CMR INSTITUTE OF TECHNOLOGY	USN									
	Internal Assessment Test III September 2023									
Sub:	**Mathematical Foundations for Computing, Proability and Statistics**						Code:	21MATCS41		
Date:	09/09/2023	Duration:	90 mins	Max Marks:	50	Sem:	IV	Branch:	**CSE/IS/AIML /AIDS**	

		Marks	OBE CO	OBE RBT
	Question 1 is compulsory and Answer any 6 from the remaining questions.		CO	RBT
1	Define tautology. Determine whether the following compound statement is a tautology or not $\{(p \vee q) \to r\} \leftrightarrow \{\neg r \to \neg(p \vee q)\}$	[8]	CO1	L1,L3
2	Give direct proof and proof by contradiction for the statement "If n is an odd integer then n+9 is an even integer	[7]	CO1	L3
3	Prove the logical equivalence $p \vee [p \wedge (p \vee q)] \equiv p$ without using truth table.	[7]	CO1	L3

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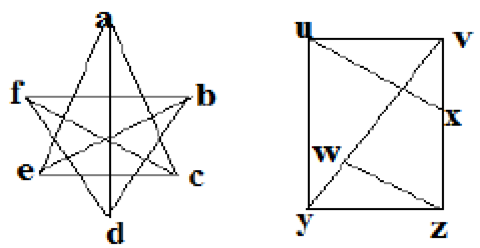
4	Let $A=\{1,2,3,4,6\}$ and R be a relation on A defined by aRb if and only if " a is a multiple of b". Write down the relation R, relation matrix $M(R)$ and draw its digraph.	[7]	CO2	L3
5	Let $A = \{1, 2, 3, 4\}$ *and* $B = \{1, 2, 3, 4, 5, 6\}$ i) How many functions are there from A to B? How many of these are one-to-one? How many are onto? ii) How many functions are there from B to A? How many of these are onto? How many are one-to-one	[7]	CO2	L3
6	Define equivalence relation. R={(1,1),(2,2),(3,3),(4,4)(1,2),(2,1),(3,4),(4,3)} be a relation on A. Show that R is an equivalence relation	[7]	CO2	L1,L3
7	Draw the Hasse diagram representing the positive divisors of 36.	[7]	CO2	L3
8	Define Graph isomorphism. Determine whether the following graphs are isomorphic or not.  	[7]	CO2	L1,L3

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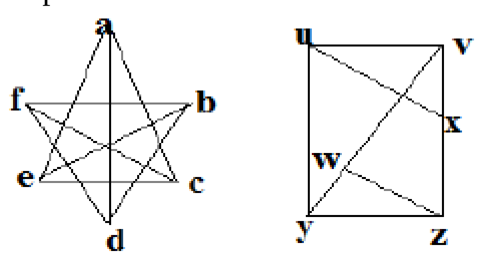
8	Define Graph isomorphism. Determine whether the following graphs are isomorphic or not. 	[7]	CO2	L1,L3

① Soln Tautology :

A compound preposition which is true for all possible truth values of its component is known as Tautology.

$$\{(p \vee q) \rightarrow r\} \leftrightarrow \{\sim r \rightarrow \sim (p \vee q)\}$$

p	q	r	$p \vee q$	$(p \vee q \rightarrow r)$	$\sim r$	$\sim (p \vee q)$
0	0	0	0	1	1	1
0	0	1	0	1	0	1
0	1	0	1	0	1	0
0	1	1	1	1	0	0
1	0	0	1	0	1	0
1	0	1	1	1	0	0
1	1	0	1	0	1	0
1	1	1	1	1	0	0

$\{\sim r \rightarrow \sim (p \vee q)\}$	$\{(p \vee q) \rightarrow r\} \leftrightarrow \{\sim r \rightarrow \sim (p \vee q)\}$
1	1
1	1
0	1
1	1
0	1
1	1
0	1
1	1

→ Here all the truth table is true

∴ Hence it is a tautology

(2)
Soln Given that
If n is an odd integer then $n+9$ is an even iteger

direct proof

p: n is an odd integer

q: $n+9$ is an even integer

Hypothesis : let us assume that n is odd integer
$\therefore$ $n = 2K+1$ for some integer K

Analysis! Consequently
$n = 2K+1$
$n+9 = 2K+1+9$
$= 2K+10$, it is divisible by 2
$\therefore$ $n+9$ is even integer

Conclusion $p \to q$ true

proof by contradiction

p : n is an odd integer
q : n+9 is an even integer

S.T : $p \to q$ is true

Hypothesis : $p \to q$ is false (let)

i.e. p is true and q is false

Analysis : q is false, then n+9 is odd so

$n+9 = 2K+1$

$n = 2K-8$, which is divisible by 2

so n is even

Hence our assumption is wrong

so $p \to q$ is True

H.P.

(3)

Soln $p \vee [p \wedge (p \vee q)] \equiv p$

To prove it:-

By absorption law $\Rightarrow$ $p \vee [p \wedge (p \vee q)] \Leftrightarrow p \vee p$

By idempotent law $\Rightarrow$ $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$

Hence proved

(4)

Soln Given that

$A = \{1, 2, 3, 4, 6\}$

R be a relation on A defined by aRb if "a is Multiple of b".

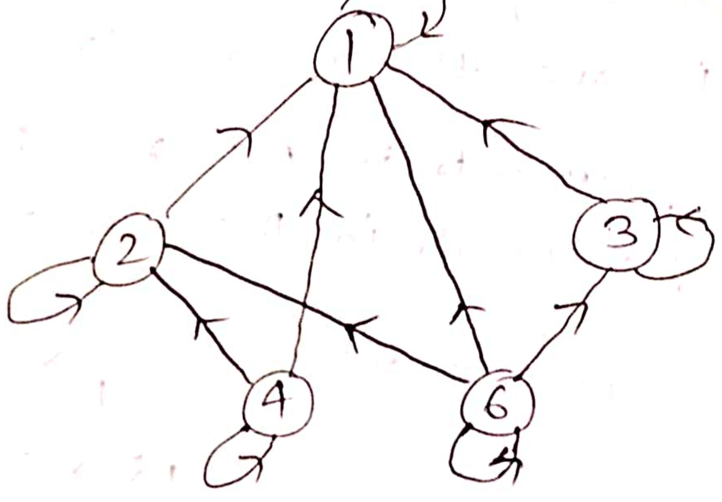
So

$R = \{(1,1), (2,1)\ (3,1)(3,3)\ (2,2)\ (4,1)\ (4,2)\ (4,4)\ (6,1)\ (6,2)\ (6,3)(6,6)\}$

So relation matrix M(R) =

$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 6 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Diagraph

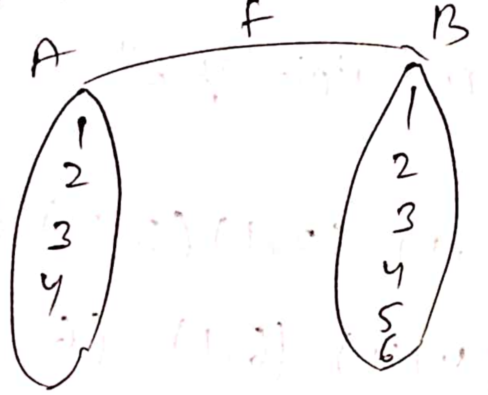


(5)

Soln Given

$A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$

function is mapped from A to B



(i) No. of function :

Every element of A has 6 choice to be mapped with element of B

so total no. of function = $6 \times 6 \times 6 \times 6 = 1296$

ONE - to - ONE

we know that

Every element of A has unique image in B and every element of f(a) has unique preimage in A then it is one - to - one function

so No. of elements in $A = r = 4$

No. of elements in $B = m = 6$

No. of one - one function = ${}^6C_4 \times 4!$

$= 15 \times 24$

$= \boxed{360}$ Ans

onto function :

for onto function if $|A| < |B|$ then it never be onto

so in given question

no of elements in A = 4

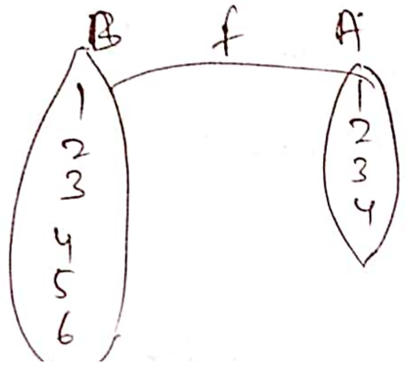
no. of elements in B = 6

so Here $|A| < |B|$ $\{4 < 6\}$

so $\boxed{\text{No. of onto function} = 0}$ Ans

(ii) $A = \{1, 2, 3, 4\}$ $B = \{1, 2, 3, 4, 5, 6\}$

function mapped from B to A



one to one function

let function mapped from A to B

$$f : A \to B$$

if $|A| > |B| \neq$ one-to one

so Here

no. of elements in B $= 6$

no. of elements in A $= 4$

function mapped from $B \to A$

so $|B| > |A|$

so No. of one-to-one function $= 0$ Ans

No of onto functions from B to A

$= p(n,m) - \sum_{k=0}^{n} (-1)^k \, {}^nC_{n-k} \, (n-k)^m$ where

$|B| = m = 6 \qquad |A| = n = 4$

$= \sum_{k=0}^{4} 4C_{4-k} \, (-1)^k \, (4-k)^6$

$= (4C_4 \times 1 \times 4^6) + 4C_3 \times (-1) \times (4-1)^6$

$+ 4C_2 \times 1 \times (4-2)^6 + 4C_1 \times (-1) \times 1^6$

$+ 4C_0 \, (1) \, (0)^6$

$= 1 \times 4^6 + (-4)(3^6) + 6 \times 2^6 + 4(-1) \times 1$

$= 4096 - 2916 + 384 - 4 = 1560$

* A Relation is said to be equivalence only when it is Reflexive, Symmetric and transitive.

$R = \{(1,1)\ (2,2)\ (3,3), (4,4), (1,2)\ (2,1)$
$(3,4)\ (4,3)\}$

~~for Ref~~
The elements present are $A = \{1, 2, 3, 4\}$

→ for Reflexive there should be (aRa) for each element of A

∵ (1,1), (2,2) (3,3) & (4,4) is present

∴ It is Reflexive.

for symmetric if aRb (a, b) is present then there must be (b, a) present

∵ for (1, 2), (2, 1) is present
for (3, 4), (4, 3) is present

∴ It is Symmetric

for transitive if (a, b) & (b, c) is present then there must be (a, c) present

$\therefore$ for $(1,2)$ & $(2,1)$, $(1,1)$ is present

for $(3,4)$ & $(4,3)$, $(3,3)$ is present

$\therefore$ It is transitive also.

Since given Relation R is Reflexive, Symmetric & Transitive.

$\therefore$ It is an equivalence Relation.

(7)

Soln Hasse diagram representing the positive divisors of 36

Let A be set consisting of positive divisor of 36

$A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

1 R (all memembers of A)

2 R (2, 4, 6, 12, 18, 36)

3 R (3, 6, 9, 12, 18, 36)

4R (4, 12, 36)

6R (6, 12, 18, 36)

9R (18, 36)

12R 36 ;

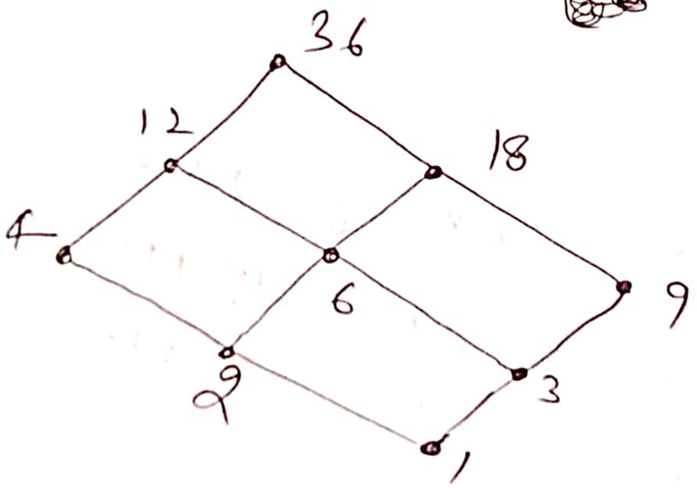
18R 36 ;

we take note of following sets

[1R2], [1R3] (12R36] (18R36] [9R18]

(6R(12, 18)] [4R12] [2R4] [3R{(6,12)]

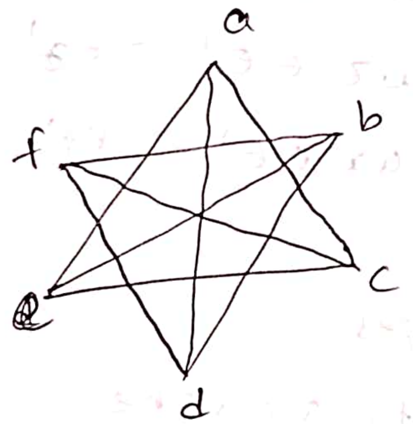
Hasse diagram :

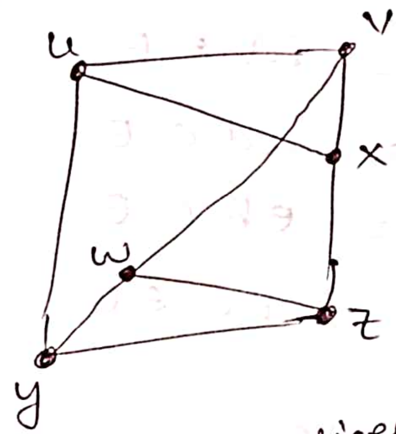


⑧

Soln Graph isomorphoism:

To Graph are said to be isomorphic, if they have same no. of vertices, Edges & same edges Connectivity, & the process of determining Graph isomorphic is known as Graph isomorphism.





No. of vertices = u, v, x, z, w, y = 6

No. of vertices = a, b, c, d, e, f = 6

No. of edges uv, vx, xz, zy, yw, yu, wv, wz, ux = 9

No. of edges = ac, ec, ae, bd, fb, fd, ad, eb, fc = 9

Graph edges Connectivity

let ae = e1
ec = e2
ae = e3
bd = e4
fb = e5
fd = e6
ad = e7
eb = e8
fc = e9

let uv = e_1'
vx = e_2'
xz = e_3'
zy = e_4'
yw = e_5'
yu = e_6'
wv = e_7'
wz = e_8'
ux = e_9'

$e_1 = ac \in E$ $\quad f(a)(c) = uv \in E' = e_1'$

$e_2 = ec \in E$ $\quad f(e)(c) = vx \in E' = e_2'$

$e_3 = ae \in E$ $\quad f(a)(e) = xz \in E' = e_3'$

$e_4 = bd \in E$ $\quad f(b)(d) = zy \in E' = e_4'$

$e_5 = fb \in E$ $\quad f(f)(b) = yw \in E' = e_5'$

$e_6 = fd \in E$ $\quad f(f)(d) = yu \in E' = e_6'$

$e_7 = ad \in E$ $\quad f(a)(d) = wv \in E' = e_7'$

$e_8 = eb \in E$ $\quad f(e)(b) = wz \in E' = e_8'$

$e_9 = fc \in E$ $\quad f(f)(c) = ux \in E' = e_9'$

Here
No. of vertice , No. of Edges
& Graph edge connectivity are same

So Graph are isomorphic