4\times4}x$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$ Row of W in to column of *x*.

X[0]=4+3+2+1 =10

X[1]=4+3($-j$)+2(-1)+1(j) =2-2j

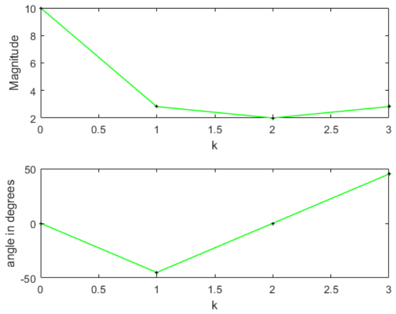
X[2]=4+3(-1)+2+1(-1)=2

X[3]=4+3(j)+2(-1)+1($-j$) =2+2j

$X(k) = \{10\angle 0\ , 2.8284\angle -45, 2\angle 0\ , 2.8284\angle 45\}$ Angles are in degrees.

|X(k)|

∠X(k)



Q3.b. To compute the 3-point IDFT of $X[k] = [7.5, 1.5 - j0.8666, 1.5 + j0.866]$ using matrix method. $x = \frac{1}{3}W^*{}_{3\times3}X$

$$W_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^1 & W_3^2 \\ 1 & W_3^2 & W_3^3 \end{bmatrix} \quad W*_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^{*1} & W_3^{*2} \\ 1 & W_3^{*2} & W_3^{*3} \end{bmatrix}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5 + j0.866 & -0.5 - j0.866 \\ 1 & -0.5 - j0.866 & -0.5 + j0.866 \end{bmatrix} \begin{bmatrix} 7.5 \\ 1.5 - j0.866 \\ 1.5 + j0.866 \end{bmatrix}$$

$\therefore x[0] = \frac{1}{3}\{7.5 + 1.5 - j0.866 + 1.65 + j0.866\}$ =3.5

$\therefore x[1] = \frac{1}{3}\{7.8 + (-0.5 + j0.866)\cdot(1.5 - j0.866) + (-0.5 - j0.866)\cdot(1.5 + j0.866)\}$ =2.5

$\therefore x[2] = \frac{1}{3}\{7.8 + (-0.5 - j0.866)\cdot(1.5 - j0.866) + (-0.5 + j0.866)\cdot(1.5 + j0.866)\}$ =1.5

Q.4 (a) Given the first 5 samples of 8-point DFT of a real 8-point sequence

$X[k] = [0.25, 0.125 - j0.3018, 0,0.125 - j0.518, 0]$ the remaining samples are computed by

X[k]=X*[8-k]

X[5]=X*[8-5]=X*[3]= $0.125 + j0.518$, X[6]=X*[8-6]=X*[2]= 0 and X[7]=X*[8-7]=X*[1]= $0.125 + j0.3018$

i) To find $x(0)$ let us use IDFT equation $x(n) = \frac{1}{8}\sum_{k=0}^{7} X(k)\, e^{j\frac{2\pi k}{8}n}$; In this equation if we substitute n=0 we get $x(0) = \frac{1}{8}\sum_{k=0}^{7} X(k)\, e^{0}$;= $\frac{1}{8}$ {X[0]+2*|X[1]|+2*|X[2]|+2*|X[3]|+X[4]}

$\therefore x(0) = \frac{1}{8}\{0.25 + 2 * 0.125 + 0 + 2 * 0.125 + 0\}$=0.938

Q4.(b) $X(k) = \frac{1}{2}\sum_{n=0}^{N-1}\left\{e^{j\frac{2\pi l n}{N}} + e^{-j\frac{2\pi l n}{N}}\right\} e^{-j\frac{2\pi nk}{N}} = \frac{1}{2}\left\{\sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(l-k)} + \sum_{n=0}^{N-1} e^{-j\frac{2\pi n}{N}(l+k)}\right\}$ now expressing its closed form of equation

$\therefore \sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(l-k)} = \frac{1-e^{j\frac{2\pi N}{N}(l-k)}}{1-e^{j\frac{2\pi}{N}(l-k)}} = 0$ for $l \neq k$ and for $l = k$ $\sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(l-k)} = \sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(0)}$=N i.e. $N\delta(l-k)$

Similarly $\sum_{n=0}^{N-1} e^{-j\frac{2\pi n}{N}(l+k)} == \frac{1-e^{j\frac{2\pi N}{N}(l+k)}}{1-e^{j\frac{2\pi}{N}(l+k)}} = 0$ for $l \neq -k$ and for $l = -k$ $\sum_{n=0}^{N-1} e^{-j\frac{2\pi n}{N}(0)}$=N i.e. $N\delta(l+k)$

$\therefore X(k) = \frac{N}{2}\{\delta(l-k) + \delta(l+k)\}$

Q.5 By definition of DFT we have $X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}$ where $W_N = e^{-j\frac{2\pi}{N}}$

$\therefore X(k) = \sum_{n=0}^{N-1} a^n\, e^{-j\frac{2\pi nk}{N}} = \sum_{n=0}^{N-1}\left(a \cdot e^{j\frac{2\pi k}{N}}\right)^n$ now expressing its closed form of equation

$X(k) = \frac{1-ae^{j\frac{2\pi N}{N}}}{1-ae^{j\frac{2\pi k}{N}}} = 0 \therefore X(k) = \frac{1-a^N}{1-ae^{j\frac{2\pi k}{N}}}$

Now for $x(n) = a^n\ for\ n = 0,1,2,3$ $X(k) = \frac{1-(0.4)^4}{1-0.4e^{j\frac{2\pi k}{4}}}$

$X(0) = \frac{1-(0.4)^4}{1-0.4} = 1.624$, $X(1) = \frac{1-(0.4)^4}{1-0.4e^{j\frac{2\pi}{4}}} = \frac{1-(0.4)^4}{1+0.4\,j}$=0.8400 - 0.3360i

$X(2) = \frac{1-(0.4)^4}{1-0.4e^{j\frac{2\pi}{4}2}} = \frac{1-(0.4)^4}{1+0.4}$=0.696, $X(3) = \frac{1-(0.4)^4}{1-0.4e^{j\frac{2\pi}{4}3}} = \frac{1-(0.4)^4}{1-0.4\,j}$=0.8400 + 0.3360i

Q.6. Two length-4 sequences are defined as below

$x(n) = \{1,0,-1,0\}, n = 0,1,2,3$, $h(n) = \{0,2,4,6\}\ for\ n = 0,1,2,3$

Now by matrix approach

$$y = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ 4 \\ 6 \end{bmatrix} \therefore y = \begin{bmatrix} 0+0-4+0 \\ 0+2+0-6 \\ 0+0+4+0 \\ 0-2+0+6 \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \\ 4 \\ 4 \end{bmatrix} \quad \therefore y(n) = \{-4,-4,4,4\}$$

Now by Stockholm's method $X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ & $H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ 4 \\ 6 \end{bmatrix}$

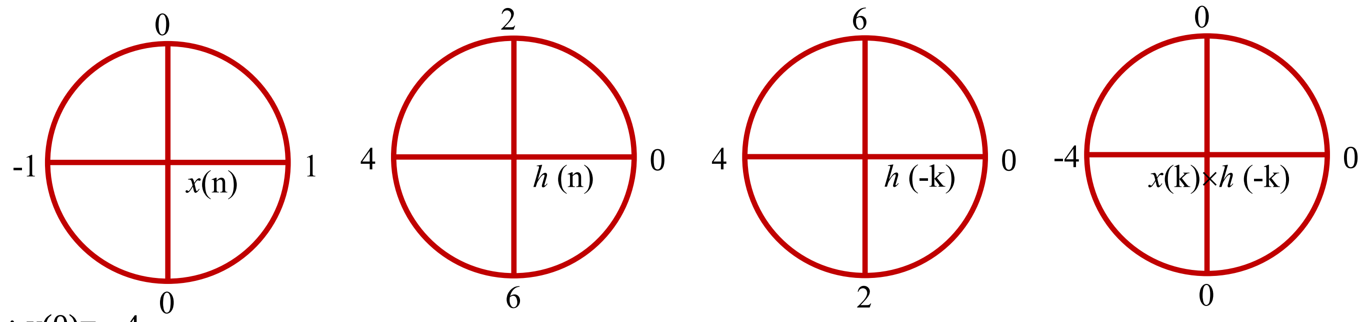
$$\therefore X = \{0,2,0,2\}\ \&H = \{12, -4+4j, -4, -4-4j\}$$

$$\therefore Y = X \cdot H = \{0, -8+8j, 0, -8-8j\}$$

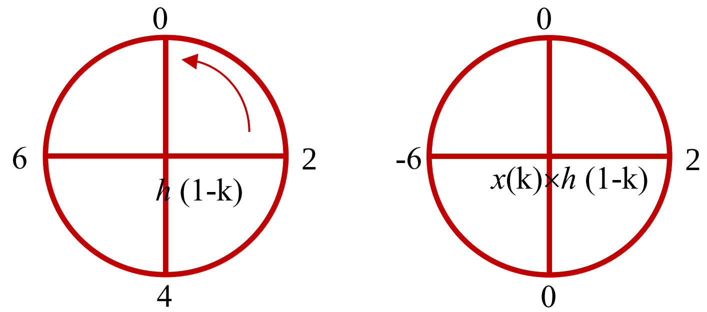
$$\therefore y = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & -j \end{bmatrix}\begin{bmatrix} 0 \\ -8+8j \\ 0 \\ -8-8j \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -8+8j-8-8j \\ -j8-8+8j-8 \\ 8j+8-8j+8 \\ -8j+8+8j+8 \end{bmatrix}$$

$$\therefore y(n) = \{-4,-4,4,4\}$$

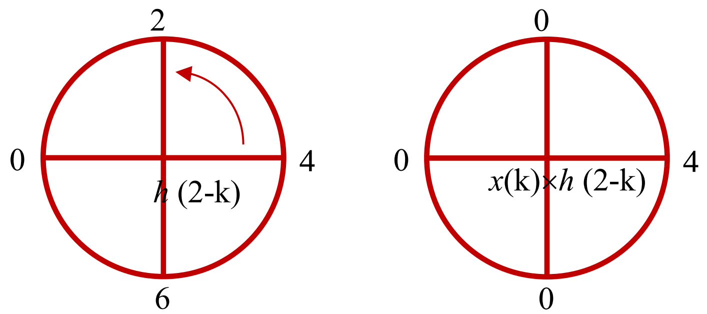
By concentric circle method



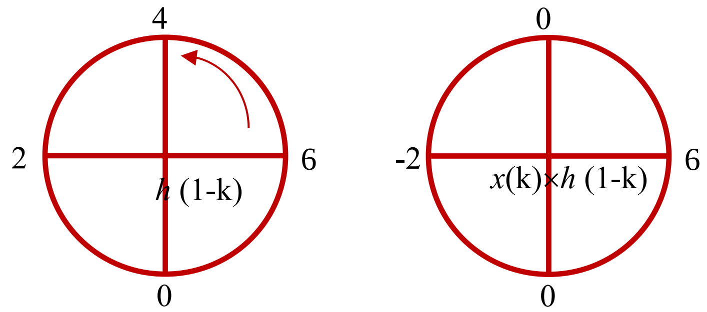
$\therefore$ y(0)= - 4



$\therefore$ y(1)= - 4



$\therefore$ y(2)= 4



$\therefore$ y(3)= 4

Q.7 If DFT of $x(n)$ is X[k], and DFT of h(n)=H[k] then DFT of $y(n)=x(n)\circledast h(n)$ is Y[k]=X[k]H[k]

$$Y[k] = DFT\{y(n)\} = \sum_{n=0}^{N-1} y(n) W_N^{nk} \quad \text{for } k = 0,1,2\ldots N-1$$

$$\text{and } y(n) = \sum_{m=0}^{N-1} x(m).h(n-m)$$

$$\therefore Y[k] = \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} x(m).h(n-m) W_N^{nk}$$

Interchanging order of summation

$$\therefore Y[k] = \sum_{m=0}^{N-1} x(m) \sum_{n=0}^{N-1} h(n-m) W_N^{nk}$$

Substitute n-m = r, for n=0, r = -m and for n=N-1,r=N-m-1. h(n) and W are both N periodic so r also varies from 0 to N-1

$$\therefore Y[k] = \sum_{m=0}^{N-1} x(m) \sum_{r=0}^{N-1} h(r) W_N^{(r+m)k}$$

$$\therefore Y[k] = \sum_{m=0}^{N-1} x(m) \sum_{r=0}^{N-1} h(r) W_N^{rk} W_N^{mk}$$

$$\therefore Y[k] = \sum_{m=0}^{N-1} x(m) W_N^{mk} H[k]$$

$$\therefore Y[k] = X[k]H[k]$$