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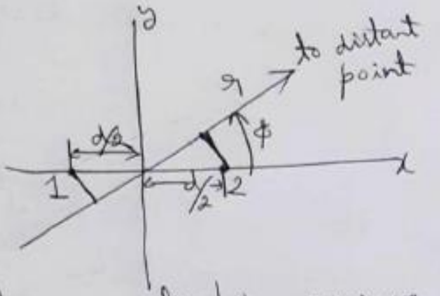
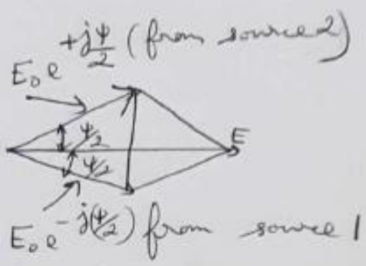
Internal Assesment Test-III								
Sub:	Microwave and Antennas						Code:	18EC63
Date:	05/07 /2023	Duration:	90 mins	Max Marks:	50	Sem: 6th	Branch:	ECE(A,B,C,D)
Answer any **FIVE FULL** Questions								

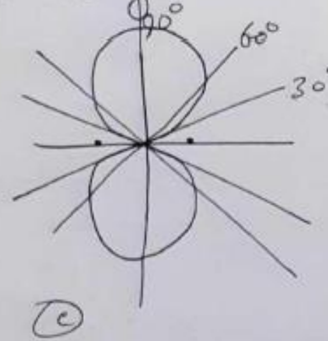
		Marks	**OBE** CO	RBT
1.	Obtain the field pattern for two sources symmetrically placed w.r.t. the origin. Two sources are fed with equal amplitude and phase signals, assume distance between the sources is given by λ/2.	[10]	CO5	L2





(a) Two isotropic sources separated by distance d and located symmetrically w.r.t origin.

(b)



Field pattern of two isotropic point sources of equal amplitude and same phase as located in fig (a)

(c)

→ Let's consider two isotropic point sources, 1 and 2, separated by a distance d and located symmetrically w.r.t the origin of the co-ordinates.
→ Angle φ is measured counter-clockwise from the +ve x-axis.
→ The origin of co-ordinates is taken as the reference for phase

Let d_r be the distance b/w the sources expressed in radians

i.e. $d_r = \frac{2\pi}{\lambda} d = \beta d$

The total field at a large distance r in the direction ϕ is then,

$$E = E_0 e^{-\frac{j\psi}{2}} + E_0 e^{+\frac{j\psi}{2}} \quad \text{---} (1)$$

where, $\boxed{\psi = d_r \cos\phi}$ and the amp. of the field components at the distance r is given by E_0.

From eqn. (1)

$$E = \frac{2E_0 e^{-\frac{j\psi}{2}} + E_0 e^{+\frac{j\psi}{2}}}{2}$$

$$= 2E_0 \frac{\cos\psi/2 - j\sin\frac{\psi}{2} + \cos\frac{\psi}{2} + j\sin\frac{\psi}{2}}{2}$$

$$= 2E_0 \left(\frac{2\cos\frac{\psi}{2}}{2}\right) = 2E_0 \cos\frac{\psi}{2}$$

∴ Phase of the total field is constant $= 2E_0 \cos\left(\frac{d_r}{2}\cos\phi\right)$ ---(3)

$E = \cos\left(\frac{\pi}{2}\cos\phi\right)$ if $d = \lambda/2 \Rightarrow d_r = \pi$

(note: d_r = phase introduced when move by a distance d.

$E = \cos\left(\frac{\pi}{2}\cos\phi\right)$. (4)

For maximum, $\frac{\pi}{2}\cos\phi = \pm k\pi$

~~for k=0~~, for $k=0$, $\cos\phi = 0$

$\Rightarrow \phi = \pm 90°$, ~~180°~~.

For minima,

$\frac{\pi}{2}\cos\phi = \pm(2k+1)\pi/2$

$\frac{\pi}{2}\cos\phi = \pm\pi/2$

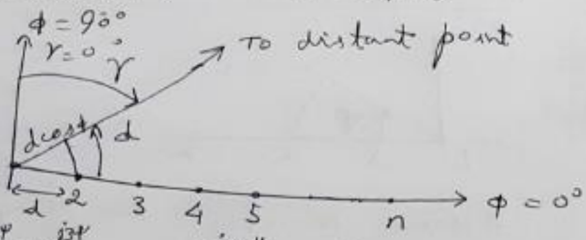
$\Rightarrow \cos\phi = \pm 1$, $\Rightarrow \phi = 0°, 180°$

2. Derive an array factor expression in case of linear array of n isotropic point sources of equal amplitude and spacing. [10] CO5 L2

Linear Arrays of n isotropic point sources of equal amplitude and spacing

$$E = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots e^{j(n-1)\psi} \quad (1)$$

$$\psi = \frac{2\pi}{\lambda} d\cos\phi + \delta = d_r \cos\phi + \delta$$



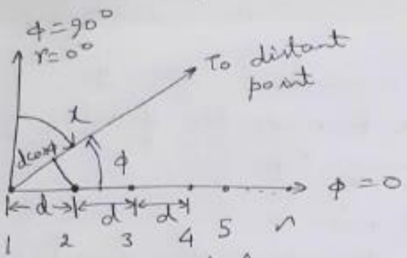
$$E e^{j\psi} = e^{j\psi} + e^{j2\psi} + e^{j3\psi} \cdots + e^{jn\psi} \quad (2)$$

$$\Rightarrow E(1 - e^{j\psi}) = 1 - e^{jn\psi}$$

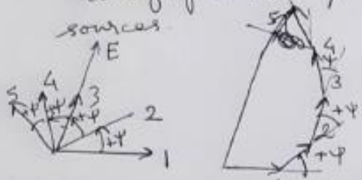
$$\Rightarrow E = \frac{1 - e^{jn\psi}}{1 - e^{j\psi}} = \frac{e^{j\psi\frac{n}{2}}\left(e^{jn\psi/2} - e^{-jn\psi/2}\right)}{e^{j\psi/2}\left(e^{j\psi/2} - e^{-j\psi/2}\right)}$$

$$e^{j(n-1)\psi/2} \frac{\sin(n\psi/2)}{\sin(\psi/2)} = e^{j\xi} \left(\frac{\sin(n\psi/2)}{\sin\psi/2}\right) \quad \text{where } \xi = \frac{(n-1)\psi}{2}$$

for $\psi = 0$, $E = n$



Arrangement of linear array of n isotropic point sources



Linear array of five isotropic point sources of equal amplitude with source 1 as the phase center

$$E = e^{\frac{j(n-1)\psi}{2}} \frac{\sin(n\psi/2)}{\sin\left(\frac{\psi}{2}\right)}$$

$$= \frac{\sin\left(\frac{n\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \angle \xi$$

Where ξ is referred to the field from source 1

where $\xi = \frac{(n-1)}{2}\psi$

$$\lim_{\psi \to 0} \frac{\sin\left(\frac{n\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} = \lim_{\psi \to 0} \frac{\frac{d}{d\psi}\left(\sin\frac{n\psi}{2}\right)}{\frac{d}{d\psi}\left(\sin\frac{\psi}{2}\right)}$$

$$= \frac{n\psi/2}{\psi/2} = n$$

when ψ is in radian

$$E = \frac{\sin(n\psi/2)}{(n\psi/2)} \Big/ \frac{\sin(\psi/2)}{(\psi/2)}$$

$\sin\left(\frac{n\psi}{2}\right) = 1$

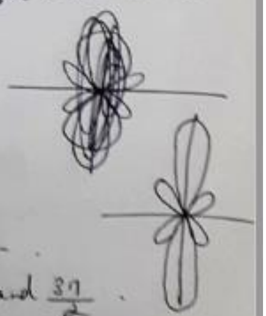
$\therefore E = n$.

This is the maximum value for E.

$\therefore$ The normalized field for $E_{max} = n$ is

$$\boxed{E = \frac{1}{n} \frac{\sin\left(\frac{n\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}} \rightarrow \text{Array factor}$$

Broadside array (Sources of same amplitude and phase) :—

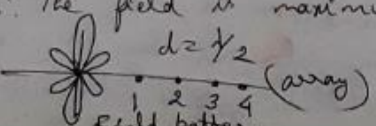


$\therefore \psi = d_r \cos\phi \;,\; \delta = 0$.

To make $\psi = 0$ (i.e. to get maxima)

$\phi = (2k+1)\pi/2$ where, $k = 0, 1, 2, 3 \ldots$

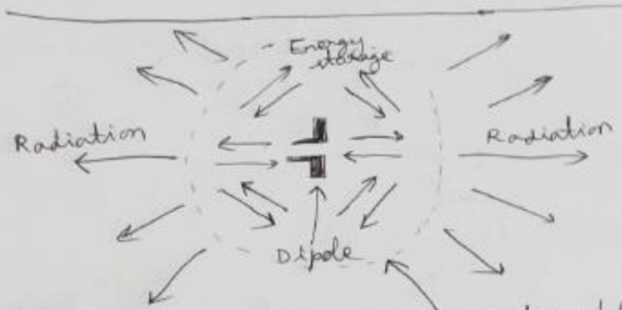
$\therefore$ The field is maximum when, $\phi = \pi/2$ and $\frac{3\pi}{2}$.



Field pattern

3. Make use of Poynting theorem to derive the expression for radiation resistance of a short dipole with uniform current. [10] CO5 L2

Radiation Resistance of short Electric Dipole



Within the radian sphere at $r = \frac{\lambda}{2\pi} = 0.16\lambda$, the situation is like that inside a resonator with high-density pulsating energy accompanied by leakage which is radiated

The average Poynting vector is given by,

$$\vec{S} = \frac{1}{2} \mathrm{Re}\,(\vec{E} \times \vec{H}^*) \quad \text{--} (1)$$

The far-field components are E_θ, H_ϕ.

∴ The radial component of the Poynting vector is,

$$S_r = \frac{1}{2} \mathrm{Re}\, E_\theta H_\phi^* \quad \text{--} (2)$$

where, E_θ and H_ϕ^* are complex.

Now, $\frac{E_\theta}{H_\phi} = Z$

$$\Rightarrow E_\theta = H_\phi Z = H_\phi \sqrt{\frac{\mu}{\epsilon}}.$$

∴ Equation (2) becomes,

$$S_r = \frac{1}{2} \mathrm{Re}\, Z H_\phi H_\phi^*$$

$$= \frac{1}{2} |H_\phi|^2 \mathrm{Re}\, Z$$

$$= \frac{1}{2} |H_\phi|^2 \sqrt{\frac{\mu}{\epsilon}} \quad \text{--} (3)$$

$I_0 \rightarrow$ maximum current,

∴ The corresponding r.m.s current $= \frac{I_0}{\sqrt{2}}$.

$$\therefore \sqrt{\frac{\mu}{\epsilon}}\ \frac{\beta^2 I_0^2 L^2}{12\pi} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R_r$$

$$\Rightarrow \sqrt{\frac{\mu}{\epsilon}}\ \frac{\beta^2 L^2}{6\pi} = R_r$$

$$\Rightarrow \boxed{R_r = \sqrt{\frac{\mu}{\epsilon}}\ \frac{\beta^2 L^2}{6\pi}}$$

For air or vacuum, $\sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 = 120\pi\ \Omega$

∴ Radiation resistance of a dipole with uniform current is,

$$R_r = \cancel{120\pi}^{\,20} \cdot \frac{\beta^2 L^2}{\cancel{6\pi}}$$

$$= 20 \cdot \frac{(2\pi)^2}{\lambda^2} \cdot L^2 = 80\pi^2 \left(\frac{L}{\lambda}\right)^2$$

$$= 80\pi^2 L_\lambda^2 = 790 L_\lambda^2\ (\Omega)$$

Let, $L_\lambda = \frac{1}{10}$.

Then $R_r = 7.9\ \Omega$

if $L_\lambda = 0.01$

$R_r = 0.079 = 0.08\ \Omega$

∴ The radiation resistance of a short dipole is small.

4. Compute peak angles, null angles, side lobe angles, HPBW, BWFN for a linear array of isotropic point sources with n =6, d = λ/2, δ = 0. [10] CO5 L3

$d = \lambda/2$

$\therefore d_r = \frac{2\pi}{\lambda} d = \frac{2\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{2} = \pi$

$\psi = d_r \cos\phi = \pi \cos\phi$

The directions of the peaks are obtained when,

$\psi = 0$

$\Rightarrow \pi \cos\phi = 0$

$\Rightarrow \cancel{\phi = \cos^{-1}(0)}$

$\Rightarrow \phi = \pm(2k+1)\frac{\pi}{2}$

for $k = 0, 1, 2 \ldots$

when, $k = 0$, $\phi = \pm\frac{\pi}{2}$

Side lobes:

$\sin\left(\frac{n\psi}{2}\right) = 1$

$\Rightarrow \frac{n\psi}{2} = \pm(2k+1)\frac{\pi}{2}$

$= \frac{n\cancel{\pi}}{2}\cos\phi = \pm(2k+1)\cancel{\pi}/\cancel{2}$

$\Rightarrow \cos\phi = \pm(2k+1)\frac{1}{n}$

$\Rightarrow \phi = \pm\cos^{-1}\left[\pm\frac{(2k+1)}{6}\right]$

$k = 0$, $\phi = \pm 80.4^\circ$, $\cancel{97.6}$ $\rightarrow k = -1$

$k = 1$, $\phi = \pm\cos^{-1}\left(\frac{1}{2}\right) = \pm 60^\circ$

$k = 2$, $\phi = \pm 33.5^\circ$ $\cancel{\pm 146.5^\circ}$

$k = -2$, $\phi = 120^\circ$.

The possible nulls are given by,

$$\sin\left(\frac{n\psi}{2}\right)=0$$

$$\Rightarrow \frac{n\psi}{2} = \pm k\pi$$

$$\Rightarrow \frac{n}{2}\, d_r \cos\phi = \pm k\pi$$

$$\Rightarrow \frac{n}{2}\, \pi \cos\phi = \pm k\pi$$

$$\Rightarrow \cos\phi = \pm \frac{2k}{n} = \pm \frac{k}{3}$$

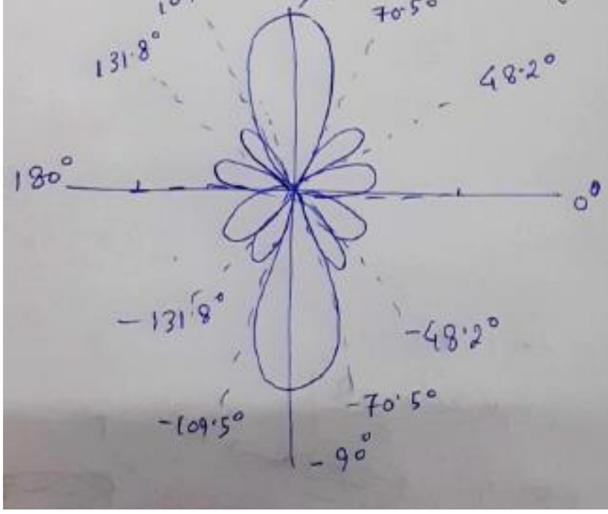
when $k=0$, $\cos\phi = 0$

$$\Rightarrow \phi = \pm 90^\circ \text{ (not possible)}$$

$k=1$, $\phi = \pm 70.5^\circ$, $\underset{k=-1}{\pm 109.5^\circ}$

$k=2$, $\phi = \pm \cos^{-1}\left(\pm \frac{2}{3}\right) = \pm 48.2^\circ$, $\underset{k=-2}{\pm 131.8^\circ}$

$k=3$, $\phi = \pm \cos^{-1}(\pm 1) = \pm 0^\circ, \pm 180^\circ$.



5. Derive the far field expression for small loop antenna. [10] CO5 L2

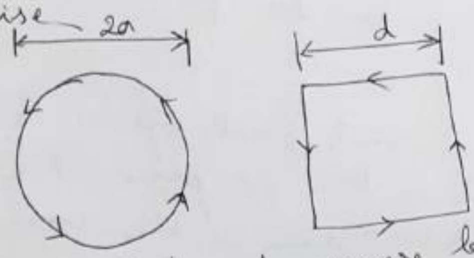
The small loop

Consider a square loop of the same area.

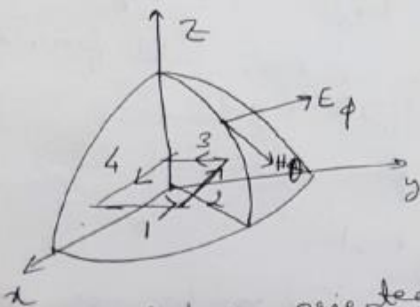
i.e. $d^2 = \pi a^2$. (1)

where, d = side length of square loop.

When loop dimensions are small compared to wavelength, the field patterns are same, but not otherwise



circular loop and square loop of equal area



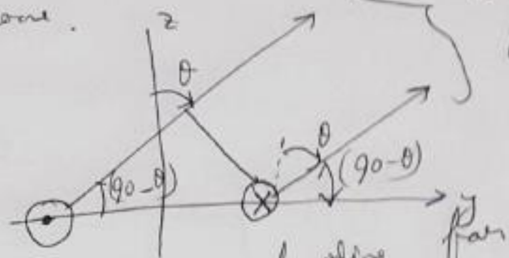
If the loop is oriented as above, its far electric field has only an E_ϕ component.

To find the far-field pattern in the yz plane, it is only necessary to consider two of the four small linear ~~antenna~~ dipoles (2 and 4).

E_θ for short dipole,

$$E_\theta = \frac{I_0 e^{j\omega(t - r/c)} L \sin\theta}{4\pi \epsilon_0}\left(\frac{j\omega}{c^2 r} + \frac{1}{c r^2} + \frac{1}{j\omega r^3}\right)$$

A cross-section through the loop in the yz plane is as follows.



Construction for finding far field of dipoles 2 & 4.

The individual small dipoles 2 and 4 are nondirectional in the yz plane, the field pattern of the loop in this plane is the same as that for two isotropic point sources.

$$\therefore E_\phi = E_{\phi_0} e^{-j\psi/2} - E_{\phi_0} e^{j\psi/2} \quad \cdots (2)$$

where, E_{ϕ_0} = electric field from individual dipole and,

$$\psi = \frac{2\pi}{\lambda} d \sin\theta = d_r \sin\theta \quad \cdots (3)$$

It follows that,

$$E_\phi = -2j E_{\phi_0} \sin\left(\frac{d_r}{2} \sin\theta\right) \cdots (4)$$

∴ The total field E_ϕ is in phase quadrature with the field E_{ϕ_0} of the individual dipole.

Now, if $d \ll \lambda$,

$$E_\phi = -2j E_{\phi_0} \frac{d_r}{2} \sin\theta$$

$$= -j E_{\phi_0} d_r \sin\theta \quad \cdots (5)$$

Here $\theta = 90^\circ$ in the dipole formula as developed with the dipole along the z-axis

In this case, dipole along x direction

$\therefore \theta = 90^\circ$.

$\therefore$ The field of this dipole is,

$$E_{\phi_0} = \frac{j\,60\pi [I]}{r} \frac{L}{\lambda} \sin(90^\circ)$$

$$= \frac{j\,60\pi [I]\, L}{r\lambda} \quad \cdots (6)$$

$$\therefore E_\phi = \frac{60\pi [I]\, L\, d_r \sin\theta}{r\lambda} \quad \cdots (7) = \frac{60\pi [I]\, d}{r\lambda}, \frac{2\pi}{\lambda} d$$

In this case $L = d$ [as they are in a square loop]

Also, $d_r = \frac{2\pi}{\lambda} d$.

And the area of the loop is $d^2 = A$

$$\therefore E_\phi = \frac{120\pi^2 [I] \sin\theta}{r} \frac{A}{\lambda^2} \quad \text{[for } E_\phi \text{ field]}$$

This is instantaneous value of the E_ϕ component of the far-field of a small loop area.

To get the peak value of the current, $[I]$ is replaced by I_0,

where, I_0 is the peak current in time on the loop.

$$\therefore H_\theta = \frac{E_\phi}{120\pi} = \frac{\pi [I] \sin\theta}{r} \frac{A}{\lambda^2}$$

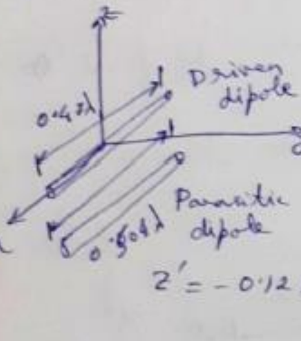
6. Explain the constructional details of the following antenna [10] CO5 L2

a. Yagi-Uda array

Yagi - Uda Array

Dipole array with only one excited dipole and other dipoles parasitically coupled to it.

- The induced currents and hence the radiation characteristics depend on the length of the dipole and the spacing b/w the dipoles.
- consider a dipole radiating in free space.
- A second dipole with its terminals short-circuited is kept ∥ to it.
- A current is established on the second dipole due to e.m. induction.



Let current on element 1 = 1A

Induced current = $0.7\angle 1$

$AF = I_1 e^{jkz_1' \cos\theta} + I_2 e^{jk}$

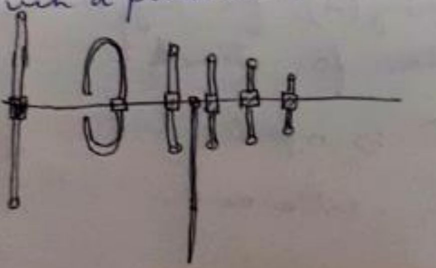
If $z_1' = 0$, $z_2' = -0.125\lambda$

θ measured from z-axis,

$AF = 1 + 0.7 e^{j\left(\frac{7\pi}{9} - \frac{\pi}{4}\right)}$

Antenna radiates more power along +z direction as compared to the -z direction.

- Parasitic element reflecting the field incident on it.
- Such a parasitic element is called a reflector.



6 element Yagi-Uda antenna with folded dipole.

A log-periodic antenna consists of four regions

a) Reflective region $(l > \lambda/2)$

b) active region $(l \approx \lambda/2)$

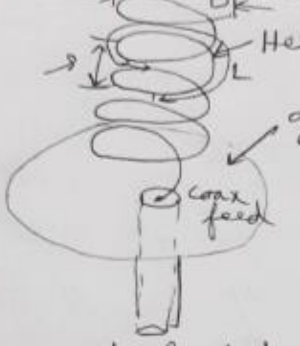
c) directive region $(l < \lambda/2)$

d) Transmission line region.

b. Axial mode helical antenna

Helical antenna

- Reduces the physical size of a wire antenna
- Wire wind over an imaginary cylinder over ground p



Right-handed helical antenna.

N turns of diameter with spacing S b/w each turn.

Let, $C = \pi D$

C → circumference of the cylindrical surfa over which the helix wound.

$L \rightarrow$ un wrapped length of one turn of the

$$L = \sqrt{S^2 + C^2}$$

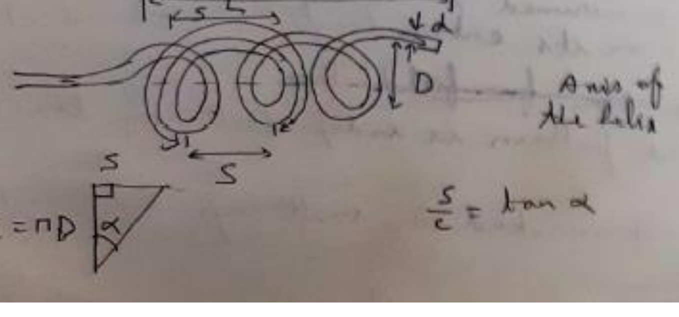
Pitch angle α : Angle b/w the tangent to the and the plane $\perp^r$ to the axis of the helix.

$$\tan\alpha = \frac{S}{C} = \frac{S}{\pi D}.$$

For $\alpha = 0°$, helix becomes a loop of N-turns.

$\alpha = 90°$, " is the same as a straight wire of length NL.

- Fed by a coaxial transmission line with the center conductor attached to the helical wire and outer conductor to the ground plane



$\frac{S}{C} = \tan\alpha$

Axial mode helix :-

Helix radiates as an endfire antenna with single main beam along the axis of the helix (+z direc

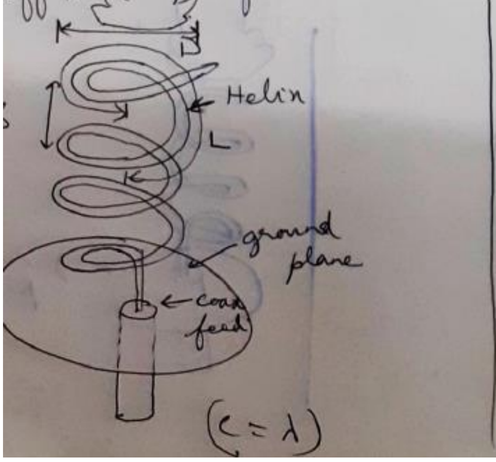
- Radiation is circularly polarized near the axis.
- Main beam narrows as turns are added to the helix. Helix circumference is on the order of a wavelength

For axial mode, $\frac{3}{4}\lambda \le C \le \frac{4}{3}\lambda$.

$$\therefore \text{BW ratio}, \ B_r = \frac{f_U}{f_L} = \frac{\frac{c}{\lambda_U}}{\frac{c}{\lambda_L}} = \frac{\frac{4}{3}}{\frac{3}{4}} = \frac{16}{9} = 1$$

- Helix carries a travelling wave that travels outward from the feed.
- The electric field associated with this travell wave rotates in a circle.
- Producing radiation nearly circularly polarize off the end of the helix.

For axial mode, $12^\circ < \alpha < 15^\circ$.



7. Find the length L, H-plane aperture, and flare angle θ_E and θ_H of pyramidal horn for which E-plane aperture is 10λ. Horn is fed by a rectangular waveguide with TE10 mode. Assume $\delta = 0.2\lambda$ in E-plane and 0.375λ in H-plane. Also find E-plane, H-plane beamwidths and directivity. [10] CO5 L3

Soln. $\delta = 0.2\lambda = \frac{\lambda}{5}$ in the E-plane

$$\therefore L = \frac{a^2}{8\delta} = \frac{100\lambda^2}{8\cdot\frac{\lambda}{5}} = \frac{100\lambda}{\frac{8}{5}} = 62.5\lambda$$

$$\theta_E = 2\tan^{-1}\left(\frac{a}{2L}\right) = 2\tan^{-1}\left(\frac{10\lambda}{2\times 62.5\lambda}\right)$$

$$= 2\tan^{-1}\left(\frac{10}{125}\right) = 9.1^\circ.$$

given $\delta = \frac{3\lambda}{8} = 0.375\lambda$ in the H-plane.

$$\therefore \theta_H = 2\cos^{-1}\frac{L}{L+\delta} = 2\cos^{-1}\frac{62.5}{62.5+0.375}$$

$$= 12.52^\circ.$$

$\therefore$ The H-plane aperture,

Now, $\theta = 2\tan^{-1}\left(\frac{a}{2L}\right)$

$$\Rightarrow \tan\left(\frac{\theta}{2}\right) = \frac{a}{2L}$$

$$\Rightarrow a_H = 2L\tan\left(\frac{\theta}{2}\right)$$

$$= 2\times 62.5\lambda \tan(6.26^\circ) = 13.7\lambda.$$

For optimum E-plane rectangular horn,

$$HPBW = \frac{56}{a_E} = \frac{56^\circ}{10} = 5.6^\circ$$

Optimum H-plane rectangular horn

$$HPBW = \frac{67}{a_H} = \frac{67}{13.7} = 4.9^\circ$$

Directivity, $D \simeq 10\log\left(\frac{7.5A_p}{\lambda^2}\right) = 10\log\left(\frac{7.5\times a_E\times a_H}{\lambda^2}\right)$

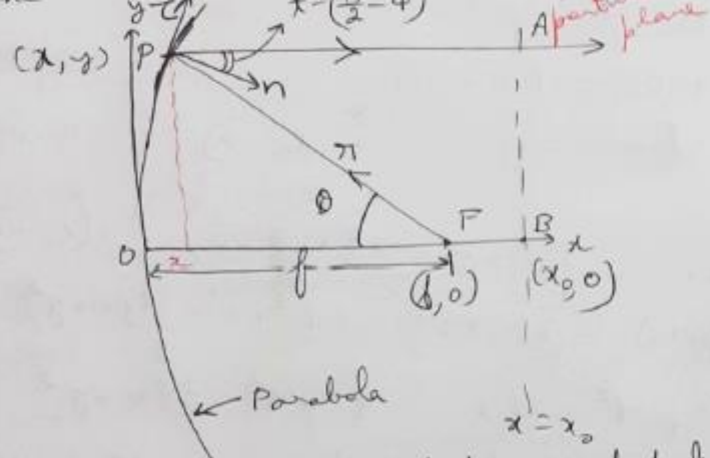
$$= 10\log(7.5\times 10\times 13.7) = 30.1\ dBi.$$

8. Explain parabolic reflector antenna with relevant diagram. [10] CO5 L2

Parabolic Reflectors

- A parabolic cylinder is a reflector shape whose cross-section in the xy-plane is a parabola.
- This cross-section is independent of z.
- It converts a cylindrical wavefront into a plane wavefront after reflection.

- Let's consider an infinite cylindrical surface excited by a line source.
- The cross-section is shown in the figure.



Parabolic cylinder reflector excited by a line source at F

- The line current source at point F radiates in all directions.

- One such ray FP is incident at a point P(x, y) on the reflector.
- The corresponding reflected ray is PA.

- To create an equi-phase front on the surface $x=x_0$, the total path length b/w the source point F and the point A on the aperture surface must be the same for each ray.
- Also, the reflected rays must be ‖ to the x-axis and Snell's law must be satisfied at the reflection point P.

Let's assume, the reflected rays are $\|^{el}$ to ...
and derive the expression for the reflector surface
which makes the path length same for each ray.

Let, the co-ordinates of point P be (x, y)
" " " line-source be $(f, 0)$

– Ray along $-x$ direction will be reflected back
point B on the aperture surface
Equating the two path lengths we get,

$$FO + OB = FP + PA \quad \text{--- (1)}$$

i.e. $$f + x_0 = \left\{(f-x)^2 + y^2\right\}^{1/2} + (x_0 - x)$$

$$\Rightarrow f + x_0 = \left(f^2 + x^2 - 2fx + y^2\right)^{1/2} + (x_0 - x)$$

$$\Rightarrow \left\{(f+x_0) - (x_0 - x)\right\}^2 = f^2 + x^2 - 2fx + y^2$$

$$\Rightarrow \cancel{f^2} + \cancel{x^2} + 2fx = \cancel{f^2} + \cancel{x^2} - 2fx + y^2$$

$$\Rightarrow \boxed{y^2 = 4fx} \rightarrow \text{eqn. of parabola} \quad \text{--- (2)}$$

with f as the focal length.
Differentiating (2) w.r.t x we get,

$$\tan\phi = \frac{dy}{dx} = \frac{2f}{y} \quad \text{--- (3)}$$

eqn. (3) is the slope of the tangent to the parabola at $P(x, y)$.
$\left(\frac{\pi}{2} - \phi\right) = t \rightarrow$ angle b/w the normal to parabola at $P(x, y)$ and the x-axis.