Internal Assessment Test 3 – Sept. 2023 QP and solutions

Sub:	Analysis of structures				Sub Code:	21CV44	Branch: Civil Engg
Date:	12.09.2023	Duration:	90 min's	Max Marks: 50	Sem / Sec:4 A	4 A	

Question number 1 is mandatory; answer any 1 full questions from Q2 and Q3. And answer any 1 full questions from Q4 and Q5		MARKS
1 (a)	Determine the fixed end moments for the beam shown in Fig.1.a.	[10]

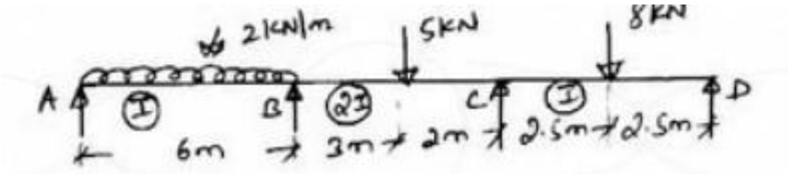
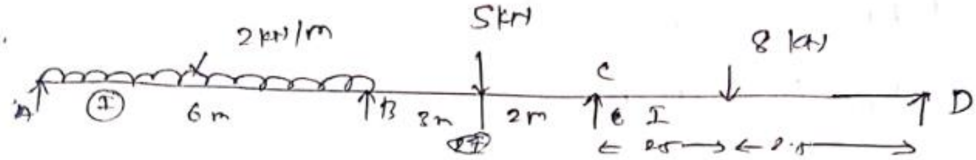


Fig.1.a

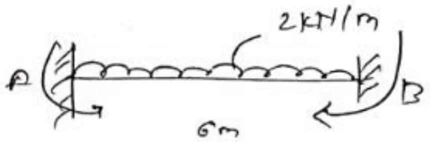
Soln

Here,



To determine the fixed end moment of beam.

① for mean beam AB



$$M_{FAB} = -\frac{wl^2}{12}$$

$$= -\frac{2 \times (6)^2}{12}$$

$$= -\frac{2 \times 36}{12} =$$

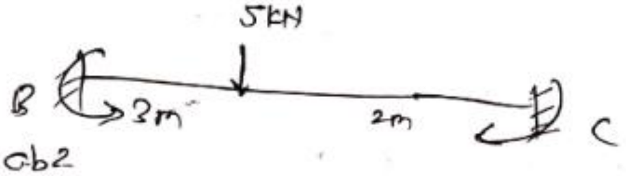
$$\therefore M_{FAB} = -6\ kN$$

$$M_{FBA} = +\frac{wl^2}{12}$$

$$= \frac{2 \times 36}{12} = +6\,kN$$

$$\therefore M_{FBA} = 6\,kN$$

For Beam BC



$$M_{FBC} = -\frac{Wab^2}{(l)^2}$$

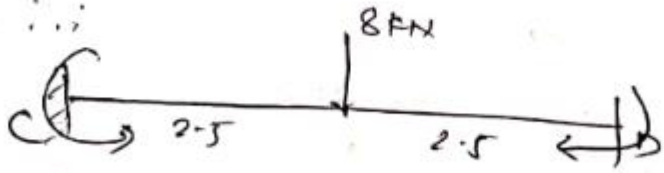
$$= -\frac{5 \times 3 \times 4}{(5)^2} = -\frac{5 \times 3 \times 4}{25}$$

$$\therefore M_{FBC} = -2.4\,kN$$

$$M_{FCB} = \frac{Wa^2b}{(l)^2} = \frac{5 \times (9) \times 2}{(5)^2} =$$

$$\therefore M_{FCB} = 3.6\,kN$$

For Beam CD



$$M_{FCD} = -\frac{wl}{8} = -\frac{8 \times 5}{8} = -5\,kN$$

$$\therefore M_{FCD} = -5\,kN$$

2 (a)	Analyse the plane frame shown in Fig.2.a by stiffness matrix method and draw BMD.	[20]

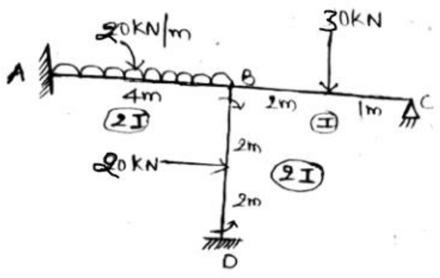


Fig.2.a

Step 1 :- FEN's :-

$$M_{FAB} = \frac{-wl^2}{12} = -\frac{20\times4^2}{12} = -26.66 \text{ kNm}$$

$$M_{FBA} = \frac{wl^2}{12} = \frac{20\times4^2}{12} = 26.66 \text{ kNm}$$

$$M_{FBC} = \frac{-wab^2}{l^2} = \frac{-30\times2\times1^2}{3^2} = -6.667 \text{ kNm}$$

$$M_{FCB} = \frac{wa^2b}{l^2} = \frac{30\times2^2\times1}{3^2} = 13.333 \text{ kNm}$$

$$M_{FBD} = \frac{wl}{8} = \frac{20\times4}{8} = 10 \text{ kNm}$$

$$M_{FDB} = -\frac{wl}{8} = \frac{-20\times4}{8} = -10 \text{ kNm}$$

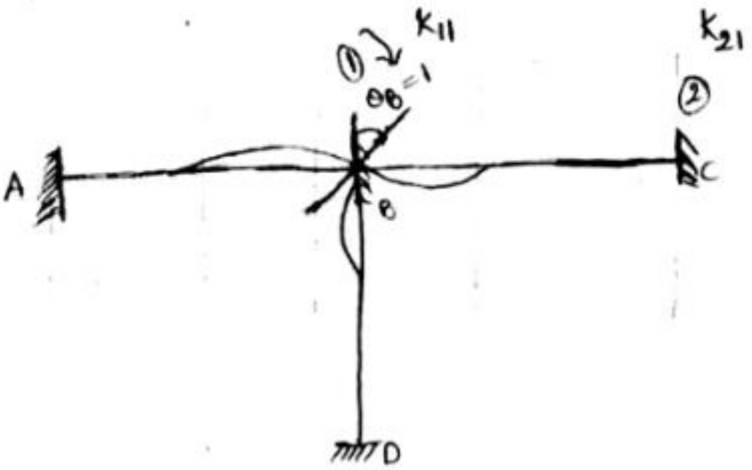
Step 2 :- $[\Delta] = \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix}$

$\Rightarrow [P] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow [P_L] = \begin{bmatrix} 29.993 \\ 13.333 \end{bmatrix}$

Step 3 :- $[k] = \begin{bmatrix} & \end{bmatrix}_{2\times2}$

⇒ Apply unit rotation at Co-ordinate (1)



$$k_{11} = \left(\frac{4EI}{l}\right)_{\text{for } BA,\, BC,\, BD}$$

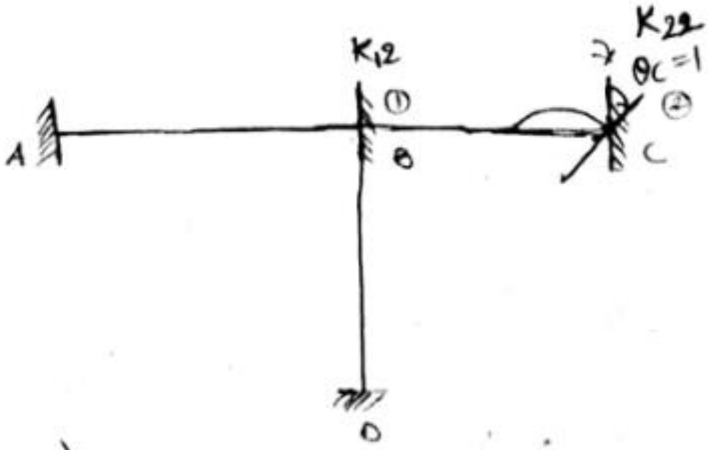
$$= \frac{4 \times E \times 2I}{4} + \frac{4 \times E \times I}{3} + \frac{4 \times E \times 2I}{4}$$

$$= 2EI + 2EI + 1.333EI$$

$$k_{11} = 5.333EI$$

$$k_{21} = \left(\frac{2EI}{l}\right)_{CB} = \frac{2 \times E \times I}{3} = 0.667EI$$

Apply unit rotation at Co-ordinate ②



$$\Rightarrow \quad K_{12} = \left(\frac{2EI}{l}\right)_{BC}$$

$$= \frac{2}{3} EI = 0.667 EI$$

$$K_{22} = \left(\frac{4EI}{l}\right)_{BC} = \frac{4}{3} EI = 1.333 EI$$

$$[K] = \begin{bmatrix} 5.333EI & 0.667EI \\ 0.667EI & 1.333EI \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 5.333 & 0.667 \\ 0.667 & 1.333 \end{bmatrix}^{-1} \begin{bmatrix} -29.993 \\ -13.333 \end{bmatrix}$$

$\therefore \theta_B = \frac{-4.665}{EI}$; $\theta_C = \frac{-7.665}{EI}$

Step 4:- Slope - deflection equations :-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

$$= -26.66 + \frac{4EI}{4}\left[2 \times 0 + \left(\frac{-4.665}{EI}\right)\right]$$

$$\Rightarrow -26.66 - 4.665$$

$$\boxed{M_{AB} = -31.325 \text{ kNm}}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$$

$$= 26.66 + EI\left[2\left(\frac{-4.665}{EI}\right) + 0\right]$$

$$= 26.66 - 9.33$$

$$\boxed{M_{BA} = 17.33 \text{ kNm}}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\delta}{l}\right]$$

$$= -6.667 + \frac{2EI}{3}\left[2\left(-\frac{4.665}{EI}\right) - \frac{7.665}{EI} - 0\right]$$

$$\boxed{M_{BC} = -17.997 \text{ KNm}}$$

$$M_{BD} = 10 + \frac{4EI}{4}\left[2\left(-\frac{4.665}{EI}\right) + 0\right]$$

$$\boxed{M_{BD} = 0.67 \text{ kNm}}$$

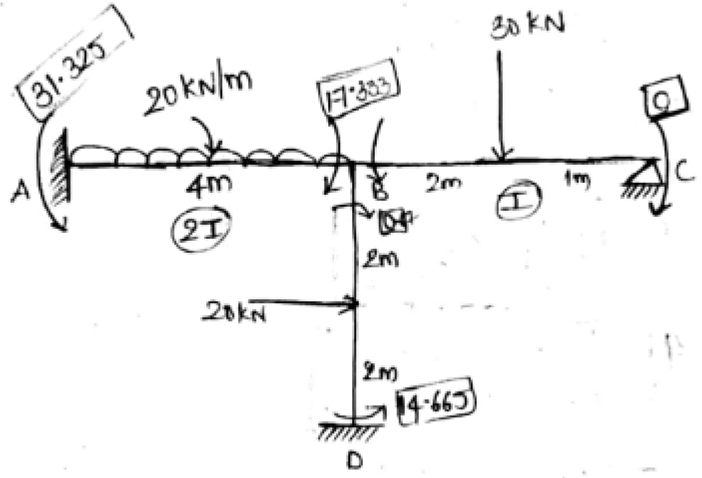
$$M_{CB} = 13.33 + \frac{2EI}{3}\left[2\left(\frac{-7.665}{EI}\right) - \frac{4.665}{EI}\right]$$

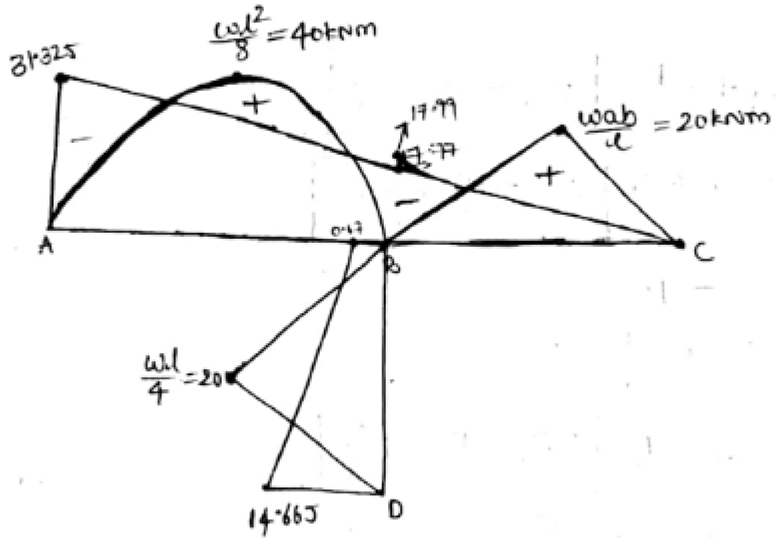
$$= 13.33 - 10.22 - 3.11$$

$$\boxed{M_{CB} = 0 \text{ KNm}}$$

$$M_{DB} = -10 + EI\left[0 + \left(\frac{-4.665}{EI}\right)\right]$$

$$\boxed{M_{DB} = -14.665 \text{ KNm}}$$

<table>
<tr><td></td><td>BMD:-



</td><td></td></tr>
<tr><td>3.(a)</td><td>List the differences between Stiffness matrix method and Flexibility matrix methods used for the analysis of structures</td><td>[05]</td></tr>
</table>

Q) Difference between stiffness method & flexibility.

stiffness method	flexibility.
1) It is used to study both statically determinate and indeterminate structures	1) It requires a different procedure to study both statically determinate and indeterminate structures.
2) This method yields displacement and forces directly.	2) In this method, the displacements are not obtained directly, so it requires some extra steps.
3) In this, we determine the degree of kinematic indeterminacy (D_K).	3) In this we determine the degree of static indeterminacy (D_S).
4) In this, we calculate reactions and moments	4) In this, we calculate deflection and rotations.
5) Also known as displacement	5) Also known as force method
6) $D_S > D_K$	6) $D_S < D_K$

3 (b) Using stiffness matrix method determine the displacement at joint B in a pin jointed frame shown in fig 3.b. [15]

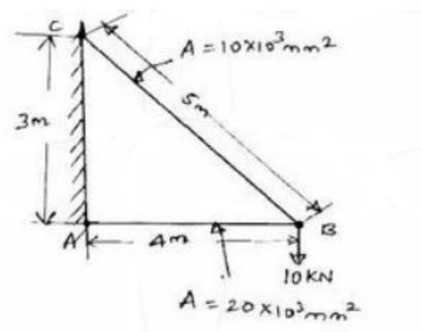
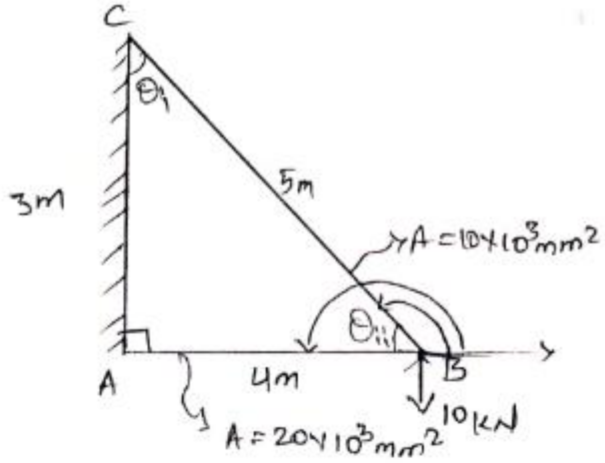


Fig.3.b



Staticall indermacy is = 2

Horizontal force $= F_1 = 0$.

Vertical force $= F_2 = -10$ kN.

Formula of stifness matrix method.

$$[P] = [K][\Delta]$$

Where. $[P] = \begin{bmatrix} F_{11} \\ F_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \end{bmatrix}$

$$[K] = \frac{AE}{L}\begin{bmatrix} \cos^2\theta & \cos\theta.\sin\theta \\ \sin\theta.\cos\theta & \sin^2\theta \end{bmatrix}$$

$$[\Delta] = \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}$$

we need to find Δ.

$$\therefore [\Delta] = [K]^{-1}[P]$$

	To find θ_{BC}, θ_{BA} $\tan\theta_i = \frac{3}{4}$ $\theta_i = \tan^{-1}(3/4) = 36.87^\circ$ $\theta_{ii} = 180^\circ - 90^\circ - \theta_i$ $= 53.13^\circ$ $\theta_{BC} = 180^\circ - \theta_{ii}$ $\quad\theta_{BA} = 180^\circ$. $= 180^\circ - 53.13^\circ$ $= 126.87^\circ$	

members	AE/L	θ	$AE/L\cos^2\theta$	$AE/L\sin^2\theta$	$AE/L\sin\theta.\cos\theta$
BC	22E	126.87°	7.9200E	14.079E	−10.560011E
BA	5E	180°	5E	0	0
Total	27E		Σ=12.92E	Σ=14.079E	Σ=−10.56001E

$$[K] = 27E\begin{bmatrix} 12.92E & -10.56001E \\ -10.56001E & 14.079E \end{bmatrix}$$

$$[\Delta] = \begin{bmatrix} 348.3 & -285.12027 \\ -285.12027 & 380.133 \end{bmatrix}\begin{bmatrix} 0 \\ -10 \end{bmatrix}$$

$$[\Delta] = \begin{bmatrix} 2851.2027 \\ -3801.33 \end{bmatrix} = \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}$$

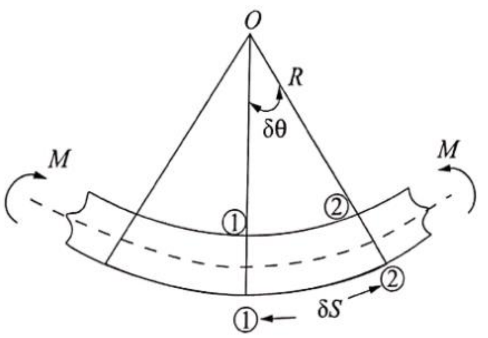
$$P_{AB} = \frac{AE}{L}\left[(\Delta_{Ax} - \Delta_{Bx})\cos\theta + (\Delta_{Ay} - \Delta_{By})\sin\theta\right].$$

$P_{AB} = 2851.2027$ kN

4 (a)	Derive an expression for strain energy store in a prismatic element subjected to pure bending.	[8]
	Any action which cause stress in a body induces strain energy. In the previous section, we have derived the expression for strain energy due to the action of direct force either tension or compression. There are other actions, viz. bending, shear and torsion also produces strain energy. The strain energy due to bending is derived as follows. (Fig. 6.2) An initially straight beam is subjected to a uniform bending moment as shown in the Fig. (6.2). Due to the application of the bending moment, two normal sections in the straight beam are deformed as (1) − (1) and (2) − (2). Let δS be the curved distance and $\delta\theta$ be its subtended angle at the centre of curvature O as shown in the figure. As moment can be replaced by equal and opposite forces	

with appropriate lever arm, external work is expressed in terms of the above said forces. Strain energy due to external loads can be expressed as

$$U = \frac{1}{2}\sum W\Delta \tag{6.7}$$



i.e., half the sum of the products of the external forces and their respective displacement in their own lines of action gives the internal work or the strain energy. The logical reasoning is extended as

$$\delta U_b = \frac{1}{2}M\delta\theta \tag{6.8}$$

where δU_b is the strain energy due to bending.
If R is the radius of curvature of the element due for the action of M,

$$R\delta\theta = \delta S \tag{6.9}$$

From Eqns. (6.8) and (6.9):

$$\delta U_b = \frac{1}{2}M\frac{\delta S}{R} \tag{6.10}$$

From theory of bending, the curvature is expressed as

$$\frac{1}{R} = \frac{M}{EI} \tag{6.11}$$

Thus,

$$\delta U_b = \frac{M^2\delta S}{2EI} \tag{6.12}$$

On integration, we obtain the total strain energy due to the bending as

$$U_b = \int \frac{M^2 ds}{2EI} \tag{6.13}$$

In the above two sections, the strain energy due to direct forces and bending was discussed and the relevant expressions were derived. These expressions will be used to determine the deflections of frames and beams in this chapter. It is recommended for the readers to know about strain energy due to shear as well as due to torsion. The strain energy due to the above structural actions are available in standard references.

4 (b) A simply supported beam of span '*l*' is subjected to a point load 'P' as shown in fig 4.b, Find the strain energy stored in the beam and the deflection under point load by using strain energy method. [12]

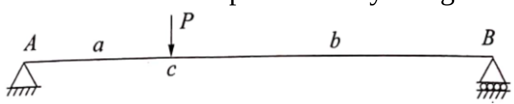


Fig.4.b

SOLUTION

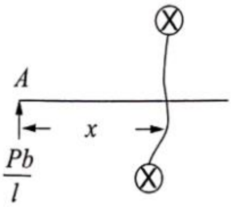
$$\sum V = 0; \quad V_A + V_B = P$$

$$\sum M = 0; \quad Pa - V_B\, l = 0$$

$$V_B = \frac{Pa}{l}$$

$$V_A = \frac{Pb}{l}$$

In the length of AC, the moment at a distance x from A is $\frac{Pbx}{l}$. Hence, the strain energy for this portion is



$$U_{AC} = \int_0^a \frac{M^2 dx}{2EI}$$

$$U_{AC} = \frac{1}{2EI}\int_0^a M^2 dx$$

$$= \frac{1}{2EI}\int_0^a \left(\frac{Pbx}{l}\right)^2 dx$$

$$= \frac{P^2b^2}{2l^2EI}\int_0^a x^2 dx$$

$$= \frac{P^2b^2}{2EIl^2}\left[\frac{x^3}{3}\right]_0^a$$

$$\boxed{U_{AC} = \frac{P^2a^3b^2}{6EIl^2}}$$

Similarly for the length BC, with B as the origin the strain energy for the portion BC can be obtained as

$$U_{BC} = \frac{1}{2EI}\int_0^b \left(\frac{Pax}{l}\right)^2 dx$$

$$\boxed{U_{BC} = \frac{P^2a^2b^3}{6EIl^2}}$$

Hence, the total strain energy

$$U = U_{AC} + U_{BC}$$
$$= \frac{P^2a^3b^2}{6EIl^2} + \frac{P^2a^2b^3}{6EIl^2}$$
$$U = \frac{P^2a^2b^2}{6EIl}$$
$$\therefore \quad \Delta = \frac{2U}{P}$$
$$\boxed{\Delta_C = \frac{P\,a^2b^2}{3EIl}}$$

5(a) Determine the vertical deflection at joint E in a pin jointed frame shown in fig 5.a by using strain energy method. Take cross sectional area of each member as 1000 mm^2 and E as 200 kN/mm^2 [20]

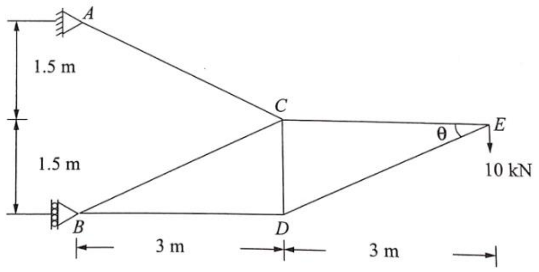
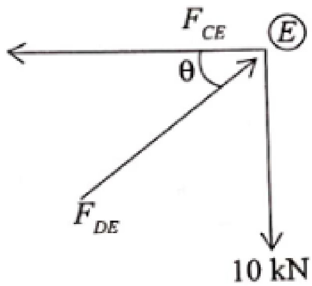


Fig.5.a

Joint E

Resolving all the forces horizontally



$$\sum H = 0;$$
$$F_{DE}\cos\theta - F_{CE} = 0$$

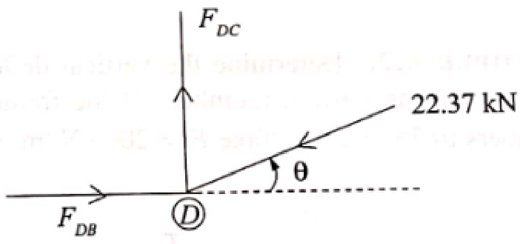
Resolving all the forces vertically

$$\sum V = 0; \qquad F_{DE}\sin\theta - 10 = 0$$

Using $\sin\theta = 0.447$ and $\cos\theta = 0.894$;

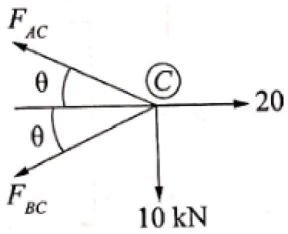
$$F_{DE} = 22.37 \text{ kN}$$
$$F_{CE} = 20 \text{ kN}$$

Joint D



$$\sum H = 0; \quad F_{DB} - 22.37\cos\theta = 0$$
$$F_{DB} = 22.37 \times 0.894 = 20 \text{ kN}$$
$$\sum V = 0; \quad F_{DC} - 22.37\sin\theta = 0$$
$$F_{DC} = 22.37 \times 0.447 = 10 \text{ kN}$$

Joint C



$$\sum H = 0; \quad -F_{AC}\cos\theta - F_{BC}\cos\theta + 20 = 0$$
$$0.894F_{AC} + 0.894F_{BC} = 20$$
$$\sum V = 0; \quad +F_{AC}\sin\theta - F_{BC}\sin\theta - 10 = 0$$
$$0.447F_{AC} - 0.447F_{BC} = 10$$

Solving the above equations

$$F_{AC} = 22.37 \text{ kN}, \; F_{BC} = 0$$

Member	Length (mm)	Area (mm²)	*P*(kN)	P^2l/A
BD	3000	1000	−20	1200.0
DE	3354	1000	+22.37	1678.4
CE	3000	1000	+20.00	1200.0
CA	3354	1000	−22.37	1678.4
CB	3354	1000	00.00	–
CD	1500	1000	+10.00	150.0
				5906.8

In a framed structure $U = \Sigma \dfrac{P^2L}{2AE}$ and

hence from Eqn. 6.15; $\Delta = \dfrac{1}{WE}\Sigma\dfrac{P^2L}{A}$

$$\boxed{\Delta_E = \frac{5906.8}{10 \times 200} = 2.95 \text{ mm}}$$