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INTERNAL ASSESSMENT TEST – II

Sub	DIGITAL SIGNAL PROCESSING							Code	21EC42
Date	07/08/2023	Duration	90 mins	Max Marks	50	Sem	V	Branch	ECE

Answer any 5 full questions

		Marks	CO	RBT
1	a) State and Prove Rayleigh's Energy theorem. b) If x (n) = {1, 2,-2, 1,-2,-3,-1, 2} find the value of $\sum_{k=0}^{7}\|X[k]\|^2$ without computing DFT.	[6] [4]	CO2	L2
2	Using FFT algorithm find 8-point IDFT of a sequence X(k)={6,2-j3.414, 0, 2+j0.586,2, 2-j0.586,0, 2+j3.414}	[10]	CO3	L3
3	Derive an expression for frequency response of linear phase FIR filter for symmetric impulse response with M even. Further express the impulse response in Z-domain.	[10]	CO4	L1

		Marks	CO	RBT
4	Find the circular cross correlation between the sequences x(n)={1,2,-2,1} and y(n)={1,1,-1,4} use Stockholm's method (DFT and IDFT).	[10]	CO2	L3
5	Find the output y(n) of a filter whose impulse response h(n)={3,2,1} and the input x(n) is{2,1,-1,-2,-3, 5,6,-1,2, 0,2,1}. Use overlap save method with N=6.	[10]	CO3	L3
6	a) Derive the radix-2 decimation in time FFT algorithm and draw the signal flow graph for eight point DFT computation. b) Find the number of complex multiplications and additions required to compute 128 point DFT using (i) Direct DFT (ii) FFT (iii) what is the speed improvement factor	[7] [3]	CO2 CO2	L2 L3
7	Find DFT of a sequence x (n) = {1, 2, 2, 1} using 8-point DITFFT algorithm.	[10]	CO3	L3
8	An FIR has the unit impulse response h(n)={1,2} and the input x(n) is{1,-1,2,1,2, -1,1,3}. Use overlap add method with N=6 and compute y(n)	[10]	CO3	L3

DSP-IAT-2 SEM-4 [2022-23]

SOLUTION

Q1[a] State and Prove Rayleigh's Energy Theorem.

Sol: Rayleigh's Energy theorem is also called as the Parseval's Energy Theorem & is stated as given below:

Consider a finite length energy signal $x(n), 0 \le n \le N-1$ then according to Rayleigh's Energy Theorem,

$\Rightarrow$ if 'E' is the energy of $x(n)$, then

$$E = \sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$

Proof: $\rightarrow$

$$E = \sum_{n=0}^{N-1} |x(n)|^2 = \sum_{n=0}^{N-1} x(n) \cdot x^*(n)$$

$$= \sum_{n=0}^{N-1} \left[\frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}kn}\right] x^*(n)$$

[using IDFT equation for $x(n)$]

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) \sum_{n=0}^{N-1} x^*(n) \cdot e^{j\frac{2\pi}{N}kn} \longrightarrow (1)$$

Also we know that:

$$\sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N}kn} = X(k) \quad ; \quad 0 \le k \le N-1.$$

Taking conjugate on both sides of the above expression, we get

$$\Rightarrow \sum_{n=0}^{N-1} x^*(n) \cdot e^{j\frac{2\pi}{N}kn} = X^*(k) \; ; \quad 0 \le k \le N-1 \longrightarrow (2)$$

$\therefore$ using (2), (1) can be written as :

$$E = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot X^*(k)$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2 .$$

Hence Proved.

[c b] If $x(n) = \{1, 2, -2, 1, -2, -3, -1, 2\}$, find the value of $\sum_{k=0}^{7} |X(k)|^2$ without computing DFT.

Sol: we.K.T. $\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2 .$

$$\therefore \sum_{k=0}^{N-1} |X(k)|^2 = N \cdot \sum_{n=0}^{N-1} |x(n)|^2 .$$

$$= N \cdot \left[1^2 + 2^2 + (-2)^2 + 1^2 + (-2)^2 + (-3)^2 + (-1)^2 + (2)^2\right]$$

Here $N = 8$.

$$\therefore \sum_{k=0}^{7} |X(k)|^2 = 8\left[28\right] = 224$$

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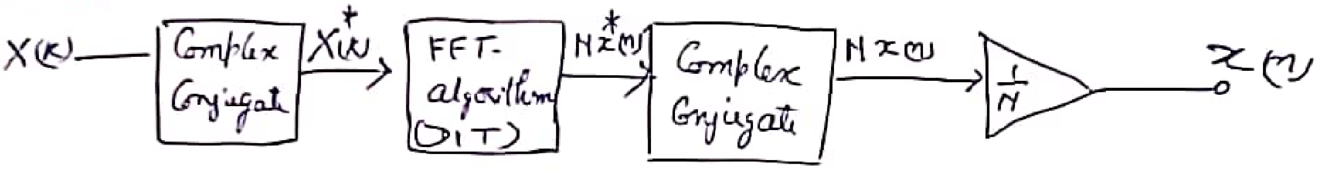
Q2] Using FFT algorithm find 8-point IDFT of a sequence.

$$X(k) = \{6,\ 2 - j3.414,\ 0,\ 2 + j0.586,\ 2,\ 2 - j0.586,\ 0,\ 2 + j3.414\}$$

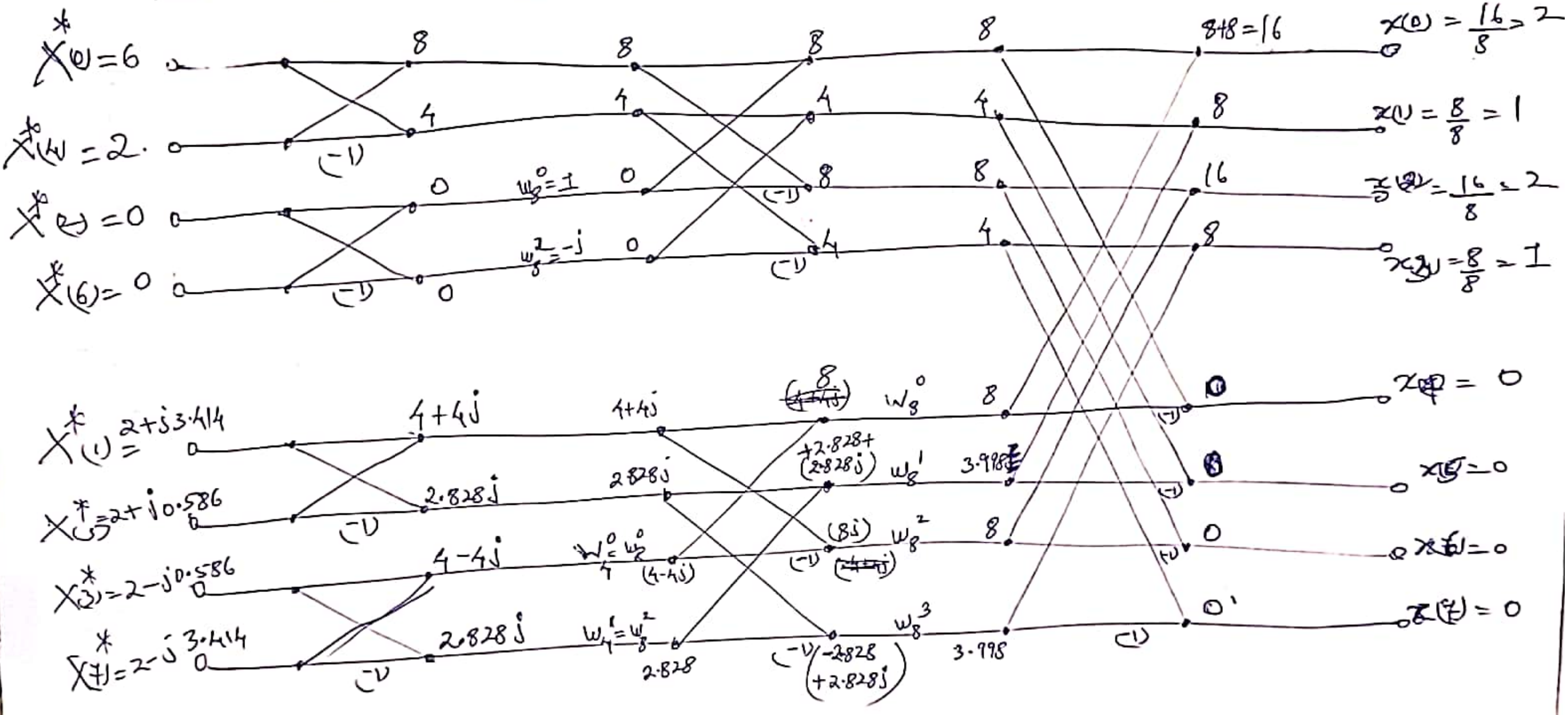
Sol:



i.e. $$x(n) = \frac{1}{N}\left\{DFT\left[X^*(k)\right]\right\}^*$$

Here $N = 8$.

Q2: Solution for 8-pt-IDFT



0 0 0	0 0 0	x(0)
0 0 1	1 0 0	x(4)
0 1 0	0 1 0	x(2)
0 1 1	1 1 0	x(6)
1 0 0	0 0 1	x(1)
1 0 1	1 0 1	x(5)
1 1 0	0 1 1	x(3)
1 1 1	1 1 1	x(7)

$W_8^0=1$, $W_8^2=-j$, $W_8^1=0.707-j0.707$, $W_8^3=-0.707-j0.707$

$X(K)=$

$\left(\underset{x(0)}{6}, \underset{x(1)}{2-j3.414}, \underset{x(2)}{0}, \underset{x(3)}{2+j0.586}, \underset{x(4)}{2}, \underset{x(5)}{2-j0.586}, \underset{x(6)}{0}, \underset{x(7)}{2+j3.414}\right)$.

Finally $\Rightarrow$ $x(n)=\{2,1,2,1,0,0,0,0\}$

$x(n)=\frac{1}{N}\left\{DFT\left[X^*(k)\right]\right\}^*$

Q3: Derive an expression for frequency response of linear Phase FIR filter for symmetric impulse response with 'M' even. Further express the impulse response in Z-domain.

Sol: Type-II FIR filter, M - even i.e. FIR filter of length 'M' & h(k) — symmetric

For a FIR filter of length 'M', the o/p sequence is expressed as:

$$y(n) = \sum_{k=0}^{M-1} h(k) \cdot x(n-k) \longrightarrow (I)$$

And its corresponding frequency response is expressed as:

$$H(e^{j\omega}) = \sum_{k=0}^{M-1} h(k) \cdot e^{-j\omega k} \longrightarrow (2)$$ [using Fourier Transform].

Here we are considering a Type-II symmetrical FIR filter.

i.e. 'M' is even in nature.

& $h(k) = h(M-1-k)$ for $k = 0, 1, \ldots, M-1$.
(∵ it is symmetrical in nature).

Eg: If we take $M=4$ & the sequence is as shown below:

Here $h(0) = h(4-1-0) = h(3)$

& $h(1) = h(4-1-1) = h(2)$.

If we divide the given sequence in to two equal sets with the 1st set containing element from ~~M=0 to the~~ $k=0$ to $k=\frac{M}{2}-1$ and from $k=\frac{M}{2}+1$ to $k=(M-1)$ then.

eq-(2) can be rewritten as:

$$\cancel{y(n) = \sum_{k=0}^{M-1} h(k) \cdot x(n-k)} \;\&\; H(e^{j\omega}) = \sum_{k=0}^{\frac{M}{2}-1} h(k)\, e^{-j\omega k} + \underbrace{\sum_{k=\frac{M}{2}}^{M-1} h(k) \cdot e^{-j\omega k}}_{2^{nd}\text{ Part}}$$

Now substituting $k = (M-1-k)$ in 2^{nd} part of above equation we get-

$k = \frac{M}{2} \longrightarrow k = M-1-\frac{M}{2} = \frac{M}{2}-1$ &

$k = (M-1) \longrightarrow k = (M-1)-(M-1) = 0$.

$$\Longrightarrow H(e^{j\omega}) = \sum_{k=0}^{\frac{M}{2}-1} h(k)\cdot e^{-j\omega k} + \sum_{k=\frac{M}{2}-1}^{k=0} h(M-1-k)\cdot e^{-j\omega(M-1-k)}$$

$$= \sum_{k=0}^{\frac{M}{2}-1} h(k)\cdot e^{-j\omega k} + \sum_{k=\frac{M}{2}-1}^{k=0} h(M-1-k)\cdot e^{-j\omega(M-1-k)}$$

$$= \sum_{k=0}^{\frac{M}{2}-1} h(k)\cdot e^{-j\omega k} + \sum_{k=0}^{\frac{M}{2}-1} h(M-1-k)\cdot e^{-j\omega(M-1-k)}.$$

Also w.k.T. $h(k) = h(M-1-k)$

$$\Longrightarrow H(e^{j\omega}) = \sum_{k=0}^{\frac{M}{2}-1} h(k)\cdot e^{-j\omega k} + \sum_{k=0}^{\frac{M}{2}-1} h(k)\cdot e^{-j\omega(M-1-k)}$$

$$= \sum_{k=0}^{\frac{M}{2}-1} h(k)\left[e^{-j\omega k} + e^{-j\omega(M-1-k)}\right]$$

$$= \sum_{k=0}^{\frac{M}{2}-1} h(k)\left[e^{-j\omega k}\times 1 + e^{-j\omega(M-1)}\cdot e^{j\omega k}\right]$$

$$= \sum_{k=0}^{\frac{M}{2}-1} h(k)\left[e^{-j\omega k}\cdot e^{\frac{-j\omega(M-1)}{2}}\cdot e^{\frac{+j\omega(M-1)}{2}} + e^{-j\omega(M-1)}\cdot e^{j\omega k}\right]$$

$$= \sum_{k=0}^{\frac{M}{2}-1} h(k)\cdot e^{\frac{-j\omega(M-1)}{2}}\cdot\left[e^{-j\omega k}\cdot e^{\frac{j\omega(M-1)}{2}} + e^{\frac{-j\omega(M-1)}{2}}\cdot e^{j\omega k}\right]$$

$$H(e^{j\omega}) = \sum_{k=0}^{\frac{M}{2}-1} h(k)\cdot e^{\frac{-j\omega(M-1)}{2}}\left[e^{j\omega\left(\frac{M-1}{2}-k\right)} + e^{-j\omega\left(\frac{M-1}{2}-k\right)}\right] \longrightarrow (A)$$

$$= e^{\frac{-j\omega(M-1)}{2}} \sum_{k=0}^{\frac{M}{2}-1} h(k)\cdot 2\cdot \cos\left[\omega\left(\frac{M-1}{2} - k\right)\right]$$

$$\Rightarrow H(e^{j\omega}) = e^{-j\omega\frac{(M-1)}{2}} \cdot 2 \sum_{k=0}^{\frac{M}{2}-1} h(k) \cos\left[\omega\left(\frac{M-1}{2} - k\right)\right] \longrightarrow (3)$$

On comparing eq (3) with

$$H(e^{j\omega}) = e^{j\angle(\theta)} |H_r(\omega)|$$

we get the real valued frequency response expressed as:

$$\boxed{\begin{aligned} H_r(\omega) &= 2 \cdot \sum_{k=0}^{\frac{M}{2}-1} h(k) \cdot \cos\left[\omega\left(\frac{M-1}{2} - k\right)\right] \\ \angle\theta &= \angle -\omega\left(\frac{M-1}{2}\right) \end{aligned}}$$

$$\therefore H(e^{j\omega}) = e^{-j\omega\left(\frac{M-1}{2}\right)} \cdot H_r(\omega).$$

$$\Rightarrow \boxed{H(e^{j\omega}) = \begin{cases} e^{-j\omega\left(\frac{M-1}{2}\right)} \cdot |H_r(\omega)| & \text{for } H_r(\omega) \geq 0 \\ e^{-j\omega\left(\frac{M-1}{2} + \pi\right)} \cdot |H_r(\omega)| & \text{for } H_r(\omega) < 0 \end{cases}}$$

To obtain the impulse response in Z-domain, we substitute $e^{j\omega} = z$ in eq-(A)

Eq (A) is

$$\Rightarrow H(e^{j\omega}) = \sum_{k=0}^{\frac{M}{2}-1} h(k) \cdot e^{-j\omega\left(\frac{M-1}{2}\right)} \cdot \left[e^{j\omega\left(\frac{M-1}{2} - k\right)} + e^{-j\omega\left(\frac{M-1}{2} - k\right)} \right]$$

Now substituting $e^{+j\omega} = z$ in the above equation we get

$$\Rightarrow H(z) = \sum_{k=0}^{\frac{M}{2}-1} h(k) \cdot \left(e^{j\omega}\right)^{-\left(\frac{M-1}{2}\right)} \cdot \left[\left(e^{j\omega}\right)^{\left(\frac{M-1}{2}-k\right)} + \left(e^{j\omega}\right)^{-\left(\frac{M-1}{2}-k\right)}\right]$$

$$= \sum_{k=0}^{\frac{M}{2}-1} h(k) \cdot z^{-\left(\frac{M-1}{2}\right)} \cdot \left[z^{\left(\frac{M-1}{2}-k\right)} + z^{-\left(\frac{M-1}{2}-k\right)}\right]$$

$$\boxed{H(z) = z^{-\left(\frac{M-1}{2}\right)} \cdot \sum_{k=0}^{\frac{M}{2}-1} \left[z^{\left(\frac{M-1}{2}-k\right)} + z^{-\left(\frac{M-1}{2}-k\right)}\right]}$$

Q4. Find the circular cross correlation between the sequences $x(n) = (1, 2, -2, 1)$ and $y(n) = (1, 1, -1, 4)$. use Stockholms method of DFT and IDFT.

Sol:

We have $x(n) = [1, 2, -2, 1]$

$$\therefore X(K) = \overset{(W_4)}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}} \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} = [2, 3-j, -4, 3+j]$$

Illy $y(n) = (1, 1, -1, 4)$

$\frac{10}{42}$

$$\therefore \quad Y(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \\ 4 \end{bmatrix}$$

$$= [5, \; 2+3j, \; -5, \; 2-3j]$$

Now $Y^*(k) = [5, \; 2-3j, \; -5, \; 2+3j]$

$$X(k) \cdot Y^*(k) = [10, \; 3-11j, \; 20, \; 3+11j]$$

$$\text{IDFT of } X(k) \cdot Y^*(k) = \frac{1}{4} \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 10 \\ 3-11j \\ 20 \\ 3+11j \end{bmatrix}$$

$$= [9, 3, 6, -8]$$

$\therefore$ the circular cross-correlation between $x(n)$ and $y(n)$ is given by the sequence

$$[9, 3, 6, -8]$$

5. Find the output $y(n)$ of a filter whose impulse response $h(n) = \{3, 2, 1\}$ and the input $x(n)$ is $\{2, 1, -1, -2, -3, 5, 6, -1, 2, 0, 2, 1\}$.

use overlap save method with $N = 6$.

Sol: $h(n) = \{3, 2, 1\}$

$N = l_{x_n} + l_h - 1$

$6 = l_{x_1} + 3 - 1$

$\Rightarrow l_{x_1} = 6 - 3 + 1 = 4.$

$l_h = 3$, $N = 6$,

a) $x_1(n)$:- contains $(l_h - 1)$ zeroes in the beginning & remaining samples are from $x(n)$.

$\Rightarrow x_1(n) = (0, 0, 2, 1, -1, -2)$

b) Last (l_h-1) samples of $x_1(n)$ becomes the first (l_h-1) samples of $x_2(n)$.

$\Rightarrow x_2(n) = (-1, -2, -3, 5, 6, -1)$

c) ~~By~~

&

$x_3(n) = (6, -1, 2, 0, 2, 1)$

d) The last (l_h-1) samples of $x_4(n)$ are non-zero, hence we create one more subsequence $x_5(n)$.

$x_4(n) = (2, 1, 0, 0, 0, 0)$

e) Now $h(n)$ is appended by 3 zeroes to make it of length 'N' = 6.

$$\Longrightarrow h(n) = (3, 2, 1, 0, 0, 0)$$

f) Next we calculate

$$y_1(n) = h(n) \circledast x_1(n)$$

$$\Longrightarrow y_1(n) = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 & 2 \\ 2 & 3 & 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ -1 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 + (-4) \\ -2 \\ 6 \\ 4 + 3 \\ 2 + 2 + (-3) \\ 1 - 2 - 6 \end{bmatrix} = \begin{bmatrix} -5 \\ -2 \\ 6 \\ 7 \\ 1 \\ -7 \end{bmatrix}$$

$$\text{IIly } y_2(n) = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 & 2 \\ 2 & 3 & 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \\ -3 \\ 5 \\ 6 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3+6-2 \\ -2-6-1 \\ -1-4-9 \\ -2-6+15 \\ -3+10+18 \\ 5+12-3 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ -14 \\ 7 \\ 25 \\ 14 \end{bmatrix} \quad \therefore y_2(n) = \begin{bmatrix} 1, -9, -14, 7, 25, 14 \end{bmatrix}$$

$$H_y\, y_3^{T} = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 & 2 \\ 2 & 3 & 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 6 \\ -1 \\ 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 18+2+2 \\ 12-3+1 \\ 6-2+6 \\ -1+4 \\ 2+6 \\ 4+3 \end{bmatrix} = \begin{bmatrix} 22 \\ 10 \\ 10 \\ 3 \\ 8 \\ 7 \end{bmatrix}$$

$$H_y\, y_4^{T} = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 & 2 \\ 2 & 3 & 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 4+3 \\ 2+2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow y_1(n) = \cancel{-5}, \cancel{-2}, 6, 7, 1, -7$

$y_2(n) = \qquad \cancel{-1}, \cancel{-9}, -14, 7, 25, 14$

$y_3(n) = \qquad \cancel{22}, \cancel{10}, 19, 3, 8, 7$

$y_4(n) = \qquad \cancel{6}, \cancel{7}, 4, 1, 0, 0$

$$y(n) = \{6, 7, 1, -7, -14, 7, 25, 14, 19, 3, 8, 7, 4, 1\}$$

. . .

Q6 :→ [a] Derive the radix-2 decimation in time FFT algorithm and draw the signal flowgraph for eight point DFT computation.

[b] Find the number of complex multiplications and additions required to compute 128 point DFT using (1) Direct DFT (2) FFT

(3) What is the speed improvement factor?

Sol: :→

[a] N-pt-DFT of $x(n)$ is given by

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn} \longrightarrow (1)$$

$; \; 0 \le k \le N-1. \;\; \& \;\; W_N = e^{-j\frac{2\pi}{N}}.$

The N-point DFT given by eq-(1) can be split into two $\frac{N}{2}$ point DFTs corresponding to even-indexed and odd indexed samples of $x(n)$ as shown:

$$X(K) = \sum_{n=\text{even}} x(n) W_N^{Kn} + \sum_{n-\text{odd}} x(n) W_N^{Kn}.$$

$0 \le K \le N-1$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2Kn} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{K(2n+1)}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_{\frac{N}{2}}^{Kn} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{2Kn} W_N^{K}.$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_{\frac{N}{2}}^{Kn} + W_N^{K} \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_{\frac{N}{2}}^{Kn}$$

$$= G(K) + W_N^{K} H(K) \longrightarrow (2)$$

where $$G(K) = \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_{\frac{N}{2}}^{Kn} \longrightarrow (3)$$

& $$H(K) = \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_{\frac{N}{2}}^{Kn} \longrightarrow (4)$$

Note that, if $0 \le K \le N-1$, then $G(K)$ and $H(K)$ represent $\frac{N}{2}$-point DFTs.

$\therefore$ $$G(K+\tfrac{N}{2}) = G(K) \longrightarrow (5)$$

$$H(K+\tfrac{N}{2}) = H(K) \longrightarrow (6)$$

(from periodicity property of DFT) $\Rightarrow$

$\therefore$ eq. 2 can be written as

$$X(k) = G(k) + W_N^k H(k) \longrightarrow 7$$

$0 \le k \le \frac{N}{2}$.

$$X(k+\tfrac{N}{2}) = G(k+\tfrac{N}{2}) + W_N^{(k+\frac{N}{2})} \cdot H(k+\tfrac{N}{2})$$

; $0 \le k \le \frac{N}{2}$.

$$= G(k) - W_N^k H(k) \longrightarrow 8$$

(using Symmetry property of W_N.)

Hence, we have divided an N-Point DFT into two $\frac{N}{2}$ -point DFTs.

Let N be = 8.

Then G(k) becomes 4-point DFT of the sequence containing even indexed samples of x(n) i.e.

$[x(0), x(2), x(4), x(6)]$

and H(k) becomes a 4 point DFT of the sequence containing odd indexed samples of x(n) i.e.

$[x(1), x(3), x(5), x(7)]$.

Using 7 we can write $\Rightarrow$

$$X(0) = G(0) + W_8^0 H(0) \longrightarrow 9$$

$$X(1) = G(1) + W_8^1 H(1) \longrightarrow 10$$

$$X(2) = G(2) + W_8^2 H(2) \longrightarrow 11$$

$$X(3) = G(3) + W_8^3 H(3) \longrightarrow 12$$

using eq. (8) we may write

$$X(4) = G(0) - W_8^0 H(0) \longrightarrow (13)$$

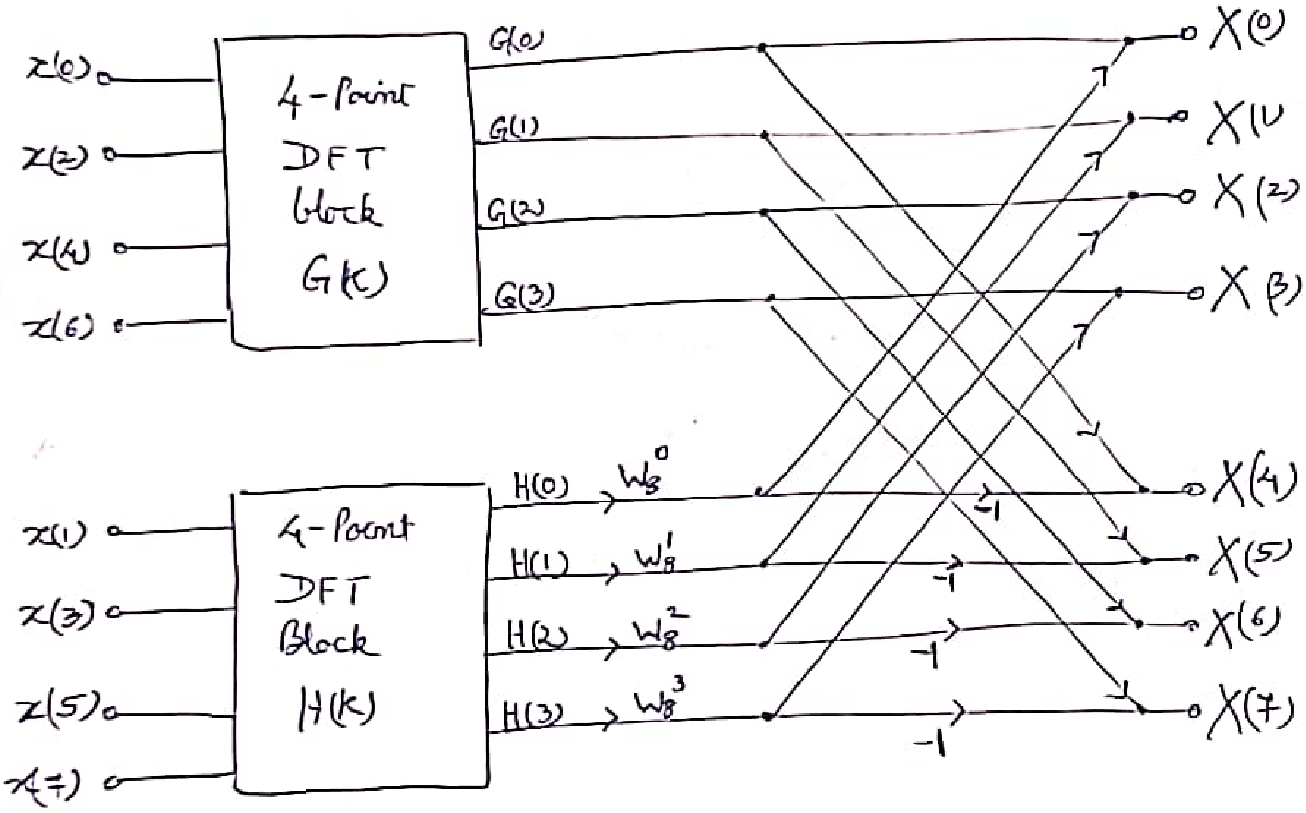
$$X(5) = G(1) - W_8^1 H(1) \longrightarrow (14)$$

$$X(6) = G(2) - W_8^2 H(2) \longrightarrow (15)$$

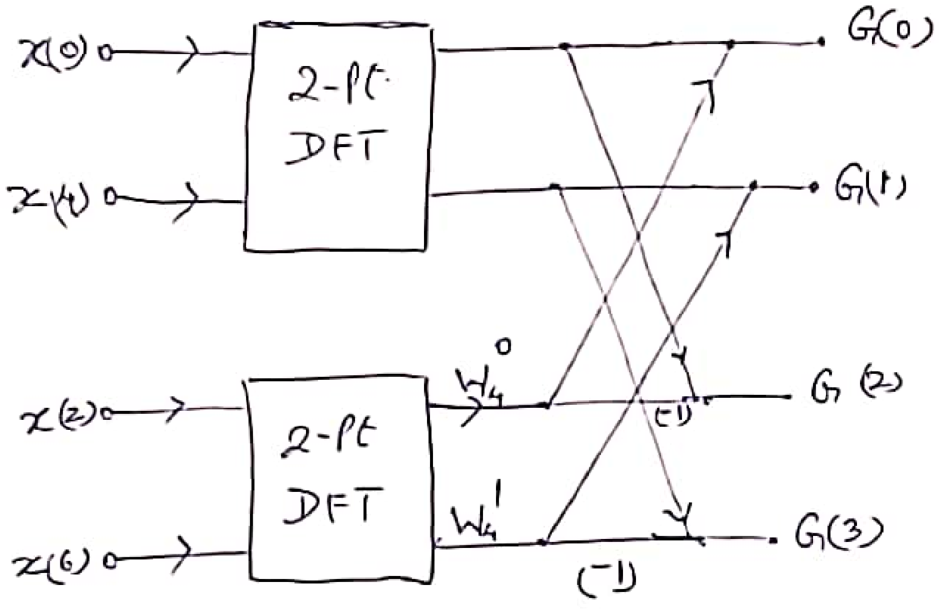
$$X(7) = G(3) - W_8^3 H(3) \longrightarrow (16)$$

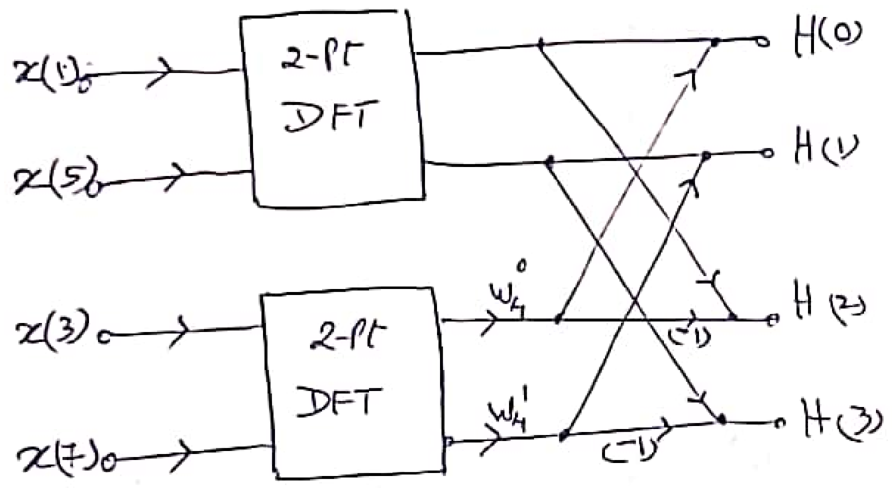
Eq. (9) to (16) may be represented by the following diagram

$\Longrightarrow$

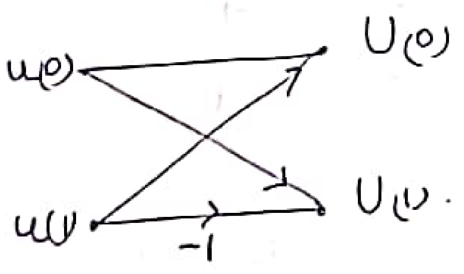


In the 2nd stage of decimation, we divide each $\frac{N}{2}$ - point DFT into two $\frac{N}{4}$ point DFTs i.e. each 4-point DFT will be divided into two 2-point DFTs as shown further: $\Longrightarrow$

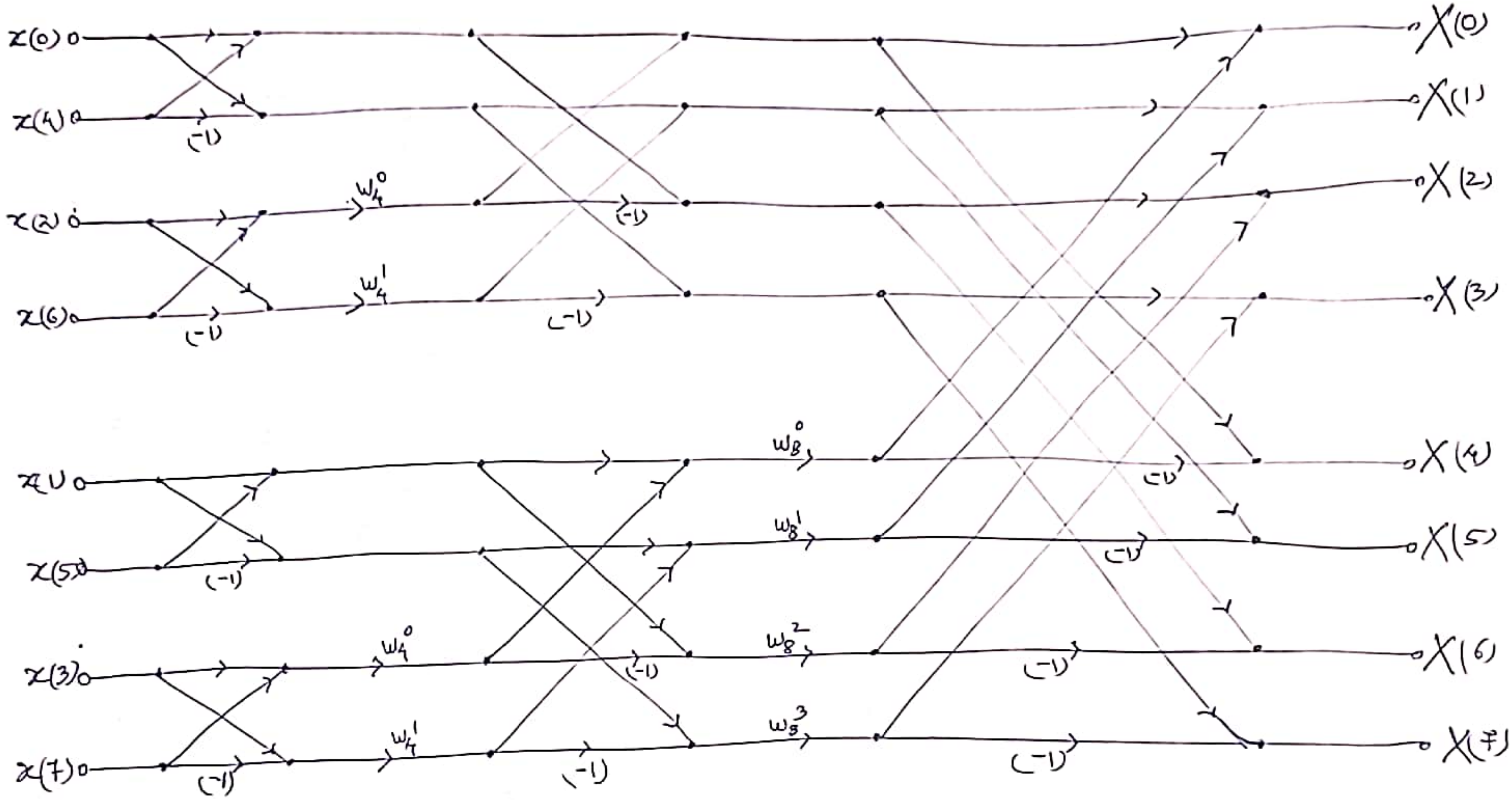




Each 2-Pt DFT can be calculated as follows:



Final Signal Flow graph to complete 8-pt-DFT can be drawn as shown below:



[b] For a 128-Point DFT:

$\Rightarrow N = 128$

$\therefore$ No of Complex Multiplications $= N^2 = 128^2 = \underline{\underline{16384}}$

No of Complex Additions $= N(N-1)$

$= 128 \times 127 = \underline{\underline{16256}}$

For a 128-Pt. FFT :-

No of Complex multiplications $= \frac{N}{2} \log_2 N$

$= \frac{128}{2} \cdot \log_2 (128)$

$= \frac{128}{2} \cdot \frac{\log_{10} 128}{\log_{10} 2} =$

$= 437.5 \approx \underline{\underline{438}}$

IIIy No of Complex Additions $= N \log_2 N$

$= 128 \cdot \frac{\log_{10} 128}{\log_{10} 2} = \underline{\underline{896}}$

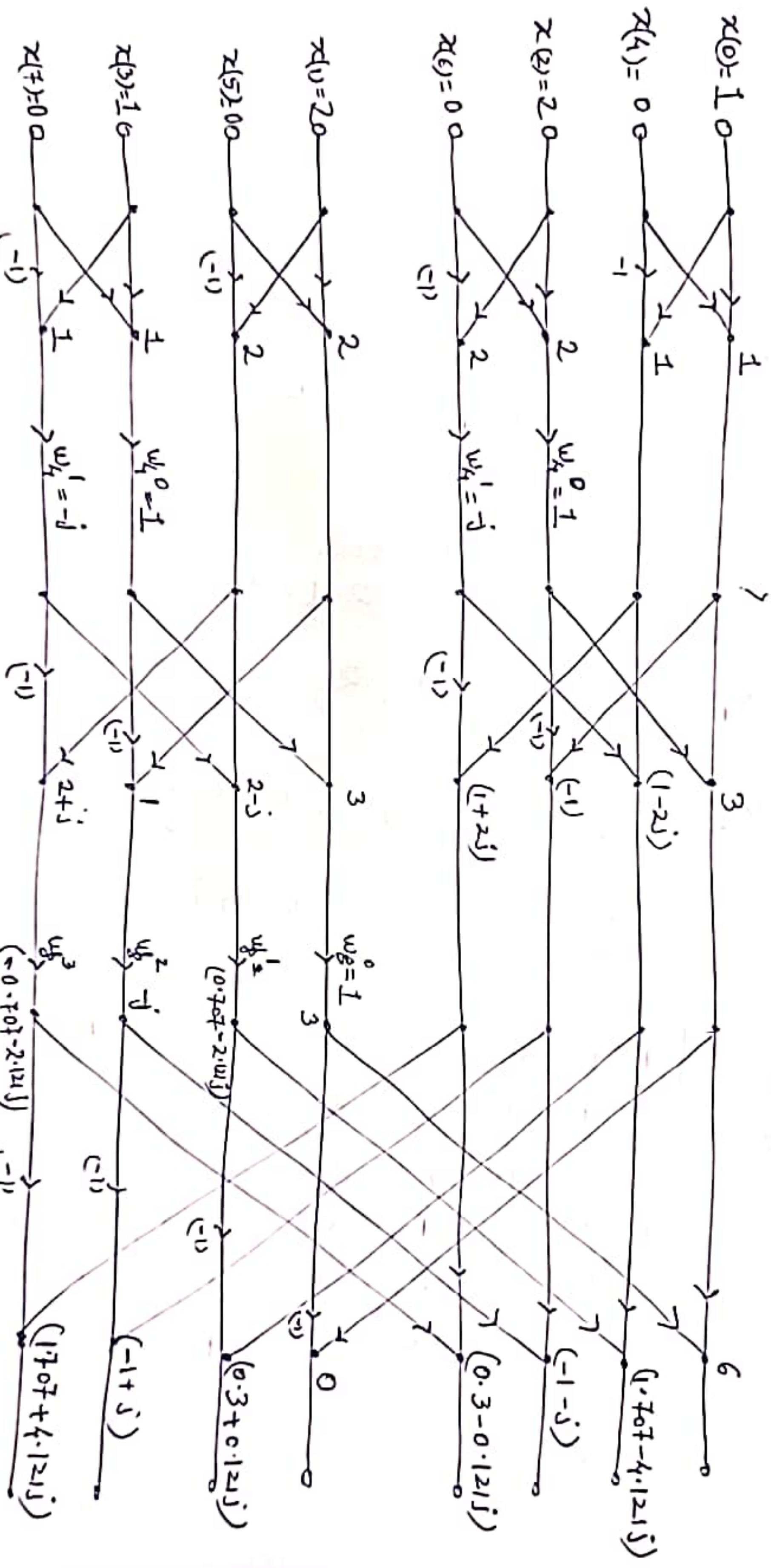
$\therefore$ Speed Improvement factor in terms of multiplication $= \frac{16384}{438} \approx \underline{\underline{37.4}}$.

Speed Improvement factor in terms of additions $= \frac{16256}{896} \approx = 18.14$.

Overall Speed Improvement factor $= \frac{(16384 + 16256)}{(438 + 896)} = \underline{\underline{24.46}}$

Q.7 Find DFT of a sequence $x(n) = \{1, 2, 2, 1\}$ using 8-pt-DIT-FFT algorithm.

$N = 8 \quad \therefore x(n) = \{\underset{0}{1}, \underset{1}{2}, \underset{2}{2}, \underset{3}{1}, \underset{4}{0}, \underset{5}{0}, \underset{6}{0}, \underset{7}{0}\}$.



$[W_8^0 = 1,\ W_8^2 = -j,\ W_8^1 = 0.707 - j0.707,\ W_8^3 = -0.707 - j0.707]$

$\therefore X(K) = [6,\ 1.707 - 4.121j,\ -1-j,\ 0.3 - 0.121j,\ 0,\ 0.3 + 0.121j,\ -1+j,\ 1.707 + 4.121j]$

Scanned by CamScanner

Q-8 : An FIR has the unit impulse response $h(n) = \{1, 2\}$ and the input $x(n)$ is $\{1, -1, 2, 1, 2, -1, 1, 3\}$. Use overlap add method with $N = 6$ and compute $y(n)$.

Sol: Given :

$$x(n) = \{1, -1, 2, 1, 2, -1, 1, 3\}$$

Here $N = 6$

$l_h = 2$. $\therefore l_{x_n} = ?$

WKT: $N = l_{x_n} + l_h - 1$

$6 = l_{x_n} + 2 - 1$

$\Longrightarrow l_{x_n} = 6 - 2 + 1 = 5$.

$\therefore$ No of zeroes to be padded in each Subsequence $= 2 - 1 = 1$.

$\therefore x_1(n) = \{1, -1, 2, 1, 2, 0\}$

$x_2(n) = \{-1, 1, 3, 0, 0, 0\}$

$h(n) = \{1, 2, 0, 0, 0, 0\}$

$\therefore y_1(n) =$

$y_1(n) = x_1(n) \circledast h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2-1 \\ -2+2 \\ 4+1 \\ 2+2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 5 \\ 4 \\ 4 \end{bmatrix}$$

$\therefore y_1(n) = \{1, 1, 0, 5, 4, 4\}$

IIly

$$y_2(n) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -2+1 \\ 2+3 \\ 6 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 5 \\ 6 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore y_2(n) = \{-1, -1, 5, 6, 0, 0\}$

$\therefore$ Output :

$y_1(n) = 1 \quad 1 \quad 0 \quad 5 \quad 4 \quad 4$

$y_2(n) = \qquad\qquad\qquad\qquad -1 \quad -1 \quad 5 \quad 6 \quad 0 \quad 0$

$y(n) = [1 \quad 1 \quad 0 \quad 5 \quad 4 \quad 3 \quad -1 \quad 5 \quad 6 \quad 0 \quad 0]$