Scheme of Evaluation									
Sub:	Fluid Mechanics					Sub Code:	21ME43	Branch :	
Date:	06/07/2023	Duration:	90 min's	Max Marks:	50	Sem / Sec:	IV		

Answer any 5 Questions.		MARKS
1.	Define the following fluid properties and mention its units: a. Density b. Weight density c. Surface tension d. Specific gravity of a fluid e. Capillarity	2*5
2.	What do you mean by single column manometers? Derive an expression for pressure using vertical single column manometers.	2+4+4
3.	Define buoyancy, center of buoyancy and metacenter with a neat diagram.	2+2+3+3
4.	Distinguish between : a. Steady flow and unsteady flow, b. Uniform and non-uniform flow, c. Compressible and incompressible flow, d. Laminar and turbulent flow,	2*5
5.	The right limb of a simple U-Tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. Gr. 0.9 is flowing. The center of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.	6+4
6.	A pipe contains an oil of sp. gr. 0.9. A differential manometer connected at the two points A and B shows a difference in mercury level as 15 *cm*. Find the difference of pressure at the two points.	6+4
7.	Define rate of flow or Discharge (Q). The diameters of a pipe at the sections 1 and 2 are 10*cm* and 15 *cm* respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 *m/s*. determine also the velocity at section 2.	2+4+4
	What do you understand by Total pressure and Center of pressure? Derive an expression for the force exerted on a submerged vertical plane surface by the static liquid and locate the position of center of pressure.	2+2+6

Solutions

1. Define the following fluid properties and mention its units: a. Density b. Weight density c. Specific volume d. Specific gravity of a fluid d. Compressibility

1.2.1 Density or Mass Density. Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted by the symbol ρ (rho). The unit of mass density in SI unit is kg per cubic metre, *i.e.*, kg/m^3. The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.

Mathematically, mass density is written as

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}.$$

The value of density of water is 1 gm/cm^3 or 1000 kg/m^3.

1.2.2 Specific Weight or Weight Density. Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume. Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol w.

Thus mathematically, $$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(\text{Mass of fluid}) \times \text{Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}}$$

$$= \rho \times g \qquad \left\{\because \frac{\text{Mass of fluid}}{\text{Volume of fluid}} = \rho\right\}$$

$$\therefore \qquad w = \rho g \qquad \text{...(1.1)}$$

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension. The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by Greek letter σ (called sigma). In MKS units, it is expressed as kgf/m while in SI units as N/m.

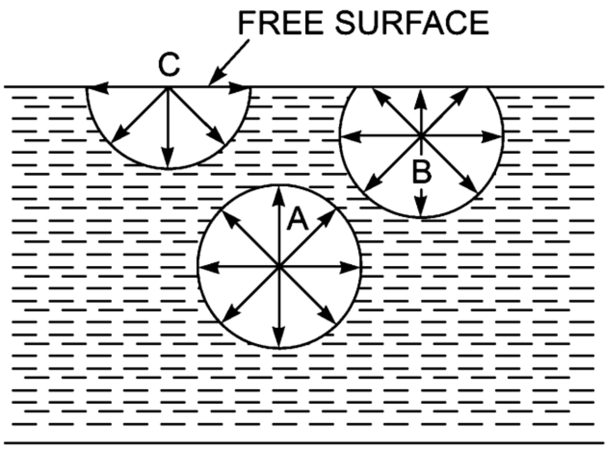


Fig. 1.10 *Surface tension.*

1.2.4 Specific Gravity. Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid. For liquids, the standard fluid is taken water and for gases, the standard fluid is taken air. Specific gravity is also called relative density. It is dimensionless quantity and is denoted by the symbol S.

Mathematically, $$S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

$$S(\text{for gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

Thus weight density of a liquid $= S \times$ Weight density of water

$= S \times 1000 \times 9.81 \text{ N/m}^3$

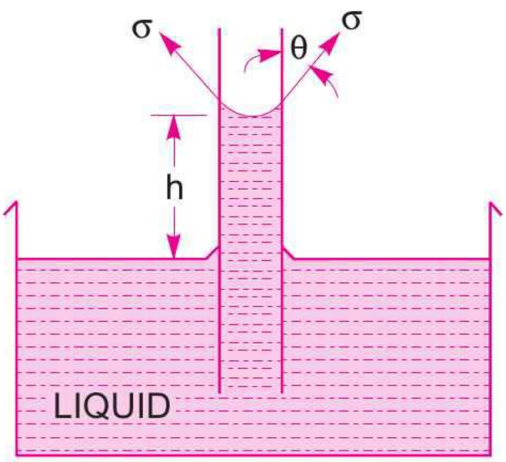
The density of a liquid $= S \times$ Density of water

$= S \times 1000 \text{ kg/m}^3$. ...(1.1$A$)

If the specific gravity of a fluid is known, then the density of the fluid will be equal to specific gravity of fluid multiplied by the density of water. For example, the specific gravity of mercury is 13.6, hence density of mercury = 13.6 × 1000 = 13600 kg/m^3.

$= 0.725 + 10.32 = \mathbf{11.045\ N/cm^2}$. **Ans.**

1.6.4 Capillarity. Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.



2. What do you mean by single column manometers? How are they used for the measurement of pressure?

2.6.3 Single Column Manometer. Single column manometer is a modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer as shown in Fig. 2.15. Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as :

1. Vertical Single Column Manometer.
2. Inclined Single Column Manometer.

1. Vertical Single Column Manometer

Fig. 2.15 shows the vertical single column manometer. Let *X-X* be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at *A*, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Let Δh = Fall of heavy liquid in reservoir

h_2 = Rise of heavy liquid in right limb

h_1 = Height of centre of pipe above *X-X*

p_A = Pressure at *A*, which is to be measured

A = Cross-sectional area of the reservoir

a = Cross-sectional area of the right limb

S_1 = Sp. gr. of liquid in pipe

S_2 = Sp. gr. of heavy liquid in reservoir and right limb

ρ_1 = Density of liquid in pipe

ρ_2 = Density of liquid in reservoir

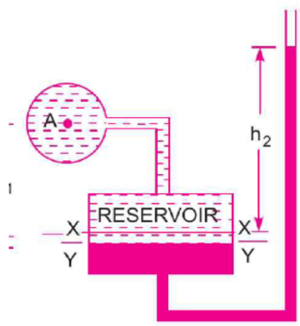


Fig. 2.15 *Vertical single column manometer.*

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$\therefore \quad A \times \Delta h = a \times h_2$

$$\therefore \quad \Delta h = \frac{a \times h_2}{A} \qquad ...(i)$$

Define buoyancy, center of buoyancy and metacenter with a neat diagram.

▶ 4.2 BUOYANCY

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.

▶ 4.3 CENTRE OF BUOYANCY

It is defined as the point, through which the force of buoyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.

▶ 4.4 META-CENTRE

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta-centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

Consider a body floating in a liquid as shown in Fig. 4.5 (*a*). Let the body is in equilibrium and G is the centre of gravity and B the centre of buoyancy. For equilibrium, both the points lie on the normal axis, which is vertical.

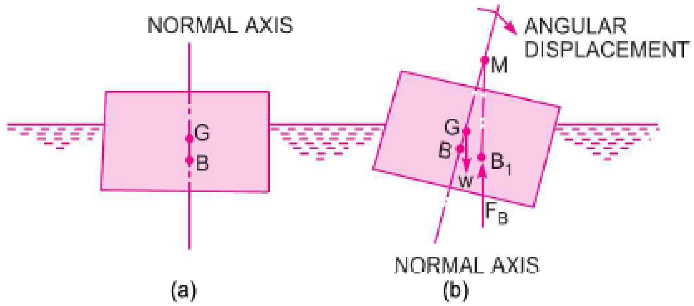


Fig. 4.5 *Meta-centre*

Let the body is given a small angular displacement in the clockwise direction as shown in Fig. 4.5 (*b*). The centre of buoyancy, which is the centre of gravity of the displaced liquid or centre of gravity of the portion of the body sub-merged in liquid, will now be shifted towards right from the normal axis. Let it is at B_1 as shown in Fig. 4.5 (*b*). The line of action of the force of buoyancy in this new position, will intersect the normal axis of the body at some point say M. This point M is called **Meta-centre.**

4. Distinguish between:

5.3.1 Steady and Unsteady Flows. Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc., at a point do not change with time. Thus for steady flow, mathematically, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where (x_0, y_0, z_0) is a fixed point in fluid field.

Unsteady flow is that type of flow, in which the velocity, pressure or density at a point changes with respect to time. Thus, mathematically, for unsteady flow

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \text{ etc.}$$

5.3.2 Uniform and Non-uniform Flows. Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (*i.e.*, length of direction of the flow). Mathematically, for uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0$$

where ∂V = Change of velocity

∂s = Length of flow in the direction S.

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus, mathematically, for non-uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0.$$

5.3.3 Laminar and Turbulent Flows. Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a *zig-zag* way. Due to the movement of fluid particles in a *zig-zag* way, the eddies formation takes place which are responsible for high energy loss. For a pipe flow, the type of flow is determined by a non-dimensional number $\frac{VD}{\nu}$ called the Reynold number,

where D = Diameter of pipe

V = Mean velocity of flow in pipe

and ν = Kinematic viscosity of fluid.

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow. If the Reynold number lies between 2000 and 4000, the flow may be laminar or turbulent.

5.3.4 Compressible and Incompressible Flows. Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid. Thus, mathematically, for compressible flow

$$\rho \neq \text{Constant}$$

Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. Mathematically, for incompressible flow

$$\rho = \text{Constant.}$$

The right limb of a simple U-Tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. Gr. 0.9 is flowing. The center of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.

Problem 2.9 *The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.*

Solution. Given :

Sp. gr. of fluid, $S_1 = 0.9$

$\therefore$ Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Sp. gr. of mercury, $S_2 = 13.6$

$\therefore$ Density of mercury, $\rho_2 = 13.6 \times 1000 \text{ kg/m}^3$

Difference of mercury level, $h_2 = 20 \text{ cm} = 0.2 \text{ m}$

Height of fluid from A-A, $h_1 = 20 - 12 = 8 \text{ cm} = 0.08 \text{ m}$

Let p = Pressure of fluid in pipe

Equating the pressure above A-A, we get

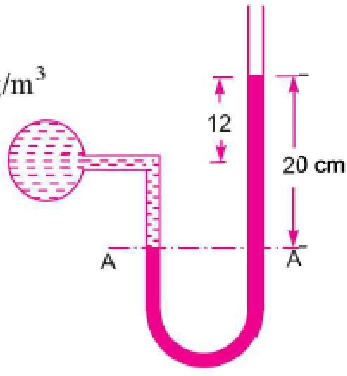


Fig. 2.10

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

or
$$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times .2$$

$$p = 13.6 \times 1000 \times 9.81 \times .2 - 900 \times 9.81 \times 0.08$$

$$= 26683 - 706 = 25977 \text{ N/m}^2 = \mathbf{2.597 \text{ N/cm}^2}. \textbf{ Ans.}$$

A pipe contains an oil of sp. gr. 0.9. A differential manometer connected at the two points A and B shows a difference in mercury level as 15 *cm*. Find the difference of pressure at the two points.

Solution. Given :

Sp. gr. of oil, $S_1 = 0.9$ $\therefore$ Density, $\rho_1 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Difference in mercury level, $h = 15 \text{ cm} = 0.15 \text{ m}$

Sp. gr. of mercury, $S_g = 13.6$ $\therefore$ Density, $\rho_g = 13.6 \times 1000 \text{ kg/m}^3$

The difference of pressure is given by equation (2.13)

or
$$p_A - p_B = g \times h(\rho_g - \rho_1)$$

$$= 9.81 \times 0.15\,(13600 - 900) = \mathbf{18688 \text{ N/m}^2}. \textbf{ Ans.}$$

Define rate of flow or Discharge (Q). The diameters of a pipe at the sections 1 and 2 are 10*cm* and 15 *cm* respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 *m/s*. determine also the velocity at section 2.

▶ 5.4 RATE OF FLOW OR DISCHARGE (Q)

It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel. For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section. Thus

(*i*) For liquids the units of Q are m^3/s or litres/s

(*ii*) For gases the units of Q is kgf/s or Newton/s

Consider a liquid flowing through a pipe in which

A = Cross-sectional area of pipe

V = Average velocity of fluid across the section

Then discharge $Q = A \times V$. ...(5.1)

Problem 5.1 *The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.*

Solution. Given :

At section 1, $D_1 = 10 \text{ cm} = 0.1 \text{ m}$

$$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(.1)^2 = 0.007854 \text{ m}^2$$

$V_1 = 5$ m/s.

At section 2, $D_2 = 15 \text{ cm} = 0.15 \text{ m}$

$$A_2 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$$

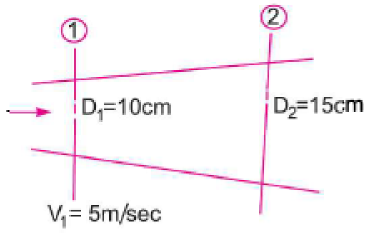


Fig. 5.2

(*i*) Discharge through pipe is given by equation (5.1)

or $Q = A_1 \times V_1$

$= 0.007854 \times 5 =$ **0.03927 m³/s. Ans.**

Using equation (5.3), we have $A_1V_1 = A_2V_2$

(*ii*) $\therefore$ $V_2 = \dfrac{A_1V_1}{A_2} = \dfrac{0.007854}{0.01767} \times 5.0 =$ **2.22 m/s. Ans.**

What do you understand by Total pressure and Center of pressure? Derive an expression for the force exerted on a submerged vertical plane surface by the static liquid and locate the position of center of pressure.

▶ 3.2 TOTAL PRESSURE AND CENTRE OF PRESSURE

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

Centre of pressure is defined as the point of application of the total pressure on the surface. There are four cases of submerged surfaces on which the total pressure force and centre of pressure is to be determined. The submerged surfaces may be :

1. Vertical plane surface,
2. Horizontal plane surface,
3. Inclined plane surface, and
4. Curved surface.

▶ 3.3 VERTICAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in Fig. 3.1.

Let A = Total area of the surface

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h^* = Distance of centre of pressure from free surface of liquid.

(*a*) **Total Pressure (F)**. The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

Consider a strip of thickness dh and width b at a depth of h from free surface of liquid as shown in Fig. 3.1

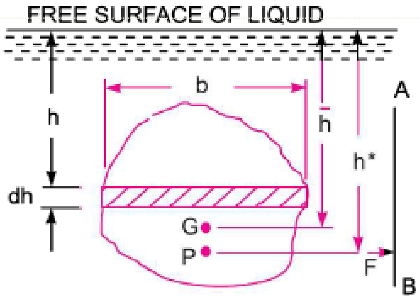


Fig. 3.1

Pressure intensity on the strip, $p = \rho g h$

(See equation 2.5)

Area of the strip, $dA = b \times dh$

Total pressure force on strip, $dF = p \times \text{Area}$

$= \rho g h \times b \times dh$

$\therefore$ Total pressure force on the whole surface,

$$F = \int dF = \int \rho g h \times b \times dh = \rho g \int b \times h \times dh$$

But $\int b \times h \times dh = \int h \times dA$

= Moment of surface area about the free surface of liquid

= Area of surface × Distance of C.G. from free surface

$= A \times \bar{h}$

$$\therefore \qquad F = \rho g A \bar{h} \qquad \text{...(3.1)}$$

For water the value of $\rho = 1000$ kg/m^3 and $g = 9.81$ m/s^2. The force will be in Newton.

(*b*) **Centre of Pressure (h*)**. Centre of pressure is calculated by using the "Principle of Moments", which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance h^* from free surface of the liquid as shown in Fig. 3.1. Hence moment of the force F about free surface of the liquid $= F \times h^*$ ...(3.2)

Moment of force dF, acting on a strip about free surface of liquid

$= dF \times h$ $\{\because \; dF = \rho g h \times b \times dh\}$

$= \rho g h \times b \times dh \times h$

Sum of moments of all such forces about free surface of liquid

$$= \int \rho g h \times b \times dh \times h = \rho g \int b \times h \times h dh$$

$$= \rho g \int b h^2 \, dh = \rho g \int h^2 \, dA \qquad (\because \; b dh = dA)$$

But $\int h^2 dA = \int b h^2 dh$

= Moment of Inertia of the surface about free surface of liquid

$= I_0$

$\therefore$ Sum of moments about free surface

$$= \rho g I_0 \qquad \text{...(3.3)}$$

Equating (3.2) and (3.3), we get

$$F \times h^* = \rho g I_0$$

But $$F = \rho g A \bar{h}$$

$\therefore$ $$\rho g A \bar{h} \times h^* = \rho g I_0$$

or $$h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A \bar{h}} \qquad \ldots(3.4)$$

By the theorem of parallel axis, we have

$$I_0 = I_G + A \times \overline{h^2}$$

where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_0 in equation (3.4), we get

$$h^* = \frac{I_G + A\overline{h^2}}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h} \qquad \ldots(3.5)$$

In equation (3.5), $\bar{h}$ is the distance of C.G. of the area of the vertical surface from free surface of the liquid. Hence from equation (3.5), it is clear that :

(*i*) Centre of pressure (*i.e.*, h^*) lies below the centre of gravity of the vertical surface.

(*ii*) The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.