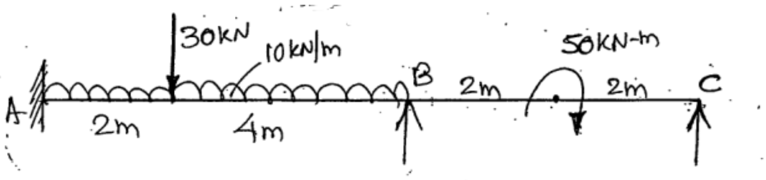
Sub: Analysis of structures-IAT-2 Solutions

Question number 1 is mandatory; answer any 2 full questions from Q2 to Q4.

1 (a) Determine the fixed end moments for the beam shown in Fig.1.a. Consider that EI is constant through the span of continuous beam



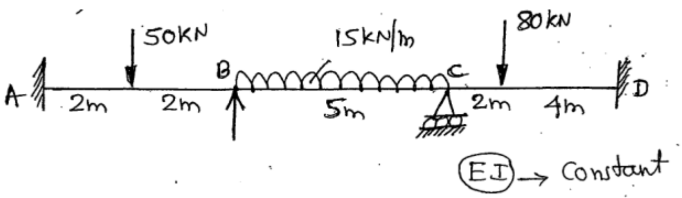
(a) FEM

$$M_{FAB} = -\frac{\omega l^2}{12} - \frac{Wab^2}{l^2} = \frac{-10\times6^2}{12} - \frac{30\times2\times4^2}{6^2} = -56.67$$

$$M_{FBA} = +\frac{\omega l^2}{12} + \frac{Wa^2b}{l^2} = +\frac{10\times6^2}{12} + \frac{30\times2^2\times4}{6^2} = +43.33$$

$$M_{FBC} = M_{FCB} = +\frac{M}{4} = +12.5 \text{ kN-m}$$

1 (b) Analyse the continuous beam shown in Fig.1.b by slope deflection method and draw BMD and EC.



Sol2

(a) FEM

$$M_{FAB} = -\frac{Wl}{8} = \frac{-50\times4}{8} = -25\,kN\text{-}m$$

$$M_{FBA} = +\frac{Wl}{8} = +25\,kN\text{-}m$$

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{15\times5^2}{12} = -31.25$$

$$M_{FCB} = +\frac{wl^2}{12} = +31.25$$

$$M_{FCD} = -\frac{Wab^2}{l^2} = -\frac{80\times2\times4^2}{6^2} = -71.11\,kN\text{-}m$$

$$M_{FDC} = +\frac{Wa^2b}{l^2} = \frac{80\times2^2\times4}{6^2} = +35.56\,kN\text{-}m$$

(b) S.D. Equation:

$\theta_A = \theta_D = 0$ ($\because$ Fixed Support)

$\delta = 0$ ($\because$ No sinking)

$$M_{AB} = \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right] + M_{FAB}$$

$$M_{AB} = \frac{2EI}{4}\left[0+\theta_B-0\right]-25 = 0.5EI\theta_B - 25 \quad \text{—(}$$

$$M_{BA} = \frac{2EI}{4}\left[2\theta_B+0-0\right]+25 = EI\theta_B + 25 \quad \text{—(II)}$$

$$M_{BC} = \frac{2EI}{5}\left[2\theta_B+\theta_C-0\right]-31.25$$

$$= 0.8EI\theta_B + 0.4EI\theta_C - 31.25 \quad \text{—(II}$$

$$M_{CB} = \frac{2EI}{5}\left[2\theta_C+\theta_B-0\right]+31.25$$

$$= 0.8EI\theta_C + 0.4EI\theta_B + 31.25 \quad \text{—(IV}$$

$$M_{CD} = \frac{2EI}{6}\left[2\theta_C + 0 - 0\right] - 71.11 = 0.667\,EI\theta_C - 71.11$$

$$M_{DC} = \frac{2EI}{6}\left[0 + \theta_C - 0\right] + 35.56 = 0.333\,EI\theta_C + 35.$$

(c) Equillibrium Condition :-

(1) At "B" $\boxed{M_{BA} + M_{BC} = 0}$ → Intermediate support

$$\left[EI\theta_B + 25\right] + \left[0.8EI\,\theta_B + 0.4EI\,\theta_C - 31.25\right] = 0$$

$$1.8EI\theta_B + 0.4EI\theta_C = 6.25 \longrightarrow \text{(I)}$$

(2) At "C" $\boxed{M_{CB} + M_{CD} = 0}$

$$\left[0.8EI\theta_C + 0.4EI\,\theta_B + 31.25\right] + \left[0.667EI\,\theta_C - 71.11\right] = 0$$

$$0.4EI\,\theta_B + 1.467\,EI\theta_C = 39.86 \longrightarrow \text{(II)}$$

Solving

$$\theta_B = -2.73/EI$$

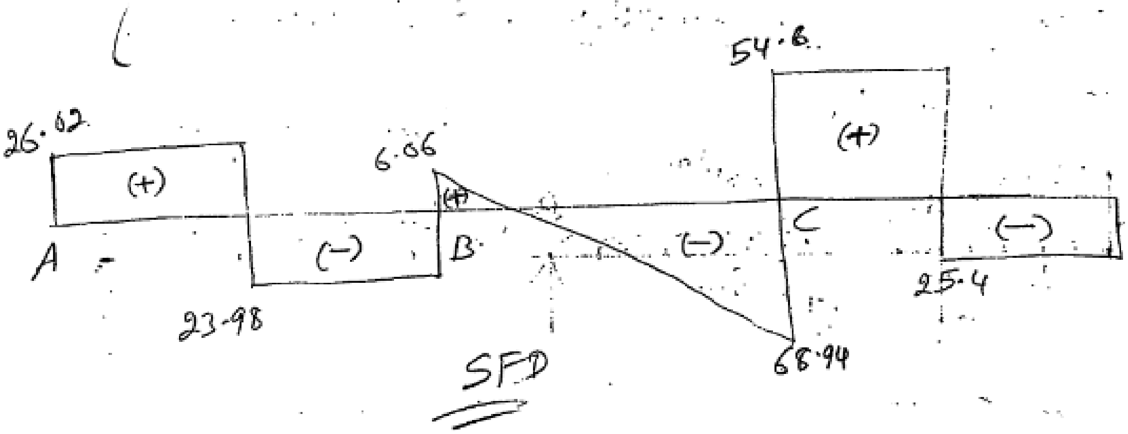
$$\theta_C = +27.91/EI$$

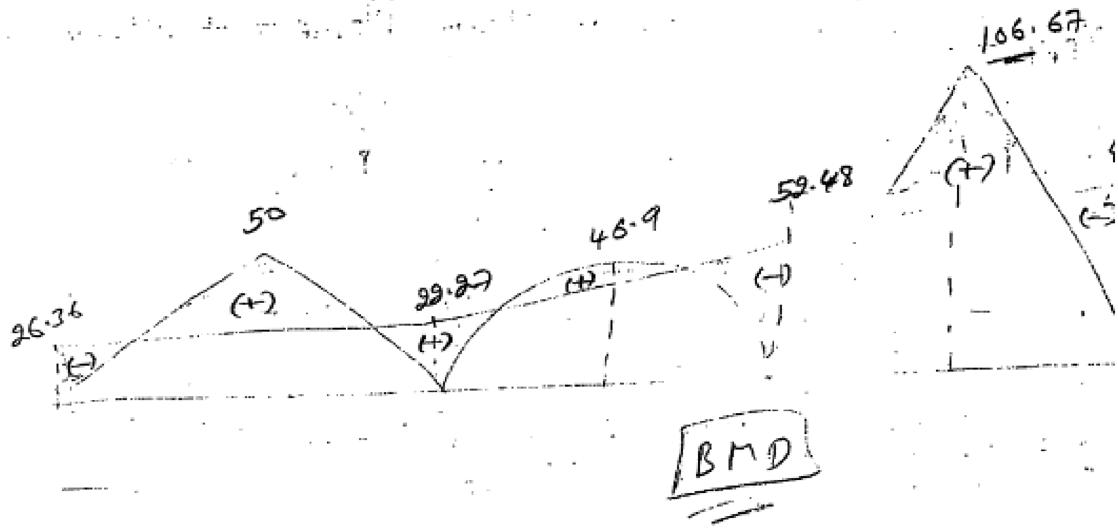
(d) Final Moments [Substitute θ value in eq^n (i) to (vi)]

$M_{AB} = -26.36$ kN-m $M_{CB} = 52.48$ kN-m

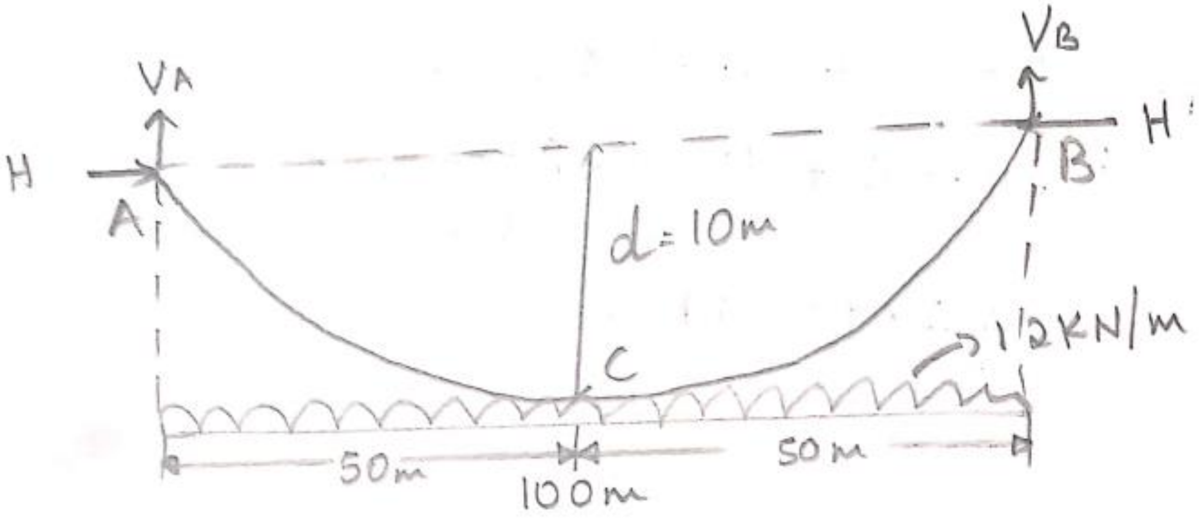
$M_{BA} = 22.27$ kN-m $M_{CD} = -52.49$ kN-m

$M_{BC} = -22.27$ kN-m $M_{DC} = 44.85$ kN-m





2 (a)	A suspension bridge of 100m span has a central dip of 10 m and supports an udl of 12 kN/m throughout the span. Calculate i) The minimum and maximum tension in cable, ii) The size of cable if the permissible stress of the cable material is 180N/mm^2 iii) Length of the cable



$$\Sigma V = 0$$

$$V_A + V_B = 12 \times 100$$

$$V_A + V_B = 1200 \text{ KN}$$

$$\Sigma M_A = 0$$

$$-V_B \times 100 + 12 \times 100 \times \frac{100}{2} = 0$$

$$V_B = 600$$

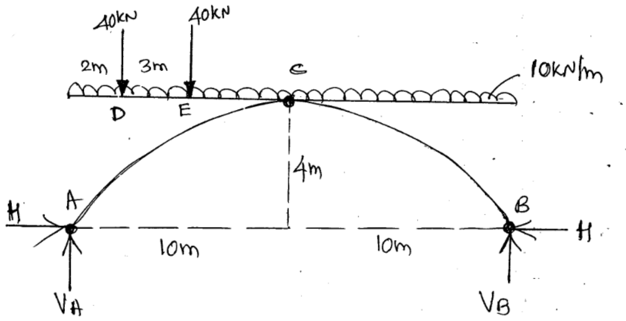
$$\therefore V_A = 600 \quad \left(\begin{array}{l} V_A + V_B = 1200 \\ V_A = 1200 - 600 \\ V_A = 600 \end{array} \right)$$

$$\Sigma M_C = \cancel{80} - 600 \times 50 + H \times 10 + 12 \times 50 \times \frac{50}{2}$$

$$H \times 10 = 15000$$

$$H = 1500 \text{ KN}$$

	Maximum tension $T_{max} = \sqrt{V^2 + H^2}$ $= \sqrt{(1200)^2 + (1500)^2}$ $T_{max} = 1920.94$ KN Minimum tensin $T_{min} = H = 1500$ KN Size of cable $\sigma = \frac{T_{max}}{\frac{\pi (d')^2}{4}}$ $(\sigma = 180\ N/mm^2)$ $180 = \frac{1920.94 \times 10^3}{\frac{\pi \times d'^2}{4}}$ $d'^2 = \frac{1920.94 \times 10^3 \times 4}{\pi \times 180}$ $d'^2 = 13587.87095$ $d' = 116.567$ mm Length of the Cable $S = l + \frac{8}{3}\left(\frac{d^2}{l}\right)$ $= 100 + \frac{8}{3}\left(\frac{10^2}{100}\right)$ $= 100 + 2.67$ $S = 102.67$ m
3 (a)	Calculate the radial shear and normal thrust at 4m from the left support of a parabolic arch shown in fig.3.a also draw bending moment diagram.



(a) Reaction

$\Sigma V = 0, \quad V_A + V_B = 10 \times 20 + 40 + 40 = 280$

$\Sigma M_A = 0, \quad 40 \times 2 + 40 \times 5 + 10 \times 20 \times \frac{20}{2} - V_B \times 20 = 0$

$\boxed{V_B = 114\,kN}$ & $\boxed{V_A = 166\,kN}$

$\Sigma M_C = 0, \quad 10 \times 10 \times 10/2 - 114 \times 10 + 4 \times H = 0$

$\therefore \boxed{H = 160\,kN}$

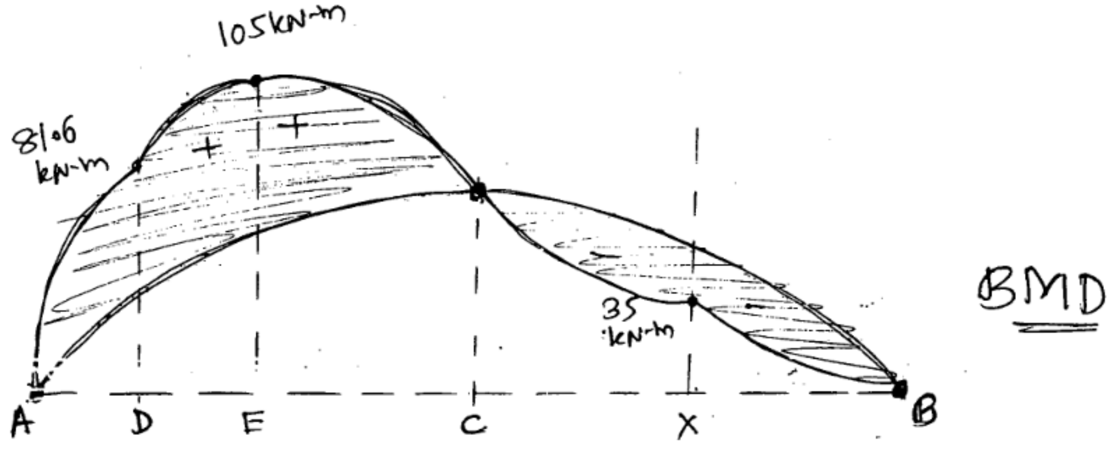
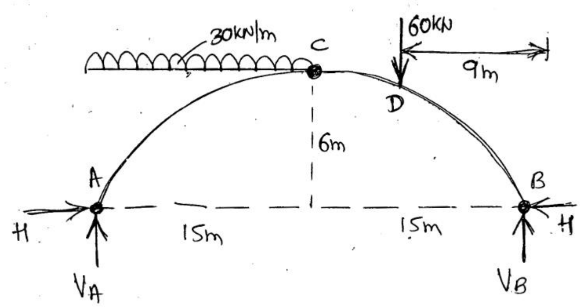
$R_A = 230.55\,kN$ | $R_B = 196.45$

$\theta_A = 46.05^\circ$ | $\theta_B = 35.47^\circ$

(b) Rise and Slope

$y = \frac{4 \times 4}{(20)^2} x(20 - x)$ $\quad \boxed{y = 0.8x - 0.04x^2}$

$\frac{dy}{dx} = \boxed{\tan\theta = 0.8 - 0.08x}$

	(C) BMD (i) $M_A = M_B = M_C = 0$ (ii) $M_D = V_A \times 2 - 10 \times 2 \times 2/2 - H \times y_D$ $= 166 \times 2 - 20 - 160\left[0.8 \times 2 - 0.04(2)^2\right] = 81.6$ kN-m (iii) $M_E = V_A \times 5 - 10 \times 5 \times 5/2 - 40 \times 3 - H \times y_E$ $= 166 \times 5 - 125 - 120 - 160\left[0.8 \times 5 - 0.04(5)^2\right] = 105$ kN-m (iv) In between "CB" Since UDL is between two hinged point, take moment exactly at mid point of CB ($x = 5$m) $M_X = 114 \times 5 - 10 \times 5 \times 5/2 - 160\left[0.8 \times 5 - 0.04(5)^2\right] = -35$ kN-m 
4 (a)	A three hinged parabolic arch is shown in fig 4.a, Compute normal thrust and radial shear at 9m from left hand support, also draw bending moment diagram. 

(a) Reaction

$\Sigma V=0, \quad V_A + V_B = 30 \times 15 + 60 = 510$

$\Sigma M_A = 0, \quad -V_B \times 30 + 30 \times 15 \times 15/2 + 60 \times 21 = 0$

$\boxed{V_B = 154.5 kN}$ & $\boxed{V_A = 355.5}$ kN

$\Sigma M_C = 0, (RHS)$

$-154.5 \times 15 + 60 \times 6 + H \times 6 = 0$ $\cancel{H = 90 kN}$

$\boxed{H = 326.25}$ kN

$\therefore R_A = \sqrt{V_A^2 + H^2} = 482.51$ kN

$\theta_A = \tan^{-1}\left(\frac{V_A}{H}\right) = 47.45^\circ$

$R_B = \sqrt{V_B^2 + H^2} = 360.98$ kN

$\theta_B = \tan^{-1}\left(\frac{V_B}{H}\right) = 25.34^\circ$

(b) Rise and Slope

$y = \frac{4 \times 6}{(30)^2} x(30-x)$ $\boxed{y = 0.8x - 0.027x^2}$

$\frac{dy}{dx} = \boxed{\tan\theta = 0.8 - 0.054x}$

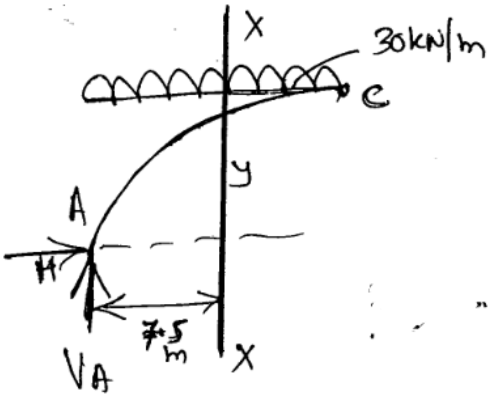
(c) BMD :-

(i) $M_A = M_B = M_C = 0$

(ii) $M_D = 154.5 \times 9 - 326.25 [y_D]$

$= 154.5 \times 9 - 326.2 [0.8 \times 9 - 0.027(9)^2] = -245$ kNm

(iii) In between AC :

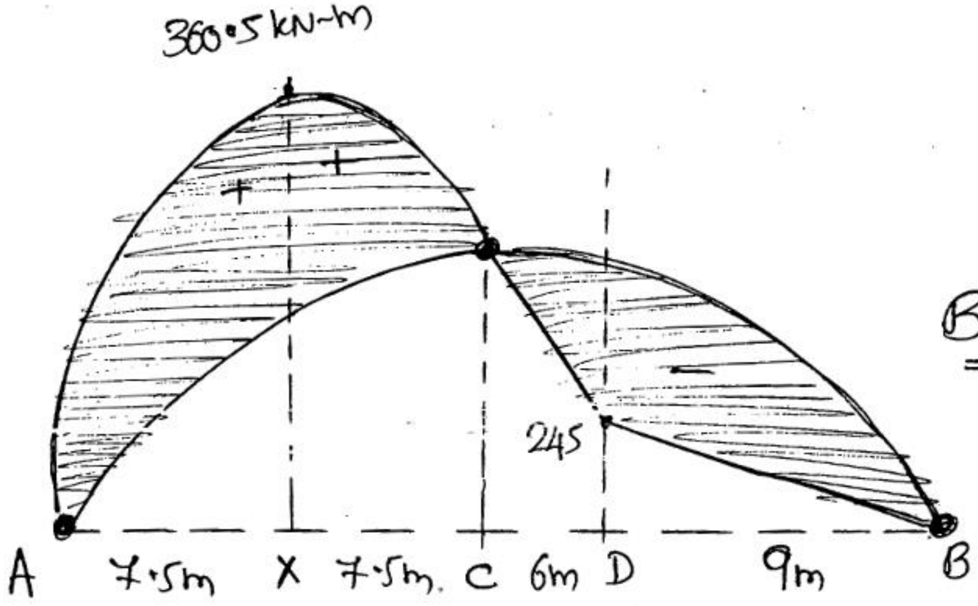


If there is a UDL between two higed suppot, then exactly at mid point you will get max. +ve moment

$$M_X = V_A \times 7.5 - 30 \times 7.5 \times \frac{7.5}{2} - H \times y_x$$

$$= 355.5 \times 7.5 - 30 \times \frac{7.5^2}{2} - 326.25\left[0.8 \times 7.5 - 0.027(7.5)^2\right]$$

$M_X = 360.5$ kN-m



BMD

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d) BM, N.T and R.S. at $x = 9m$ from left :-

$H = 326.25 kN$, $\theta = \tan^{-1}(0.8 - 0.054 \times 9) = 17.43^\circ$

Shear Force $V = V_A - 30 \times 9 = 355.5 - 270 = 85.5 kN$

$\therefore N.T = H\cos\theta + V\sin\theta = 336.88 kN$

$R.S = H\sin\theta - V\cos\theta = 16.15 kN$

$$BM\Big|_{x=9m} = 355.5 \times 9 - 30 \times 9 \times 9/2 - 326.25\left[0.8 \times 9 - 0.027(9)^2\right]$$

$$= 349 kN\text{-}m$$

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<td>5 (a)</td>
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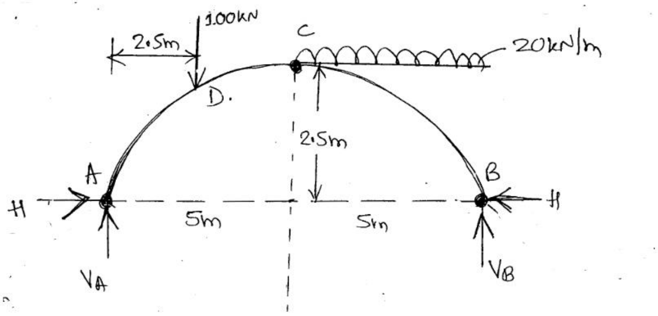
A three hinged segmental (circular) arch of 10m and central rise 2.5m supports a point load of 100 kN at left quarter span and a UDL of 20 kN/m over the right half of the span shown in figure. Determine support reactions, normal thrust and radial shear at right quarter span.



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Sol^n (a) Reactions

$\Sigma V=0, \quad V_A+V_B=100+20\times5=200$ —(i)

$\Sigma M_B=0, \quad V_A\times10-20\times5\times5/2-100\times7.5=0$

$\boxed{V_A=100\,kN}$ & $\boxed{V_B=100\,kN}$

$\Sigma M_C=0, \quad -100\times5+20\times5\times5/2+H\times2.5=0$

$\boxed{H=100\,kN}$

$\therefore R_A = 141.42\,kN \qquad \theta_A=\theta_B=45^\circ$

$R_B = 141.42\,kN$

(b) Radius & Rise eq^n: Q3

$$R=\frac{l^2+4h^2}{8h} = \frac{10^2+4(2.5)^2}{8\times2.5} = \boxed{6.25m}$$

$$R^2=x^2+(R-h+y)^2$$

$$6.25^2=x^2+(6.25-2.5+y)^2$$

$$39.06=x^2+(3.75+y)^2$$

$(39.06-x^2)=(3.75+y)^2$ Taking Under Root

$$\sqrt{(39.06-x^2)}=(3.75+y)$$

$$\therefore \boxed{y = \sqrt{(39.06 - x^2)} - 3.75}$$

$x \rightarrow$ is measured from Crown.

(c) BMD

(i) $M_A = M_B = M_C = 0$

(ii) $M_D = V_A \times 2.5 - H \times y$

$$= 100 \times 2.5 - 100\left[\sqrt{(39.06 - x^2)} - 3.75\right]$$

Put $x = 2.5m$ measured from Crown.

$M_D = 52.2$ KN-m

(iii) In between BC

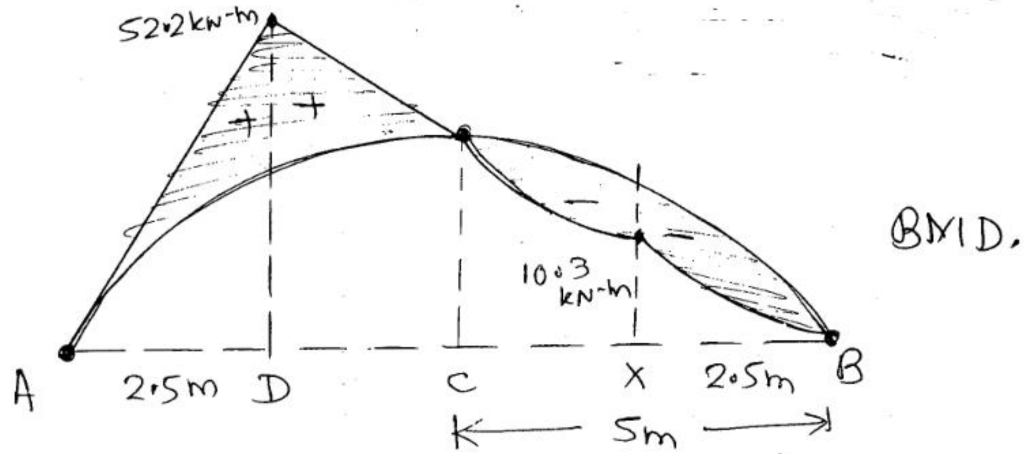
Since UDL is from hinge 'C' to 'B'

take moment exactly at midpoint of CB.

$$M_x = 100 \times 2.5 - 20 \times 2.5 \times \frac{2.5}{2} - 100\left[\sqrt{(39.06 - x^2)} - 3.75\right]$$

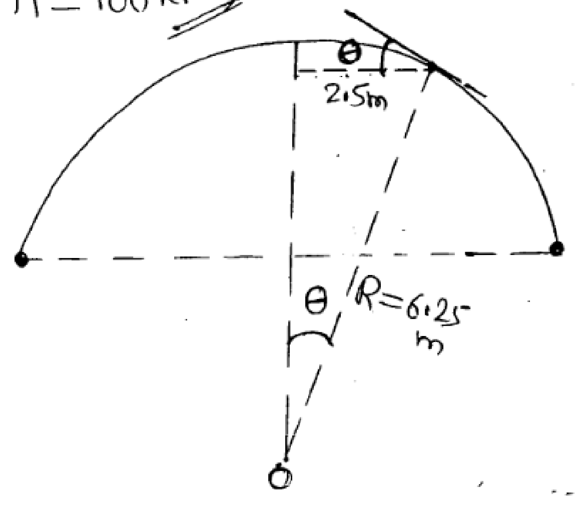
Put $x = 2.5m$

$M_X = -10.30\ kN\text{-}m$



(d) N.T and R.S at "Right Quarter" span

$H = 100\ kN$, $V = -100 + 20 \times 2.5 = -50\ kN$



$\sin\theta = \frac{2.5}{6.25}$

$\therefore \theta = 23.58°$

$NT = H\cos\theta + V\sin\theta = 71.65\ kN$

$RS = H\sin\theta - V\cos\theta = 85.82\ kN$