	USN	CMRIT

Internal Assessment Test – II AUGUST 2023

Sub:	Complex Analysis, Probability and Linear Programming							Code:	21MATME41
Date:	08-08-2023	Duration:	90 mins	Max Marks:	50	Sem:	IV	Branch:	**ME**

Question 1 is compulsory and Answer any 4 from the remaining questions.

		Marks	OBE CO	OBE RBT
1	Find an initial solution by NWC method	[10]	CO5	L3

Source	Destination				
		A	B	C	D	Supply
	I	11	13	17	14	**250**
	II	16	18	14	10	**300**
	III	21	24	13	10	**400**
	Demand	**200**	**225**	**275**	**250**	

		Marks	CO	RBT
2	Explain the **canonical form** and **standard form** of an **LPP**. Convert the following LPP to the standard form: **Minimize** $z = 3x_1 + 4x_2$ **Subject to the constraints** $2x_1 - x_2 \leq 4;\ 3x_1 + 5x_2 \geq 10;\ x_1 - 4x_2 = 12$, $x_1 \geq 0$	[10]	CO4	L3
3	Solve by simplex method, **Maximize** $z = 3x + 2y$ **Subject to the constraints** $2x + y \leq 5;\ x + y \leq 3$, $x, y \geq 0$	[10]	CO4	L3
4	Use two-phase method to solve the LPP, **Maximize** $z = 9x + 3y$ **Subject to the constraints** $4x + y \leq 8;\ 2x + y \leq 4$, $x, y \geq 0$	[10]	CO4	L3
5	Using Big – M method, solve the LPP, **Maximize** $z = 2x + y$ **Subject to the constraints** $3x + y = 3;\ 4x + 3y \geq 6;\ x + 2y \leq 3$, $x, y \geq 0$	[10]	CO4	L3
6	Using Least cost method solve the following transportation problem	[10]	CO5	L3

Origin	Destination					
		D1	D2	D3	D4	Supply
	O-I	19	30	50	10	**7**
	O-II	70	30	40	60	**9**
	O-III	40	8	70	20	**18**
	Demand	**5**	**8**	**7**	**14**	

① Find an initial basic feasible solution to the following transportation problem using the north-west corner rule.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

Soln:-

Warehouse

Plants (origin)	D	E	F	G	Supply
A	11 $\boxed{200}$	13 $\boxed{50}$	17	14	~~250~~ (50) (0)
B	16	18 $\boxed{175}$	14 $\boxed{125}$	10	~~300~~ ~~(125)~~ (0)
C	21	24	13 $\boxed{150}$	10 $\boxed{250}$	~~400~~ ~~(250)~~ (0)
Demand →	~~200~~ (0)	~~225~~ ~~(175)~~ (0)	~~275~~ ~~(150)~~ (0)	~~250~~ (0)	950 / 950

Step1:- NWC is (1,1), $x_{11} = \min(250, 200) = 200$, C_1-completes

Step2:- NWC is (1,2), $x_{12} = \min(50, 225) = 50$, R_1-Completes

(2,2), $x_{22} = \min(300, 175) = 175$, C_2-Completes

Step3:- NWC is (2,3), $x_{23} = \min(125, 275) = 125$, R_2-Completes

Step4:- NWC is (3,3), $x_{33} = \min(250, 150) = 150$, C_3-Completes

Step5:- NWC is ~~(3,4), x_{34} = min(250, 250) = 250~~,

Step6:- Allocate 14 to the cell (3,4)

We note that total allocations $= 6 = 3+4-1$

$\therefore$ The solution is basic feasible

Transportation cost $= (11\times200)+(13\times50)+(18\times175)+(14\times125)$
$+(13\times150)+(10\times250)$

$= \underline{12,200}$ /-

(2) Canonical form

- The general L.P.P. can always be expressed in the form:
- Maximize $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$
- Subject to the constraints

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \leq b_i \; ; \; i = 1, 2, \ldots, m$$

$$x_1, x_2, \ldots, x_n \geq 0,$$

by making some elementary transformations. This form of the L.P.P. is called its Canonical form and has the following characteristics:

(i) objective function is of maximization type

(ii) All constraints are of $(\leq)$ type

(iii) All variables are non-negative

The Canonical form is a format for a L.P.P. which finds its use in the Duality theory.

Standard form

The general L.P.P. Can also be put in the following form:

Maximize $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

Subject to the Constraints

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n = b_i \;;\; i = 1, 2, \ldots m$$

$$x_1, x_2, \ldots, x_n \geq 0$$

This form of the L.P.P. is called its Standard form and has the following characteristics:

(i) objective function is of maximization type

(ii) All Constraints are expressed as equations

(iii) Right hand side of each Constraints is non-negative

(iv) All variables are non-negative

Note: • Any L.P.P. Can be expressed in the standard form

As minimize $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$ is equivalent to maximize $Z' (= -Z) = -c_1x_1 - c_2x_2 - \cdots - c_nx_n$, the objective function can always be expressed in the maximization form.

• The decision variables $x_1, x_2, \ldots, x_n$ – ~~non-nega~~ assumed to be non-negative.

• If the decision variables $x_1, x_2, \ldots x_n$ are zero or negative, it can always be expressed as the difference of two ~~[illegible]~~ non-negative variables

- As x_2 is unrestricted
- Let $x_2 = x_2' - x_2''$ where $x_2', x_2'' \geq 0$
- Now the given constraints can be expressed as

$$2x_1 - (x_2' - x_2'') \leq 4 \Rightarrow 2x_1 - x_2' + x_2'' \leq 4$$

$$3x_1 + 5(x_2' - x_2'') \geq 10 \Rightarrow 3x_1 + 5x_2' - 5x_2'' \geq 10$$

$$x_1 - 4(x_2' - x_2'') = 12 \Rightarrow x_1 - 4x_2' + 4x_2'' = 12$$

- We know that Minimize $= -$(Maximize)

i.e, objective function becomes

$$\text{Maximize } z'(=-z) = -3x_1 - (4x_2' - 4x_2'')$$

$$\text{i.e, Maximize } z'(=-z) = -3x_1 - 4x_2' + 4x_2''$$

- Introducing the slack/surplus variables, the problem in the standard form becomes.

$$\text{Maximize } z'(=-z) = -3x_1 - 4x_2' + 4x_2''$$

Subject to

$$2x_1 - x_2' + x_2'' + s_1 = 4$$

$$3x_1 + 5x_2' - 5x_2'' - s_2 = 10$$

$$x_1 - 4x_2' + 4x_2'' = 12$$

where, $x_1, x_2, x_2', x_2'', s_1, s_2 \geq 0$

(3) Using simplex method solve the L.P.P.

Maximize $Z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \leq 5$

$x_1 + x_2 \leq 3$

$x_1, x_2 \geq 0$

Solution:-

→ Introduce 2 slack variables S_1 and S_2, rewrite the given L.P.P. in the standard form.

i.e., Maximize, $Z = 3x_1 + 2x_2 + 0S_1 + 0S_2$ → objective function

subject to, $2x_1 + x_2 + S_1 + 0S_2 = 5$

$x_1 + x_2 + 0S_1 + S_2 = 3$

$x_1, x_2, S_1, S_2 \geq 0$

R_1 → first row

R_2 → second row

- Total number of variables = 4
- Basic variables = 2
- Number of non-basic variables = 4 − 2 = 2

	NZV	C_B	X_B	3 x_1	2 x_2	0 S_1	0 S_2	Ratio	
R_1 →	S_1	0	5	$\boxed{2}$	1	1	0	$\frac{5}{2} = 2.5$	→ PR
R_2 →	S_2	0	3	1	1	0	1	$\frac{3}{1} = 3$	
	$Z_j - C_j$	$Z = C_B X_B =$	0	$\Delta_1 = -3$ → PC	$\Delta_2 = -2$	$\Delta_3 = 0$	$\Delta_4 = 0$		

- $Z = C_B X_B = (0 \times 5 + 0 \times 3) = 0$; $\Delta_1 = (0 \times 2 + 0 \times 1) - 3 = -3$; $\Delta_2 = (0 \times 1 + 0 \times 1) - 2 = -2$;

$\Delta_3 = (0 \times 1 + 0 \times 0) - 0 = 0$; $\Delta_4 = (0 \times 0 + 0 \times 1) - 0 = 0$

- Pivot Key = 2
- Outgoing variable = S_1
- Incoming variable = x_1

Apply $R_1' \to \frac{R_1}{2}$

	NZV	C_B	X_B	$\overset{3}{x_1}$	$\overset{2}{x_2}$	$\overset{0}{S_1}$	$\overset{0}{S_2}$	Ratio
R_1'	x_1	3	$\frac{5}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{5}{2} \times \frac{2}{1} = 5$
R_2'	S_2	0	$\frac{1}{2}$	0	$\boxed{\frac{1}{2}}$	$-\frac{1}{2}$	1	$\frac{1}{2} \times \frac{2}{1} = 1 \to$ (PR)
	$Z_j - C_j$	$Z = C_B X_B =$	$\frac{15}{2}$	$\Delta_1 = 0$	$\Delta_2 = -\frac{1}{2}$	$\Delta_3 = \frac{3}{2}$	$\Delta_4 = 0$	
					(PC)			

Apply $R_2' \to R_2 - R_1'$

- $Z = C_B X_B = \left(3 \times \frac{5}{2} + 0 \times \frac{1}{2}\right) = \frac{15}{2}$; $\Delta_1 = (3 \times 1 + 0 \times 0) - 3 = 0$
- $\Delta_2 = \left(3 \times \frac{1}{2} + 0 \times \frac{1}{2}\right) - 2 = \frac{3}{2} - 2 = -\frac{1}{2}$; $\Delta_3 = \left(3 \times \frac{1}{2} + 0 \times -\frac{1}{2}\right) - 0 = \frac{3}{2}$
- $\Delta_4 = (3 \times 0 + 0 \times 1) - 0 = 0$
- Pivot key $= \frac{1}{2}$
- Outgoing variable $= S_2$; Incoming variable $= x_2$
- Apply $R_2'' \to 2R_2'$

	NZV	C_B	X_B	$\overset{3}{x_1}$	$\overset{2}{x_2}$	$\overset{0}{S_1}$	$\overset{0}{S_2}$	Ratio
R_1''	x_1	3	2	1	0	1	-1	
R_2''	x_2	2	1	0	1	-1	2	
	$Z_j - C_j$	$Z = C_B X_B =$	8	$\Delta_1 = 0$	$\Delta_2 = 0$	$\Delta_3 = 1$	$\Delta_4 = 1$	

- Apply $R_1'' \to R_1' - \frac{R_2''}{2}$

- $Z = C_B X_B = (3 \times 2 + 2 \times 1) = 8$; $\Delta_1 = (3 \times 1 + 2 \times 0) - 3 = 0$

$\Delta_2 = (3 \times 0 + 2 \times 1) - 2 = 0$; $\Delta_3 = (3 \times 1 + 2 \times (-1)) - 0 = 1$; $\Delta_4 = (3 \times -1 + 2 \times 2) - 0 = 1$

As all the indicators are non-negative, $\boxed{Z_{max} = 8}$ and it occurs at $\boxed{x_1 = 2}$ and $\boxed{x_2 = 1}$

(4) Use two - phase method to solve the LPP

Maximize $Z = 9x_1 + 3x_2$

subject to: $4x_1 + x_2 \leq 8$

$2x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$

Sol:- · Introduce two slack variables s_1 and s_2

i.e, Maximize, $Z = 9x_1 + 3x_2 + 0s_1 + 0s_2$

STC $\quad 4x_1 + x_2 + s_1 = 8$

$2x_1 + x_2 + s_2 = 4$

$x_1, x_2, s_1, s_2 \geq 0$

put $x_1 = x_2 = 0$

$s_1 = 8$ feasible

$s_2 = 0$ feasible

Phase-I

- Assign a cost '0' to all the variables

NZV	C_B	X_B	0 x_1	0 x_2	0 S_1	0 S_2
S_1	0	8	4	1	1	0
S_2	0	4	2	1	0	1
$Z_j - C_j$	$Z^* = 0$		0	0	0	0

Since $z^* = 0$ (maximum) and no AV's appears in the optimum basis. In this case problem has a solution and to find the solution, go to phase 2.

Phase-II

Consider the final simplex table obtained at the end of phase I. Assign the actual cost of the OF.

	NZV	C_B	X_B	9 x_1	3 x_2	0 S_1	0 S_2	Ratio
R_1	S_1	0	8	4	1	1	0	$\frac{8}{4} = 2$
R_2	S_2	0	4	2	1	0	1	$\frac{4}{2} = 2 \rightarrow PR$
	$Z_j - C_j$	$Z^* = C_B X_B = 0$		$\Delta_1 = -9$	$\Delta_2 = -3$	$\Delta_3 = 0$	$\Delta_4 = 0$	

$\hookrightarrow PC$

Pivot key = 2

ObV = S_2

ICV = x_1

Apply $R_2' \longrightarrow \frac{R_2}{2}$

$R_1' \longrightarrow R_1 - 4R_2'$

NZV	C_B	X_B	$\overset{9}{x_1}$	$\overset{3}{x_2}$	$\overset{0}{s_1}$	$\overset{0}{s_2}$
s_1	0	0	0	-1	1	-2
x_1	9	2	1	$1/2$	0	$1/2$
$Z_j - C_j$	$z^* = C_B X_B = 18$		$\Delta_1 = 0$	$\Delta_2 = 3/2$	$\Delta_3 = 0$	$\Delta_4 = 9/2$

- Since all $\Delta_j \geq 0$, optimal ~~solution~~ basic feasible Solution is obtained.
- Therefore, the solution is $\boxed{\text{Max } z^* = 18}$ at $\boxed{x_1 = 2}$ and $\boxed{x_2 = 0}$

(5) Using Big-M method, solve the LPP

Minimize $Z = 2x_1 + x_2$,

Subject to : $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

Sol:- We know minimization = −(maximization)

- ~~As the~~ As the first constraint is of the type '=', we introduce an A.V a_1

$$3x_1 + x_2 + a_1 = 3$$

- As the second constraint is of the type '$\geq$', we introduce surplus variable S_1 and hence an AV a_2.

$$4x_1 + 3x_2 - S_1 + a_2 = 6$$

- As the third constraint is of the type '$\leq$', we introduce slack variable S_2

$$x_1 + 2x_2 + S_2 = 3$$

- As two A.V's a_1 and a_2 are introduced, the new OF is

$$\text{Maximize } Z' = -2x_1 - x_2 - Ma_1 - Ma_2$$

	NZV	C_B	X_B	$\overset{-2}{x_1}$	$\overset{-1}{x_2}$	$\overset{0}{S_1}$	$\overset{0}{S_2}$	$\overset{-M}{a_1}$	$\overset{-M}{a_2}$	Min Ratio
R_1	a_1	$-M$	3	$\boxed{3}$	1	0	0	1	0	$\frac{3}{3}=1 \rightarrow$ PR
R_2	a_2	$-M$	6	4	3	-1	0	0	1	$\frac{6}{4}=1.5$
R_3	S_2	0	3	1	2	0	1	0	0	$\frac{3}{1}=3$
	Z_j-C_j	$Z'=$	$-9M$	$-7M+2$	$-4M+1$	M	0	0	0	

$\hookrightarrow$ PC

- Pivot element $= 3$, $OGV = a_1$, $ICV = x_1$
- Apply $R_1' \rightarrow \frac{R_1}{3}$, $R_2' \rightarrow R_2 - 4R_1'$, $R_3' \rightarrow R_3 - R_1'$
- As the OGV is an A.V, we shall drop the column corresponding to a_1 in the next simplex table.

	NZV	C_B	X_B	$\overset{-2}{x_1}$	$\overset{-1}{x_2}$	$\overset{0}{S_1}$	$\overset{0}{S_2}$	$\overset{-M}{a_2}$	Min ratio
R_1'	x_1	-2	1	1	$1/3$	0	0	0	$1/(1/3) = 3$
R_2'	a_2	$-M$	2	0	$\boxed{5/3}$	-1	0	1	$2/(5/3) = \frac{6}{5} \rightarrow$ PR
R_3'	S_2	0	2	0	$5/3$	0	1	0	$2/(5/3) = \frac{6}{5}$
	Z_j-C_j	$Z'=-2-2M$		0	$\frac{-5M}{3}+\frac{1}{3}$	M	0	0	

$\hookrightarrow$ PC

- Pivot element $= \frac{5}{3}$, $OGV = a_2$, $ICV = x_2$
- Apply, $R_2'' \rightarrow \frac{3}{5} \times R_2'$, $R_1'' \rightarrow R_1' - \frac{1}{3}R_2''$, $R_3'' \rightarrow R_3' - \frac{5}{3}R_2''$

As the OG.V is an A.V, we can drop the column corresponding to a_2 in the next simplex table

	NZV	C_B	X_B	$\overset{-2}{x_1}$	$\overset{-1}{x_2}$	$\overset{0}{s_1}$	$\overset{0}{s_2}$
R_1''	x_1	-2	$3/5$	1	0	$1/5$	0
R_2''	x_2	-1	$6/5$	0	1	$-3/5$	0
R_3''	s_2	0	0	0	0	1	1
	$z_j - c_j$	$z' = -12/5$		0	0	$1/5$	0

As all the indicators are non-negative, simplex method is complete

$$\therefore Z'_{max} = -\frac{12}{5} \text{ at } x_1 = \frac{3}{5} \text{ and } x_2 = \frac{6}{5}$$

$$\Rightarrow Z_{min} = \frac{12}{5} \text{ at } x_1 = \frac{3}{5} \text{ and } x_2 = \frac{6}{5}$$

(6) Using least cost method solve the following transportation problem

	Destination				
origin	D1	D2	D3	D4	Supply
O-I	19	30	50	10	7
O-II	70	30	40	60	9
O-III	40	8	70	20	18
Demand	5	8	7	14	

Sol:- • Supply = 7 + 9 + 18 = 34

• Demand = 5 + 8 + 7 + 14 = 34

• As Supply = Demand ⇒ Transportation problem is balanced

	D1	D2	D3	D4	Supply
O-I	19	30	50	10 $\boxed{7}$	~~7~~ (0)
O-II	70 $\boxed{2}$	30	40 $\boxed{7}$	60	~~9~~ (2)
O-III	40 $\boxed{3}$	8 $\boxed{8}$	70	20 $\boxed{7}$	~~18~~ ~~(10)~~ ~~(3)~~ (0)
Demand	~~5~~ (2)	~~8~~ (0)	~~7~~ (0)	~~14~~ ~~(7)~~ (0)	

Step 1:- Least cost is 8, $x_{32} = \min(18, 8) = 8$, cut C_2

Step 2:- Least cost is 10, $x_{14} = \min(7, 14) = 7$, cut R_1

Step 3:- Least cost is 20, $x_{34} = \min(10, 7) = 7$, cut C_4

Step 4:- Least cost is 40, $x_{23} = \min(9, 7) = 7$, cut C_3

Step 5:- Least cost is 40, $x_{31} = \min(3, 5) = 3$, cut R_3

Step 6:- Allocate 2 to cell (2,1) i.e., x_{21}

$\therefore$ Total Cost $= (10 \times 7) + (70 \times 2) + (40 \times 7) + (40 \times 3) + (8 \times 8) + (20 \times 7)$

$= 814$