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Internal Assessment Test – I JULY 2023

Sub:	Complex Analysis, Probability and Linear Programming							Code:	21MATME41
Date:	04-07-2023	Duration:	90 mins	Max Marks:	50	Sem:	IV	Branch:	**ME**

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE	
			CO	RBT
1	Find the constant k such that $f(x)=\begin{cases}kx^2, & 0<x<3\\ 0, & \text{otherwise}\end{cases}$ is a probability density function. Also, compute i) $P(1<x<2)$ ii) $P(x\le 1)$ iii) $P(x>1)$ iv) Mean v) Variance.	[08]	CO3	L3
2	A random variable X has the following probability function: $\begin{array}{|c|c|c|c|c|c|c|c|c|}\hline x & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7\\ \hline P(x) & 0 & k & 2k & 2k & 3k & k^2 & 2k^2 & 7k^2+k\\ \hline\end{array}$ Find i) the value of k ii) $P(x<6)$ iii) $P(x\ge 6)$ iv) $P(3<x\le 6)$. Also find the probability distribution.	[07]	CO3	L3
3	The probability that a pen manufactured by a company be defective is $\frac{1}{10}$. If 12 such pens are manufactured, what is the probability that i) exactly 2 are defective ii) at least 2 are defective iii) none of them are defective.	[07]	CO3	L3
4	2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contain, i) no defective fuses ii) 3 or more defective fuses iii) at least one defective fuses.	[07]	CO3	L3
5	Out of 800 families with 5 children each, how many families would you expect to have i) 3 boys ii) 5 girls iii) either 2 or 3 boys iv) at most 2 girls.	[07]	CO3	L3
6	If x is the normal variate with mean 30 and standard deviation 5. Find the probabilities that i) $26\le x\le 40$ ii) $x\ge 45$ iii) $25<x<35$. Given that $\phi(1)=0.3413, \phi(0.8)=0.2881, \phi(2)=0.4772, \phi(3)=0.4987$.	[07]	CO3	L3
7	The length of telephone conversation in a booth has been an exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made from this booth i) ends less than 5 minutes ii) between 5 and 10 minutes.	[07]	CO3	L3
8	The marks of 100 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be i) less than 65 ii) more than 75 iii) 65 to 75.	[07]	CO3	L3

(1) Find the Constant K Such that

$$f(x) = \begin{cases} Kx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$ is a p.d.f. Also compute,

(i) $P(1 < x < 2)$ (ii) $P(x \leq 1)$ (iii) $P(x > 1)$ (iv) Mean

(v) Variance

Sol:- $\because f(x) \geq 0$ if $K \geq 0$

$\because$ we have, $\int_{-\infty}^{\infty} f(x)\,dx = 1$

i.e., $\int_0^3 Kx^2\,dx = 1 \Rightarrow \left[\frac{Kx^3}{3}\right]_0^3 = 1 \Rightarrow \left[\frac{K(9 \times 3)}{3}\right] - \left[\frac{K(0)^3}{3}\right] = 1$

$\Rightarrow K9 = 1 \Rightarrow \boxed{K = \frac{1}{9}}$

(i) $P(1 < x < 2) = \int_1^2 f(x)\,dx = \int_1^2 \frac{x^2}{9}\,dx = \left[\frac{x^3}{27}\right]_1^2 = \left(\frac{8}{27}\right) - \left(\frac{1}{27}\right) = \frac{7}{27}$

(ii) $P(x \leq 1) = \int_0^1 f(x)\,dx = \int_0^1 \frac{x^2}{9}\,dx = \left[\frac{x^3}{27}\right]_0^1 = \left(\frac{1}{27}\right) - \left(\frac{0}{27}\right) = \frac{1}{27}$

(iii) $P(x > 1) = \int_1^3 f(x)\,dx = \int_1^3 \frac{x^2}{9}\,dx = \left[\frac{x^3}{27}\right]_1^3 = \left(\frac{27}{27}\right) - \left(\frac{1}{27}\right) = \frac{26}{27}$

(iv) Mean $= \mu = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_0^3 x \cdot \frac{x^2}{9}\,dx = \left[\frac{x^4}{36}\right]_0^3 = \frac{81}{36} - 0 = \frac{81}{36}$

(7)

(v) variance $V = \int_a^b x^2 f(x)\,dx - \left[\int_a^b x f(x)\,dx\right]^2$

$$= \int_0^3 \frac{x^4}{9}\,dx - \left(\frac{9}{4}\right)^2 \quad \{\text{from (iv) Mean}\}$$

$$= \left.\frac{x^5}{45}\right|_0^3 - \frac{81}{16}$$

$$= \frac{243}{45} - \frac{81}{16}$$

$$= \frac{81}{15} - \frac{81}{16} = \frac{27}{80}$$

(2) A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

(i) Find K (ii) $P(x<6)$, $P(x \geq 6)$ and $P(3<x\leq 6)$.

Sol:- • $P(x) \geq 0$

• $\sum P(x) = 1$

i.e, $0+K+2K+2K+3K+K^2+2K^2+(7K^2+K) = 1$

i.e, $10K^2+9K-1=0$

or $(10K-1)(K+1)=0$

or $\boxed{K=\frac{1}{10}}$ and $K=-1$

$$\boxed{\begin{aligned}&10K^2+9K-1=0\\&10K^2+10K-1K-1=0\\&10K(K+1)-1(K+1)=0\\&(K+1)(10K-1)=0\end{aligned}}$$

If $K=-1$ the first condition fails and hence $K\neq -1$

$\therefore \boxed{K=\frac{1}{10}}$ —— (i)

(ii) $P(x<6) = P(0)+P(1)+P(2)+P(3)+P(4)+P(5)$

$= 0+K+2K+2K+3K+K^2$

$= 0+\frac{1}{10}+\frac{2}{10}+\frac{2}{10}+\frac{3}{10}+\frac{1}{100}$

$= \frac{81}{100} = \underline{\underline{0.81}}$

- $P(x \geq 6) = P(6) + P(7)$

$$= 2K^2 + 7K^2 + K = 9K^2 + K$$

$$= 9\left(\frac{1}{100}\right) + \frac{1}{10} = \frac{19}{100} = \underline{\underline{0.19}}$$

- $P(3 < x \leq 6) = P(4) + P(5) + P(6).$

$$= 3K + K^2 + 2K^2$$

$$= 3K + 3K^2$$

$$= \frac{3}{10} + \frac{3}{100} = \frac{3}{100} = \underline{\underline{0.33}}$$

(3) The probability that a pen manufactured by a factory be defective is 1/10. If 12 such pens are manufactured, what is the probability that
(i) exactly 2 are defective (ii) atleast 2 are defective
(iii) none of them are defective.

Sol:- Probability of a defective pen is $p = \frac{1}{10} = 0.1$

Probability of a non-defective pen is $q = 1-p = 1-0.1 = 0.9$

We have, $P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$ where $n = 12$

(i) probability (exactly 2 defectives) is $P(x=2)$

$$= {}^{12}C_{2}\,(0.1)^{2}(0.9)^{12-2}$$

$$= {}^{12}C_{2}\cdot(0.1)^{2}(0.9)^{10} = \underline{0.2301}$$

(ii) probability (atleast 2 defectives) is $1 - [P(x=0) + P(x=1)]$

$$= 1 - \left[{}^{12}C_{0}(0.1)^{0}(0.9)^{12} + {}^{12}C_{1}(0.1)^{1}(0.9)^{11}\right] = \underline{0.341}$$

(iii) probability (no defectives) is $P(x=0)$.

$$= {}^{12}C_{0}(0.1)^{0}(0.9)^{12} = (0.9)^{12}$$

$$= \underline{0.2824}$$

⑤

(4) ~~2%~~ of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains

(i) no defective fuses (ii) 3 or more defective fuses.

Sol:- p = probability of a defective fuse $= \frac{2}{100} = 0.02$

$\therefore$ mean number of defectives $\mu = m = np = 200 \times 0.02 = 4$

The poisson distribution is given by $P(x) = \frac{e^{-m} m^x}{x!}$

i.e., $P(x) = \frac{e^{-4} 4^x}{x!} = 0.0183 \cdot \frac{4^x}{x!}$ $\quad (\because e^{-4} = 0.0183)$

(i) probability of no defective fuse $= P(0) = 0.0183 \cdot \frac{4^0}{0!} = 0.0183$

(ii) probability of 3 or more defective fuses

$$= 1 - [P(0) + P(1) + P(2)]$$

$$= 1 - \left[\frac{e^{-4} 4^0}{0!} + \frac{e^{-4} 4^1}{1!} + \frac{e^{-4} 4^2}{2!}\right]$$

$$= 1 - (0.0183)\left[1 + \frac{4^1}{1!} + \frac{4^2}{2!}\right]$$

$$= 1 - (0.0183)(1 + 4 + 8)$$

$$\simeq 0.7621$$

(5) Out of 800 families with 5 children each, how many families would you expect to have

(i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys (iv) at most 2 girls.

by assuming probabilities for boys and girls to be equal.

Sol:- p = probability of having a boy $= \frac{1}{2}$

q = Probability of having a girl $= \frac{1}{2}$

Let x denote the number of boys in the family.

$P(x) = {}^nC_x\, p^x q^{n-x}$ where $n=5$.

i.e., $P(x) = {}^5C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x} = {}^5C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^{-x}$

$= {}^5C_x \left(\frac{1}{2^5}\right)$

$= \frac{{}^5C_x}{32}$

using $a^{m-n} = a^m \cdot a^{-n}$

Since we need to find the expected number in respect of 800 families we have,

$800\, P(x) = 800 \cdot \frac{{}^5C_x}{32} = 25 \cdot {}^5C_x = f(x)$ (say)

(i) We have to find $f(3)$.

$f(3) = 25 \cdot {}^5C_3 = 25 \times 10 = 250$

Expected number of families with 3 boys is 250.

using

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$${}^5C_3 = \frac{5!}{3!(5-3)!} = \frac{5\times 4\times \cancel{3\times 2\times 1}}{\cancel{3\times 2\times 1}\times 2\times 1} = \frac{5\times 4}{2\times 1} = \frac{20}{2} = 10$$

(ii) We have to find $f(0)$

$f(0) = 25\, {}^5C_0 = 25 \times 1 = 25$

Expected number of families with 5 girls is 25.

(iii) We have to find $f(2)+f(3)$

$$= 25 \cdot {}^5C_2 + 25 \cdot {}^5C_3$$

$$= 50\ {}^5C_2 = 50 \times 10 = 500.$$

(iv) At most 2 girls means that, families can have 5 boys and 0 girls or 4 boys and 1 girls or 3 boys and 2 girls.

Hence we have to find $f(5)+f(4)+f(3)$

$$= 25 \cdot {}^5C_5 + 25 \cdot {}^5C_4 + 25 \cdot {}^5C_3$$

$$= 25(1+5+10)$$

$$= 25 \times 16$$

$$= 400$$

Expected number of families with atmost 2 girls is 400.

(6) If x is a normal variate with mean 30 and Standard deviation 5 find the probability that

(i) $26 \leq x \leq 40$ (ii) $x \geq 45$

Sol:- We have standard normal variate, $z = \frac{x-\mu}{\sigma} = \frac{x-30}{5}$

(i) To find $P(26 \leq x \leq 40)$

If $x = 26$, $z = -0.8$; If $x = 40$, $z = 2$

Hence we need to find $P(-0.8 \leq x \leq 2)$

$$P(-0.8 \leq z \leq 2) = P(-0.8 \leq z \leq 0) + P(0 \leq z \leq 2)$$

$$= P(0 \leq z \leq 0.8) + P(0 \leq z \leq 2)$$

$$= \phi(0.8) + \phi(2)$$

$$= 0.2881 + 0.4772 = 0.7653$$ (17

$\therefore \boxed{P(26 \leq x \leq 40) = 0.7653}$

(ii) To find $P(x \geq 45)$

If $x = 45$, $z = 3$ and hence we have to find $P(z \geq 3)$

$$P(z \geq 3) = P(z \geq 0) - P(0 \leq z \leq 3)$$

$$= 0.5 - \phi(3)$$

$$= 0.5 - 0.4987 = 0.0013$$

$\therefore \boxed{P(x \geq 45) = 0.0013}$

(7) The length of telephone conversation in a booth has been an exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made from this booth

(i) ends less than 5 minutes

(ii) between 5 and 10 minutes

Sol:- We have, $f(x) = \alpha e^{-\alpha x}, \; x > 0$

- Mean $= \frac{1}{\alpha}$
- By data $\frac{1}{\alpha} = 5 \Rightarrow \boxed{\alpha = \frac{1}{5}}$
- Hence, $f(x) = \frac{1}{5} e^{-x/5}$

(i) $P(x<5) = \int_0^5 f(x)\, dx$

(15)

$$P(x<5) = \int_0^5 \frac{1}{5} e^{-x/5}\, dx = -\left[e^{-x/5}\right]_0^5$$

$$= 1 - e^{-1} = 0.6321$$

$$\therefore \boxed{P(x<5) = 0.6321}$$

(ii) $P(5<x<10) = \int_5^{10} f(x)\, dx = \int_5^{10} \frac{1}{5} e^{-x/5}\, dx$

$$P(5<x<10) = -\left[e^{-x/5}\right]_5^{10}$$

$$= \left(\frac{1}{e}\right) - \left(\frac{1}{e^2}\right) = 0.2325$$

$$\therefore \boxed{P(5<x<10) = 0.2325}$$

(8) The marks of 1000 Students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be

(i) less than 65 (ii) more than 75 (iii) 65 to 75

Sol:- ~~~~ Given, Mean $(\mu) = 70$, S.D $(\sigma) = 5$

Hence, $S.N.V = \frac{x-\mu}{\sigma} = \frac{x-70}{5}$

(i) To find $P(x < 65)$

- If $x = 65$, $Z = \frac{65-70}{5} = \frac{-5}{5} = -1 \Rightarrow Z = -1$

- Hence we have to find $P(Z < -1)$

$\left\{ \phi(1) = 0.3413 \text{ using table} \right\}$

$$P(Z<-1) = P(Z>1)$$
$$= P(Z \geq 0) - P(0 \leq Z \leq 1)$$
$$= 0.5 - \phi(1)$$
$$= 0.5 - 0.3413 = \underline{\underline{0.1587}}$$

∴ number of students scoring less than 65 marks

$= 1000 \times 0.1587 = 158.7 \approx \underline{159}$

(ii) <u>To find $P(x > 75)$</u>

• If $x = 75$, $z = \frac{75-70}{5} = \frac{5}{5} = 1$

• We have to find $P(z > 1)$

i.e, $P(z>1) = P(z \geq 0) - P(0 \leq z \leq 1)$

$= 0.5 - \phi(1)$

$= 0.5 - 0.3413 = 0.1587$

∴ number of students scoring more than 75 marks

$= 1000 \times 0.1587 = 158.7 \approx \underline{159}$

(iii) <u>To find $P(65 < x < 75)$</u>

• We have to find $P(-1 < z < 1)$

i.e, $P(-1 < z < 1) = P(-1 < z < 0) + P(0 < z < 1)$

$= P(0 < z < 1) + P(0 < z < 1)$

$= 2P(0 < z < 1)$

$= 2\phi(1)$

$= 2(0.3413)$

$= 0.6826$

∴ number of student scoring marks between 65 and 75

$= 1000 \times 0.6826 = 682.6 \approx \underline{683}$