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CMR INSTITUTE OF TECHNOLOGY	USN								
	Internal Assessment Test II Aug 2023							Code:	21MAT41
Sub:	**Complex analysis, , Proability and Statistical methods**								
Date:	08/08/2023	Duration:	90 mins	Max Marks:	50	Sem:	IV	Branch:	**EEE,ECE,CV**

<table>
<tr><th colspan="2" rowspan="2">Question 1 is compulsory and Answer any 6 from the remaining questions.</th><th rowspan="2">Marks</th><th colspan="2">OBE</th></tr>
<tr><th>CO</th><th>RBT</th></tr>
<tr><td>1</td><td><table><tr><td>X \ Y</td><td>-4</td><td>2</td><td>7</td></tr><tr><td>1</td><td>1/8</td><td>1/4</td><td>1/8</td></tr><tr><td>5</td><td>1/4</td><td>1/8</td><td>1/8</td></tr></table>Find correlation of X and Y also check if X and Y are independent or not</td><td>[8]</td><td>CO5</td><td>L3</td></tr>
<tr><td>2</td><td>A random variable X has density function f(x)=$f(x) = \begin{cases} kx^2 , & -3 < x < 3 \\ 0 , & otherwise \end{cases}$
Find k, $p(1 \leq x \leq 2), p(x \leq 2), p(x > 1)$</td><td>[7]</td><td>CO4</td><td>L3</td></tr>
<tr><td>3</td><td>Derive mean and variance of binomial distribution.</td><td>[7]</td><td>CO4</td><td>L3</td></tr>
</table>

4	In a quiz contest of answering 'Yes' or 'No' what is the probability of guessing atleast 6 answers correctly out of 10 questions asked? Also, find the probability of the same if there are 4 options for a correct answer.	[7]	CO4	L3
5	If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals, more than two will get a bad reaction.	[7]	CO4	L3
6	In a test on electric bulbs, it was found that the lifetime of a particular brand was distributed normally with an average life of 2000 hours & SD of 60 hours. If a firm purchases 2500 bulbs find the number of bulbs that are likely to last for a) More than 2100 hours b) Less than 1950 hours c) Between 1900 to 2100 hours Given.Ø(1.67) = 0.4525 ; $\Phi(0.83) = 0.2967$	[7]	CO4	L3
7	In a normal distribution 31% are under 45 and 8% are over 64. Find the mean and S.D of the distribution. Given.Ø(0.5) = 0.19 .Ø(1.4) = 0.42	[7]	CO4	L3
8	Define 1) Null and alternate hypothesis ,2) type I and type II error 3) level of significance	[7]	CO5	L3

$COV(X, Y)$ (c) $\rho(X, Y)$

The distribution (*marginal distribution*) of X and Y is as follows.
is distribution is obtained by adding the all the respective row entries and
o the respective column entries.

stribution of X :

x_i	1	5
$f(x_i)$	1/2	1/2

Distribution of Y :

y_j	−4	2	7
$g(y_j)$	3/8	3/8	1/4

a) $E(X) = \sum x_i\, f(x_i) = (1)(1/2) + (5)(1/2) = 3$

$$E(Y) = \sum y_j\, g(y_j)$$

$$= (-4)(3/8) + (2)(3/8) + (7)(1/4) = 1$$

Thus, $\boxed{\mu_X = E(X) = 3 \text{ and } \mu_Y = E(Y) = 1}$

(b) $E(XY) = \sum x_i\, y_j\, J_{ij}$

$$= (1)(-4)(1/8) + (1)(2)(1/4) + (1)(7)(1/8)$$

$$+ (5)(-4)(1/4) + (5)(2)(1/8) + (5)(7)(1/8)$$

$$= \frac{-1}{2} + \frac{1}{2} + \frac{7}{8} - 5 + \frac{5}{4} + \frac{35}{8} = \frac{3}{2}$$

Thus, $\boxed{E(XY) = 3/2}$

(c) $\sigma_X^2 = E(X^2) - \mu_X^2$ and $\sigma_Y^2 = E(Y^2) - \mu_Y^2$

Now, $E(X^2) = \sum x_i^2\, f(x_i)$

ie., $E(X^2) = (1)(1/2) + (25)(1/2) = 13$

Also, $E(Y^2) = \sum y_j^2\, g(y_j)$

ie., $E(Y^2) = 16(3/8) + (4)(3/8) + (49)(1/4) = 79/4$

Hence, $\sigma_X^2 = 13 - (3)^2 = 4$; $\sigma_Y^2 = (79/4) - (1)^2 = 75/4$

Thus, $\boxed{\sigma_Z = 2 \quad \text{and } \sigma_Y = \sqrt{75/4} = 4.33}$

(d) $COV(Z,Y) = E(ZY) - \mu_Z \mu_Y$

$$= (3/2) - (3)(1) = -3/2$$

$\therefore \quad \boxed{COV\ (Z\ ,Y) = -3/2}$

(e) $$\rho\ (Z\ ,Y) = \frac{COV(Z,Y)}{\sigma_Z \sigma_Y}$$

$$= \frac{-3/2}{(2)\sqrt{75/4}} = \frac{-3}{2\sqrt{75}}$$

Thus, $\boxed{\rho(Z,Y) = -0.1732}$

So $f(x_1) = 1/2$, $g(y_1) = 3/8$, $J_{ij} = 1/8$

$J_{ij} \neq f(x_1)\, g(y_1)$

$\Rightarrow$ X and Y are not independent

or dependent

2) That is, $\int_{-3}^{3} k x^2 \, dx = 1$ [Since $\int_{-\infty}^{\infty} p(x) = 1$]

or $\left[\frac{k x^3}{3}\right]_{-3}^{3} = 1 \quad \therefore \boxed{k = \frac{1}{18}}$

(i) $P(1 \le x \le 2) = \int_{1}^{2} \frac{x^2}{18} dx = \left[\frac{x^3}{54}\right]_{1}^{2} = \frac{1}{54}(8-1) = \boxed{\frac{7}{54}}$

(ii) $P(x \le 2) = \int_{-3}^{2} \frac{1}{18} x^2 \, dx = \frac{1}{18}\left[\frac{x^3}{3}\right]_{-3}^{2} = \frac{1}{54}(8+27) = \boxed{\frac{35}{54}}$

(iii) $P(x > 1) = \int_{1}^{3} \frac{1}{18} x^2 \, dx = \frac{1}{18}\left[\frac{x^3}{3}\right]_{1}^{3} = \frac{1}{54}(27-1) = \frac{26}{54} = \boxed{\frac{13}{27}}$

Proof: Mean $\mu = \sum x_i \, p(x_i)$

$$= \sum_{x_i=0}^{n} x_i \; nC_{x_i} \, p^{x_i} \, q^{n-x_i}$$

$$= 0 + nC_1 \, p^1 q^{n-1} + 2 nC_2 \, p^2 q^{n-2} + \cdots +$$

$$= np \, q^{n-1} + \frac{\cancel{2} n(n-1)}{\cancel{2}} p^2 q^{n-2} + \cdots + n$$

$$= np \left[q^{n-1} + (n-1) p \, q^{n-2} + \cdots + p^{n-}\right.$$

$$= np (q+p)^{n-1}$$

$$\boxed{\mu = np}$$ (Since $p+q=1$)

Variance: $V = \sum \underline{x_i^2}\, p(x_i) - \mu^2$

$= \sum \underline{\left(x_i (x_i - 1) + x_i\right)}\, p(x_i) - \mu^2$

$= \sum^{n} x_i (x_i - 1)\, \underline{p(x_i)} + \sum x_i\, p(x_i) - \mu^2$

expanding $x_i = 0$

$= 0 + 0 + \sum_{x_i = 2}^{n} x_i (x_i - 1)\, \underline{{}^{n}C_{x_i}\, p^{x_i} q^{n - x_i}} + \mu - \mu^2$

$= \cancel{2}\, \frac{n(n-1)}{\cancel{2!}}\, p^2 q^{n-2} + \frac{3 \times 2\, n(n-1)(n-2)\, p^3 q^{n-3}}{3!} + \cdots + n^{th}\ \text{term} + \mu - \mu^2$

$= n(n-1)\, p^2 \left[q^{n-2} + (n-2)\, p\, q^{n-3} + \cdots + p^{n-2} \right] + \mu - \mu^2$

$= n(n-1)\, p^2\, (q + p)^{n-2} + \mu - \mu^2$

$= (n^2 - n)\, p^2 + \mu - \mu^2 \quad (\because q + p = 1)$

$= n^2 \cancel{p^2} - np^2 + np - \cancel{n^2 p^2}$

$= np(1 - p)$

$\sigma^2 = npq$

$\therefore \sigma = \sqrt{npq}$

Ⓐ In a quiz contest of answering Yes (or) no what is the probability of guessing atleast 6 answers correctly out of 10 Questions asked ?

Ⓑ Also find the probability of the same if there are 4 options for the correct answer ?

Soln : How you will decide ito Binomial

$P(x) = nC_x \, p^x q^{n-x}$ ($x \rightarrow$ correct answer

$p(x) \rightarrow$ probability of cho

correct answer)

$n = 10$ (Questions)

$P =$ Prob of success = Answering correctly $= 1/2$

$q = 1 - p = 1 - 1/2 = 1/2$

$\therefore P(x) = 10C_x \, (1/2)^x \, (1/2)^{10-x}$

$= 10C_x \, (1/2)^{x+10-x}$

$= 10C_x \, (1/2)^{10}$

p(Guessing atleast 6 answers correctly) $= p(x \geq 6)$

$= p(x=6) + p(x=7) + p(x=8) + p(x=9) + p(x=10)$

$= 10C_6 (1/2)^{10} + 10C_7 (1/2)^{10} + 10C_8 (1/2)^{10} + \cdots + 10C_{10} (1/2)^{10}$

$p(x \geq 6) =$ (?)

—x—

3) If 4 options are given Im UA

here $p = 1/4$ & $q = 1 - 1/4 = 3/4$

$\therefore p(x) = 10C_x (1/4)^x (3/4)^{10-x}$

$= 10C_x (1/4)^{10} 3^{10-x}$

$p(x \geq 6) = p(6) + p(7) + p(8) + p(9) + p(10)$

$= (1/4)^{10} \left[10C_6 3^{10-6} + 10C_7 3^{10-7} + \cdots + 10C_{10} 3^{10-10} \right]$

$= 0.0197.$

5)

$P = 0.001$ $\mu = np.$

$n = 2000$ $\mu = 2000 \times 0.001 = \boxed{2} = m$

$\therefore \boxed{P = 0.001}$ & $\boxed{m = 2}$

$$P(x) = \frac{e^{-m} m^{x}}{x!} = \frac{e^{-2} 2^{x}}{x!}$$

$$P(x > 2) = 1 - P(x \le 2)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[\frac{e^{-2} 2^{0}}{0!} + \frac{e^{-2} 2^{1}}{1!} + \frac{e^{-2} 2^{2}}{2!}\right]$$

$$= 1 - 0.6766$$

$$= 0.3233$$

6c

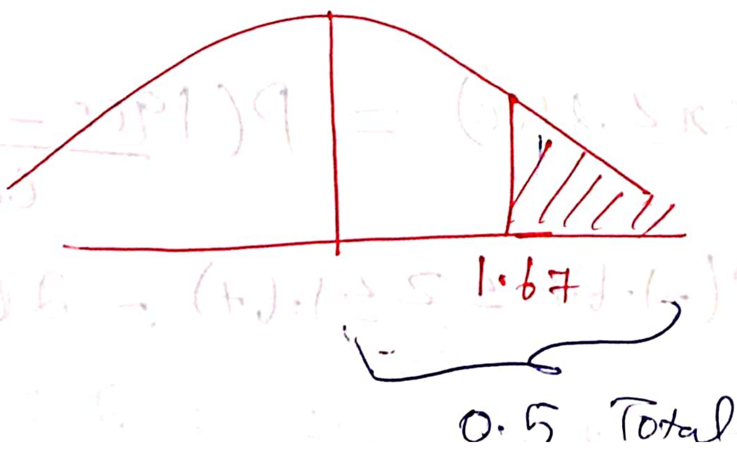
In a test on electric bulbs it was found that the lifetime of a particular brand was distributed normally, with an average of 2000 hours and. σ 60 hours. If a firm purchases 2500 bulbs. Find the number of bulbs that are likely to last for (i) More than 2100 hours (ii) less than 1950 hrs (iii) between 1900 to 2100 hours.

Soln :

Given $\mu = 2000$ $\sigma = 60$, $z = \frac{x-\mu}{\sigma}$

$z = \frac{x-2000}{60}$

① $p(x > 2100) = p\left(z > \frac{2100-200}{60}\right) = p(z > 1.67)$



$= \text{Total } P(0 \text{ to } 1.67)$

$= 0.5 - \Phi(1.67)$

$= 0.5 - 0.4525$

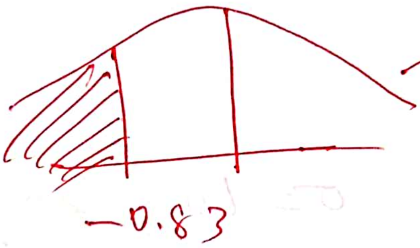
$= 0.0475$

∴ No of bulbs that are likely to last for more than 2100 hours is

$2500 \times 0.0475 = 118.95 \sim 119$ bulbs

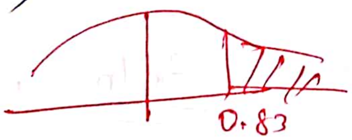
approx

② $P(x < 1950) = P\left(z < \frac{1950-2000}{60}\right)$



$= P(z < -0.83)$

$= P(z > 0.83)$



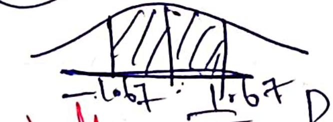
$= 0.5 - \phi(0.83) = 0.5 - 0.2967$

$= 0.2033$

∴ For 2500 bulbs $= 2500 \times 0.2033 = 508.25$

~ 508 bulbs

③ $P(1900 \leq x \leq 2100) = P\left(\frac{1900-2000}{60} \leq z \leq \frac{2100-200}{60}\right)$



$= P(-1.67 \leq z \leq 1.67) = 2\phi(1.67)$.

$= 2 \times 0.4525 = 0.405$

For 2500 bulbs.

$\boxed{\therefore 2500 \times 0.405 = 2262.5 \sim 2263}$

7) By data, $P(x<45) = 0.31$ and $P(x>64) = 0.08$

We have, s.n.v $z = \dfrac{x-\mu}{\sigma}$

When $x = 45$, $z = \dfrac{45-\mu}{\sigma} = z_1$ (say)

$x = 64$, $z = \dfrac{64-\mu}{\sigma} = z_2$ (say)

So we have,

$$P(z<z_1) = 0.31 \text{ and } P(z>z_2) = 0.08$$

ie., $0.5+\phi(z_1) = 0.31$ and $0.5-\phi(z_2) = 0.08$

$\Rightarrow$ $\phi(z_1) = -0.19$ and $\phi(z_2) = 0.42$

Refering to the normal probability tables we have

$$0.1915(\approx 0.19) = \phi(0.5) \text{ and } 0.4192(\approx 0.42) = \phi(1.4)$$

$\therefore$ $\phi(z_1) = -\phi(0.5)$ and $\phi(z_2) = \phi(1.4)$

$\Rightarrow$ $z_1 = -0.5$ and $z_2 = 1.4$

ie., $\dfrac{45-\mu}{\sigma} = -0.5$ and $\dfrac{64-\mu}{\sigma} = 1.4$

or $\mu - 0.5\sigma = 45$ and $\mu + 1.4\,\sigma = 64$

By solving we get $\mu = 50$ and $\sigma = 10$

Thus, $\boxed{\text{Mean} = 50 \text{ and S.D} = 10}$

less than 35% marks and 99% of

5.4 Test of Hypothesis

n order to arrive at a decision regarding the population through a sample f the population, we have to make certain assumption referred to as **ypothesis** *which may or may not be true.* Much depends on the framing of ypothesis.

he hypothesis formulated for the purpose of its rejection under the ssumption that it is true is called the **Null Hypothesis** denoted by H_0.

Any hypothesis which is complimentary to the null hypothesis is called **Alternative Hypothesis** denoted by H_1.

Examples

(1) To test whether a process B is better than a process A we can formulate the hypothesis as *there is no difference between the process A and B.*

(2) To test whether there is a relationship between two variates we can formulate the hypothesis as *there is no relationship between them.*

5.41 Errors : Type-I and Type-II

In a test process there can be four possible situations of which two of the situations leads to the two types of errors and the same is presented as follows.

	Accepting the hypothesis	Rejecting the hypothesis
Hypothesis true	Correct decision	Wrong decision (Type I error)
Hypothesis false	Wrong decision (Type II error)	Correct decision

In order to minimize both these types of errors we need to increase the sample size. It is further important to note that acceptance or non acceptance of a hypothesis is purely based on the information revealed by the sample and what is indicated by a particular sample may not always be true in respect of the population.

A region which amounts to the rejection of null hypothesis is called critical region or region of rejection.

5.42 Significance level

The probability level, below which leads to the rejection of the hypothesis is known as the *significance level.* This probability is conventionally fixed at 0.05 or 0.01 being 5% or 1%. These are called *significance levels.*

We feel confident in rejecting a hypothesis at 1% level of significance than at 5% level of significance. 5% level of significance can also be understood as, the probability of committing errors of either types, (Type I or Type II) is 0.05.

5.43 Tests of significance and Confidence intervals

The process which helps us to decide about the acceptance or rejection of the hypothesis is called the *test of significance.*

Let us suppose that we have a normal population with mean μ and S.D σ. If $\overline{x}$ is the sample mean of a random sample of size n the quantity z defined by

$$z = \frac{\overline{x} - \mu}{(\sigma/\sqrt{n})} \quad \ldots (1)$$

is called the *Standard Normal Variate.* (*S.N.V*)

From the table of normal areas we find that 95% of the area lies between $z = -1.96$ and $z = +1.96$. In other words we can say with 95% confidence that z lies between -1.96 and $+1.96$. Further 5% level of significance is denoted by $z_{0.05}$. We can write the verbal statement in the mathematical form as follows.