Scheme Of Evaluation

Internal Assessment Test III – July 2023

Sub:	DIGITAL COMMUNICATION							Code:	18EC61
Date:	04/07/2023	Duration:	90 mins	Max Marks:	50	Sem:	VI	Branch:	ECE

Note: Answer Any 5Questions

Question #		Description	Marks Distribution		Max Marks
1		Explain Binary Frequency Shift Keying (BFSK) with neat block diagrams of modulator and demodulator. Explain the decision logic at the demodulator.		10	10
		• Modulator • Demodulator • Basis Function • Constellation Diagram • Decision Logic	2 2 2 2 2		
2		Draw the block diagram of Binary Phase Shift Keying (BPSK) demodulator. Explain the decision logic. Derive an expression for probability of error.		10	10
		• Receiver • Decision Rule • Probability of Error	2 2 6		
3		Explain Quadrature Phase Shift Keying (QPSK) with neat block diagrams of modulator and demodulator. Explain the decision logic at the demodulator.		10	10
		• Modulator • Demodulator • Basis Function • Constellation Diagram • Decision Logic	2 2 2 2 2		
4	a	Consider a signal space with the following basis functions. $\Phi_1(t) = \begin{cases} \frac{1}{\sqrt{2}} \text{ for } 0 \le t \le 2 \\ 0 \text{ otherwise} \end{cases}$ and $\Phi_2(t) = \begin{cases} \frac{1}{\sqrt{2}} \text{ for } 0 \le t < 1 \\ -\frac{1}{\sqrt{2}} \text{ for } 1 \le t \le 2 \\ 0 \text{ otherwise} \end{cases}$ Plot the signals with coordinates $(\mathbf{2\sqrt{2}, -2\sqrt{2}})$ and $(\mathbf{-2\sqrt{2}, 2\sqrt{2}})$.			10
		• $x_1(t)$ • $x_2(t)$	2.5 2.5	05	

4	b	Prove that the energy of a signal is equal to squared length of the signal vector representing it in the signal space diagram.		05	
		• Basis functions and Coordinates • Expression for Energy	2 3		
5		Obtain a set of orthonormal basis functions for the following set of signals. $x_1(t) = \begin{cases} 3 \text{ from } 0 \le t \le 3 \\ 0 \quad \text{otherwise} \end{cases}$ and $x_2(t) = \begin{cases} 3 \text{ from } 0 \le t \le 1 \\ 0 \quad \text{otherwise} \end{cases}$ and $x_3(t) = \begin{cases} 3 \text{ from } 1 \le t \le 3 \\ 0 \quad \text{otherwise} \end{cases}$ Express the signals as a linear combination of basis functions. Draw the signal space diagram (Constellation Diagram).		10	10
		• Basis Function $\phi_1(t)$ • Basis Function $\phi_2(t)$ • Linear Combination • Constellation Diagram	2 2 3 3		
6		What is the need of matched filter in digital communication receiver? Derive the impulse response of a filter matched to the signal $x(t)$.		10	10
		• Need of matched filter Decision Rule • Impulse Response of LTI system • Convolution • Impulse Response of Matched filter	2 2 3 3		

SOLUTIONS

1	In binary frequency shift keying (FSK) bit '1' and bit '0' are represented by the following symbols. Bit 1: $s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t)$, $0 \le t \le T_b$ $f_1 = \frac{n}{T_b}$ n - nonzero integer T_b - bit duration.

Bit 0:

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t), \quad 0 \le t \le T_b$$

$f_2 = \frac{m}{T_b}$

m- non zero integer

$m \neq n$.

To find basis functions

$$\int_0^{T_b} s_1(t)\, s_2(t)\, dt = 0$$

$\Rightarrow s_1(t)$ and $s_2(t)$ are orthogonal to each other.

$\therefore$ Basis function $\phi_1(t) = \frac{s_1(t)}{\sqrt{E_b}}$

$$= \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t)$$

$0 \le t \le T_b$

Basis function $\phi_2(t) = \frac{s_2(t)}{\sqrt{E_b}}$

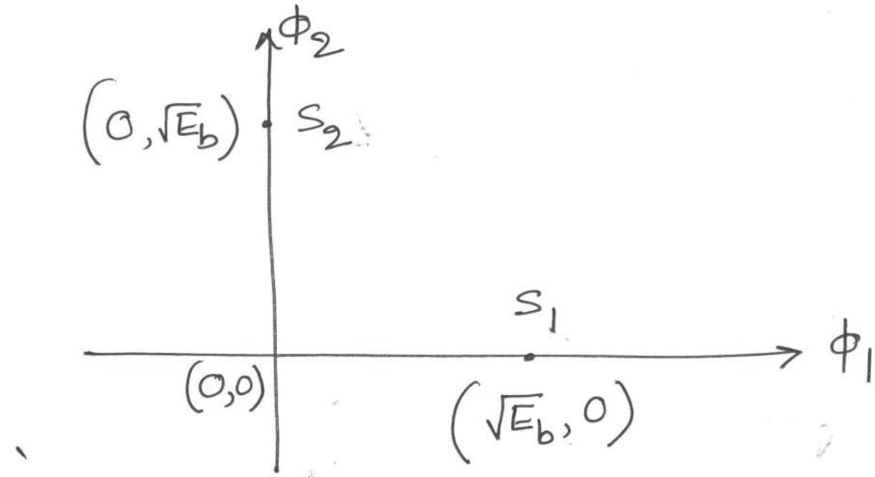
$$= \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t)$$

$0 \le t \le T_b$

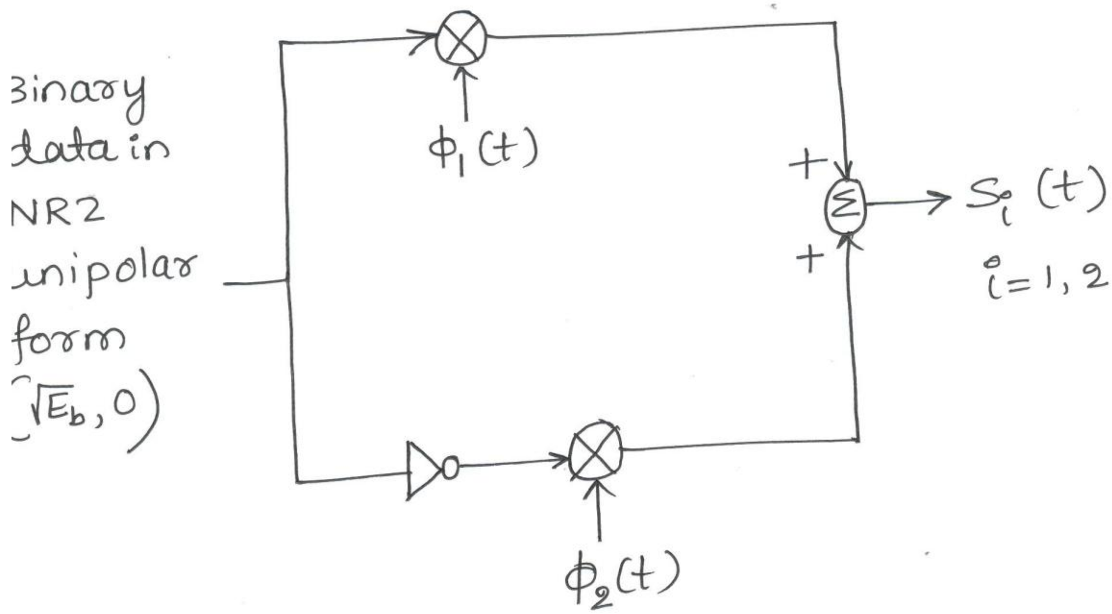
$$\therefore \quad s_1(t) = \sqrt{E_b}\,\phi_1(t) + 0\,\phi_2(t) \quad , 0 \le t \le T_b$$

$$s_2(t) = 0\,\phi_1(t) + \sqrt{E_b}\,\phi_2(t) \, , 0 \le t \le T_b$$

Constellation diagram



Block diagram of transmitter



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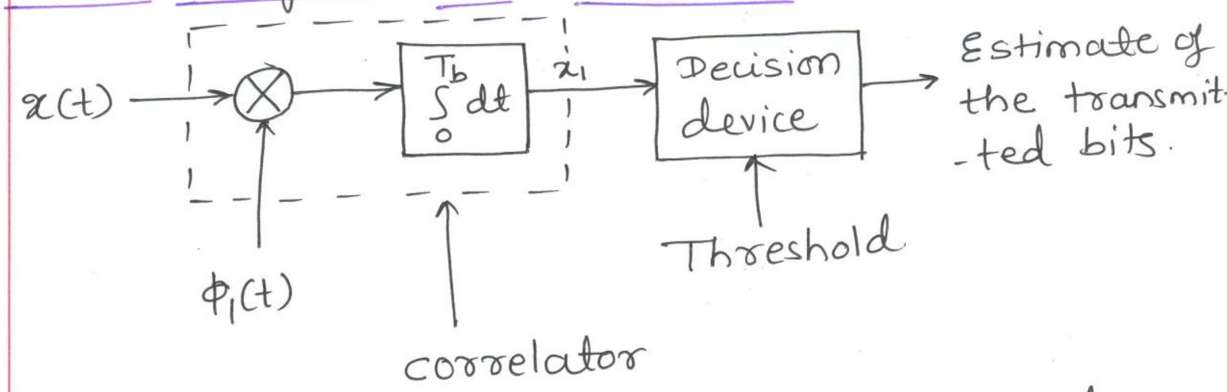
Block diagram of receiver



Let $x(t)$, $0 \le t \le T_b$ be the received signal.

$$x(t) = S_i(t) + w(t), \quad 0 \le t \le T_b \qquad \ldots (1)$$
$$i = 1, 2$$

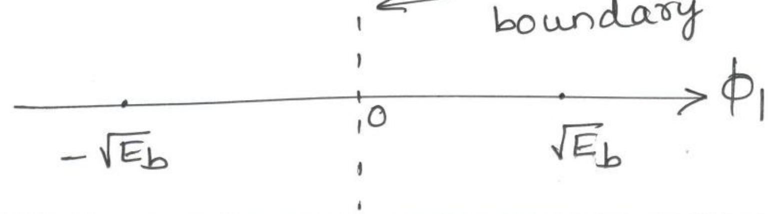
where $w(t)$ represents additive, white Gaussian noise with zero mean and PSD $\frac{N_0}{2}$ ie. variance $\frac{N_0}{2}$.

Decision logic

Let x_1 be the output of the correlator.

If $x_1 > 0$, decide in favor of bit 1

If $x_1 < 0$, decide in favor of bit 0



Probability of error

Suppose that bit '0' was transmitted.
ie, $s_2(t)$ was transmitted.
Then, from the block diagram of the receiver, we may write,

output of the correlator,

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt$$

$$= \int_0^{T_b} \left[s_2(t) + w(t)\right] \phi_1(t)\,dt$$

$$= \int_0^{T_b} s_2(t)\,\phi_1(t)\,dt + \int_0^{T_b} w(t)\,\phi_1(t)\,dt$$

$$= -\sqrt{E_b} + w_1 \quad \cdots \; (2)$$

coordinate of $s_2(t)$.

Mean of X_1 when '0' was transmitted,

$$\mu = E\left[X_1\right]$$

$$= -\sqrt{E_b} \quad \cdots \; (3)$$

Variance of X_1 when '0' was transmitted,

$$\sigma^2 = VAR[W_1]$$
$$= \frac{N_0}{2} \quad \cdots (4)$$

($\because$ variance does not change by the addi-tion of a constant to a random varia-ble)

$\therefore$ Probability density function (PDF) of output of correlator when bit '0' was transmitted,

$$f_{X_1}\left(x_1/0\right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}$$
$$= \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x_1+\sqrt{E_b})^2}{N_0}} \quad \cdots (5)$$

Wrong decision is made when $s_2(t)$ was transmitted and $x_1 > 0$.

$\therefore$ Probability of error when bit '0' was transmitted,

$$P_e(0) = P\left(X_1 > 0 / 0\right)$$
$$= \int_0^\infty f_{X_1}\left(x_1/0\right) dx_1$$

$$= \int_0^\infty \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x_1+\sqrt{E_b})^2}{N_0}} dx_1 \quad \text{---} (6)$$

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We know that,

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-\frac{z^2}{2}} dz \quad \ldots (7)$$

Let us represent $P_e(0)$ in terms of Q function.

Put $$\frac{(x_1+\sqrt{E_b})^2}{N_0} = \frac{z^2}{2} \quad \ldots (8)$$

ie, $$\frac{x_1+\sqrt{E_b}}{\sqrt{N_0}} = \frac{z}{\sqrt{2}}$$

$$\therefore \frac{dx_1}{\sqrt{N_0}} = \frac{dz}{\sqrt{2}}$$

$$\therefore dx_1 = \sqrt{\frac{N_0}{2}}\, dz \quad \ldots (9)$$

when $x_1 = 0$, $z = \sqrt{\frac{2E_b}{N_0}}$ ... (10)

when $x_1 = \infty$, $z = \infty$ ... (11)

Using (8), (9), (10), (11), we may write (6) as

$$P_e(0) = \int_{\sqrt{\frac{2E_b}{N_0}}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{z^2}{2}} \sqrt{\frac{N_0}{2}}\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{2E_b}{N_0}}}^{\infty} e^{-\frac{z^2}{2}}\, dz$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \ldots (12)$$

Similarly, we may prove that, probability of error when bit '1' was transmitted,

$$P_e(1) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \ldots (13)$$

∴ Average probability of error

$$= \frac{1}{2} P_e(0) + \frac{1}{2} P_e(1)$$

(Assuming equiprobable 0s & 1s)

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \ldots (14)$$

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one of the four equally spaced values (30)
such as $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

For this set of values, we may define the transmitted signal as

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right], \quad 0 \le t \le T$$

$$i = 1, 2, 3, 4$$

$$f_c = \frac{n}{T}$$

n-non-zero integer

Here, T is the symbol duration and E is the energy of each symbol.

Each possible value of the phase corresponds to a pair of bits (dibit).

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t)$$

$$0 \le t \le T$$

$E = 2E_b$ and $T = 2T_b$ $\quad\quad$ $i = 1, 2, 3, 4.$

Basis functions are

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \quad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \quad 0 \le t \le T$$

$\therefore S_i(t) = \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right]\phi_1(t)$

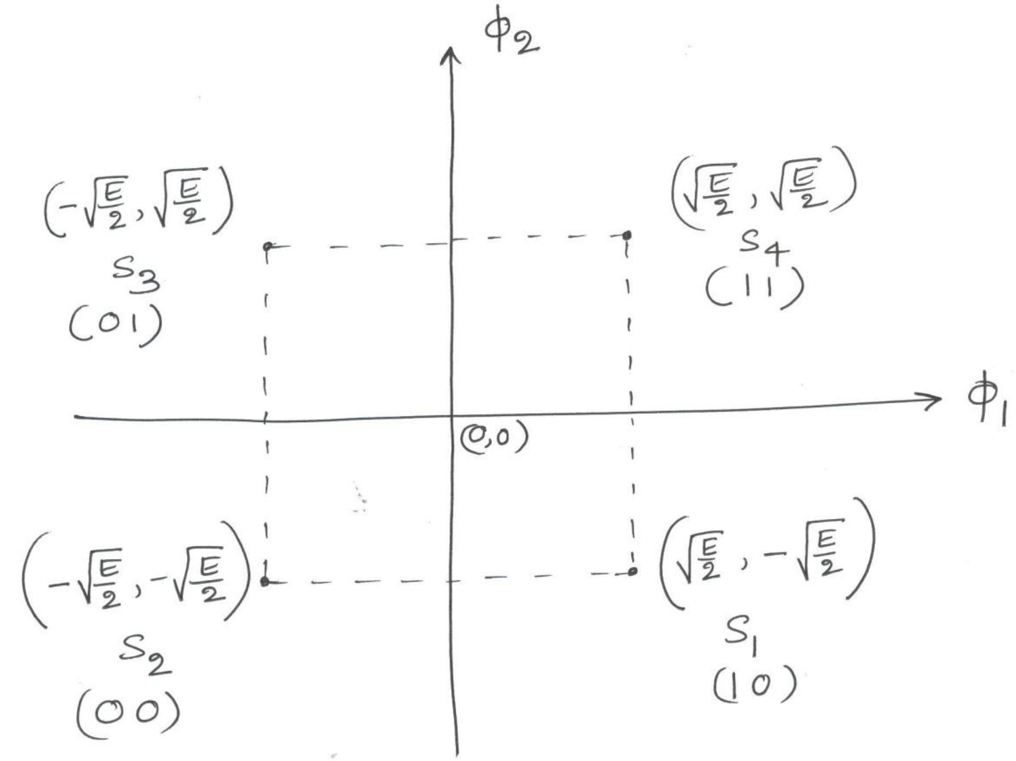
$- \sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right]\phi_2(t)$, $0 \le t \le T$

$\therefore$ The coordinates of message points are

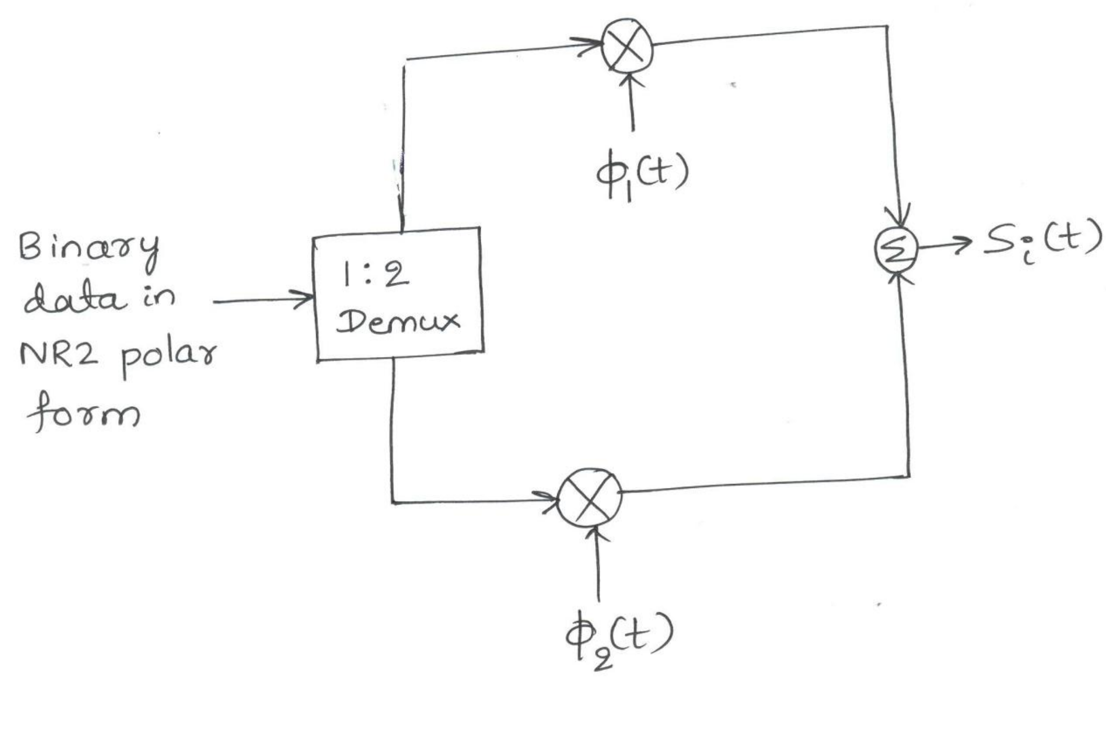
$$\begin{bmatrix} \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right] \\ -\sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right] \end{bmatrix} \quad i = 1, 2, 3, 4.$$

i	phase	coordinates	dibits
1	$\frac{\pi}{4}$	$\sqrt{\frac{E}{2}}, -\sqrt{\frac{E}{2}}$	10
2	$\frac{3\pi}{4}$	$-\sqrt{\frac{E}{2}}, -\sqrt{\frac{E}{2}}$	00
3	$\frac{5\pi}{4}$	$-\sqrt{\frac{E}{2}}, \sqrt{\frac{E}{2}}$	01
4	$\frac{7\pi}{4}$	$\sqrt{\frac{E}{2}}, \sqrt{\frac{E}{2}}$	11

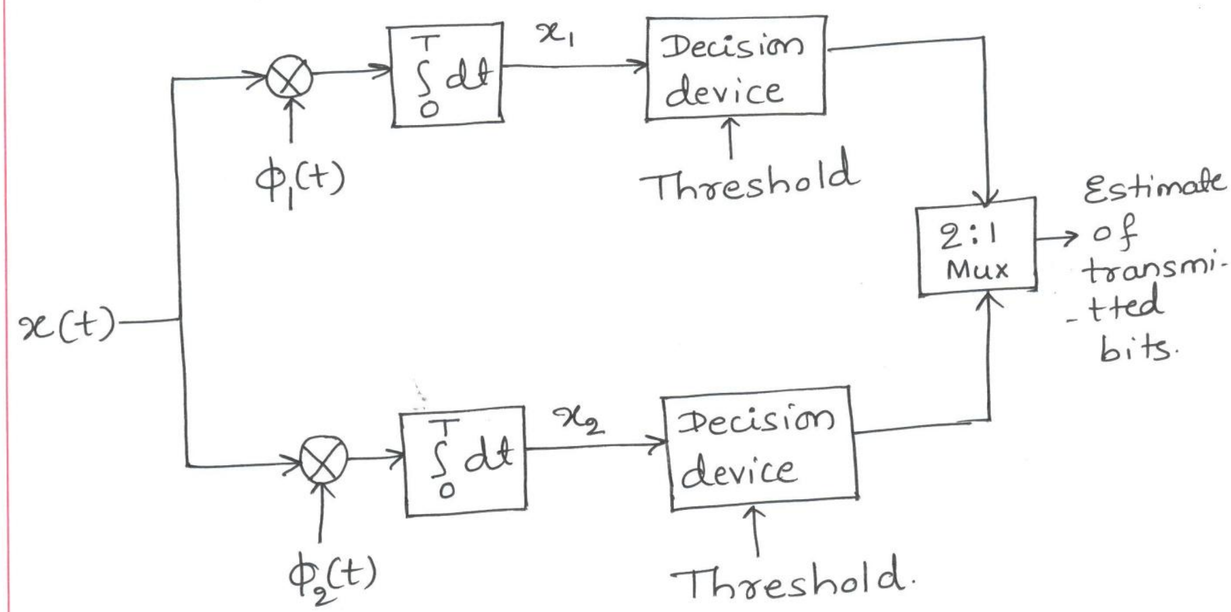
Based on these coordinates, signal space diagram of QPSK system may be drawn as follows.



Block diagram of transmitter



Block diagram of receiver



Let $x(t)$, $0 \le t \le T$ be the received symbol.

$$x(t) = s_i(t) + \omega(t) \quad , \quad 0 \le t \le T$$
$$i = 1, 2, 3, 4$$

where $\omega(t)$ represents additive, white Gaussian noise with zero mean and PSD $\frac{N_0}{2}$ ie, variance $\frac{N_0}{2}$.

Probability of error.

Suppose that $s_4(t)$ was transmitted.

From the block diagram of receiver, we have

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