

1. Derive Euler's equation of motion along a stream line and deduce Bernoulli's equation. State the assumptions made.

► **6.3 EULER'S EQUATION OF MOTION**

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as :

Consider a stream-line in which flow is taking place in s -direction as shown in Fig. 6.1. Consider a cylindrical element of cross-section dA and length ds . The forces acting on the cylindrical element are:

1. Pressure force $p dA$ in the direction of flow.
2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds \right) dA$ opposite to the direction of flow.
3. Weight of element $\rho g dA ds$.

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element \times acceleration in the direction s .

$$\begin{aligned} \therefore \quad & p dA - \left(p + \frac{\partial p}{\partial s} ds \right) dA - \rho g dA ds \cos \theta \\ & = \rho dA ds \times a_s \end{aligned} \quad \dots(6.2)$$

where a_s is the acceleration in the direction of s .

$$\begin{aligned} \text{Now} \quad a_s &= \frac{dv}{dt}, \text{ where } v \text{ is a function of } s \text{ and } t. \\ &= \frac{\partial v}{\partial s} \frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v \partial v}{\partial s} + \frac{\partial v}{\partial t} \left\{ \because \frac{ds}{dt} = v \right\} \end{aligned}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore \quad a_s = \frac{v \partial v}{\partial s}$$

Substituting the value of a_s in equation (6.2) and simplifying the equation, we get

$$- \frac{\partial p}{\partial s} ds dA - \rho g dA ds \cos \theta = \rho dA ds \times \frac{\partial v}{\partial s}$$

Fig. 6.1 Forces on a fluid element.

Dividing by $\rho ds dA$, $-\frac{\partial p}{\rho \partial s} - g \cos \theta = \frac{v \partial v}{\partial s}$

$$\text{or} \quad \frac{\partial p}{\rho \partial s} + g \cos \theta + v \frac{\partial v}{\partial s} = 0$$

But from Fig. 6.1 (b), we have $\cos \theta = \frac{dz}{ds}$

$$\therefore \quad \frac{1}{\rho} \frac{dp}{ds} + g \frac{dz}{ds} + \frac{v dv}{ds} = 0 \quad \text{or} \quad \frac{dp}{\rho} + g dz + v dv = 0$$

$$\text{or} \quad \frac{dp}{\rho} + g dz + v dv = 0 \quad \dots(6.3)$$

Equation (6.3) is known as Euler's equation of motion.

► 6.4 BERNOULLI'S EQUATION FROM EULER'S EQUATION

Bernoulli's equation is obtained by integrating the Euler's equation of motion (6.3) as

$$\int \frac{dp}{\rho} + \int g dz + \int v dv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\therefore \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

or
$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

or
$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \quad \dots(6.4)$$

Equation (6.4) is a Bernoulli's equation in which

$$\frac{p}{\rho g} = \text{pressure energy per unit weight of fluid or pressure head.}$$

$$v^2/2g = \text{kinetic energy per unit weight or kinetic head.}$$

$$z = \text{potential energy per unit weight or potential head.}$$

► 6.5 ASSUMPTIONS

The following are the assumptions made in the derivation of Bernoulli's equation :

- (i) The fluid is ideal, *i.e.*, viscosity is zero (ii) The flow is steady
(iii) The flow is incompressible (iv) The flow is irrotational.

2. Define Similitude and explain the following:

- i) Geometric similarity ii) Kinematic similarity iii) Dynamic similarity

Similitude is defined as the similarity between the model and its prototype in every respect, which means that the model and prototype have similar properties or model and prototype are completely similar. Three types of similarities must exist between the model and prototype. They are

1. Geometric Similarity, 2. Kinematic Similarity, and 3. Dynamic Similarity.

1. Geometric Similarity. The geometric similarity is said to exist between the model and prototype. The ratio of all corresponding linear dimensions in the model and prototype are equal.

Let L_m = Length of model, b_m = Breadth of model,

D_m = Diameter of model, A_m = Area of model,

\forall_m = Volume of model,

and $L_p, b_p, D_p, A_p, \forall_p$ = Corresponding values of the prototype.

For geometric similarity between model and prototype, we must have the relation,

$$\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r \quad \dots(12)$$

where L_r is called the scale ratio.

For area's ratio and volume's ratio the relation should be as given below :

$$\frac{A_p}{A_m} = \frac{L_p \times b_p}{L_m \times b_m} = L_r \times L_r = L_r^2 \quad \dots(12)$$

and
$$\frac{\forall_p}{\forall_m} = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{b_p}{b_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3 \quad \dots(12)$$

2. Kinematic Similarity. Kinematic similarity means the similarity of motion between model and prototype. Thus kinematic similarity is said to exist between the model and the prototype if the ratio of the velocity and acceleration at the corresponding points in the model and at the correspond

points in the prototype are the same. Since velocity and acceleration are vector quantities, hence only the ratio of magnitude of velocity and acceleration at the corresponding points in model and prototype should be same ; but the directions of velocity and accelerations at the corresponding points in the model and prototype also should be parallel.

Let V_{p_1} = Velocity of fluid at point 1 in prototype,
 V_{p_2} = Velocity of fluid at point 2 in prototype,
 a_{p_1} = Acceleration of fluid at point 1 in prototype,
 a_{p_2} = Acceleration of fluid at point 2 in prototype, and

$V_{m_1}, V_{m_2}, a_{m_1}, a_{m_2}$ = Corresponding values at the corresponding points of fluid velocity and acceleration in the model.

For kinematic similarity, we must have

$$\frac{V_{p_1}}{V_{m_1}} = \frac{V_{p_2}}{V_{m_2}} = V_r \quad \dots(11)$$

where V_r is the velocity ratio.

For acceleration, we must have $\frac{a_{p_1}}{a_{m_1}} = \frac{a_{p_2}}{a_{m_2}} = a_r \quad \dots(12)$

where a_r is the acceleration ratio.

Also the directions of the velocities in the model and prototype should be same.

3. Dynamic Similarity. Dynamic similarity means the similarity of forces between the model and prototype. Thus dynamic similarity is said to exist between the model and the prototype if the ratio of the corresponding forces acting at the corresponding points are equal. Also the directions of corresponding forces at the corresponding points should be same.

Let $(F_i)_p$ = Inertia force at a point in prototype,
 $(F_v)_p$ = Viscous force at the point in prototype,
 $(F_g)_p$ = Gravity force at the point in prototype,
and $(F_i)_m, (F_v)_m, (F_g)_m$ = Corresponding values of forces at the corresponding point in model.

Then for dynamic similarity, we have

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} \dots = F_r, \text{ where } F_r \text{ is the force ratio.}$$

Also the directions of the corresponding forces at the corresponding points in the model and prototype should be same.

3. Sketch and derive the relation for actual discharge through an orifice meter.

6.7.2 Orifice Meter or Orifice Plate. It is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter. It also works on the same principle as that of venturimeter. It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe. The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter.

A differential manometer is connected at section (1), which is at a distance of about 1.5 to 2.0 times the pipe diameter upstream from the orifice plate, and at section (2), which is at a distance of about half the diameter of the orifice on the downstream side from the orifice plate.

Let p_1 = pressure at section (1),
 v_1 = velocity at section (1),
 a_1 = area of pipe at section (1), and

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Fig. 6.12. Orifice meter.

p_2, v_2, a_2 are corresponding values at section (2). Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

or
$$\left(\frac{p_1}{\rho g} + z_1 \right) - \left(\frac{p_2}{\rho g} + z_2 \right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But
$$\left(\frac{p_1}{\rho g} + z_1 \right) - \left(\frac{p_2}{\rho g} + z_2 \right) = h = \text{Differential head}$$

$$\therefore h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{or} \quad 2gh = v_2^2 - v_1^2$$

or
$$v_2 = \sqrt{2gh + v_1^2} \quad \dots(i)$$

Now section (2) is at the vena-contracta and a_2 represents the area at the vena-contracta. If a_0 is the area of orifice then, we have

$$C_c = \frac{a_2}{a_0}$$

where C_c = Co-efficient of contraction

$$\therefore a_2 = a_0 \times C_c \quad \dots(ii)$$

By continuity equation, we have

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2}{a_1} v_2 = \frac{a_0 C_c}{a_1} v_2 \quad \dots(iii)$$

Substituting the value of v_1 in equation (i), we get

$$v_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 v_2^2}{a_1^2}}$$

or
$$v_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^2 v_2^2 \text{ or } v_2^2 \left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2gh$$

$$\therefore v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$\therefore \text{ The discharge } Q = v_2 \times a_2 = v_2 \times a_0 C_c \quad \{ \because a_2 = a_0 C_c \text{ from (ii)} \}$$

$$= \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}} \quad \dots(iv)$$

The above expression is simplified by using

$$C_d = C_c \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$\therefore C_c = C_d$$

Substituting this value of C_c in equation (iv), we get

$$Q = a_0 \times C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2} C_c^2}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$= \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}} \quad \dots(6.13)$$

where C_d = Co-efficient of discharge for orifice meter.

4. Derive an expression for discharge through a rectangular notch.

► 8.3 DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR

The expression for discharge over a rectangular notch or weir is the same.

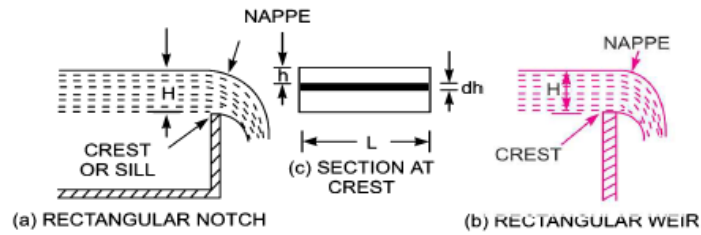


Fig. 8.1 Rectangular notch and weir.

Consider a rectangular notch or weir provided in a channel carrying water as shown in Fig. 8.1.

Let

H = Head of water over the crest

L = Length of the notch or weir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface of water as shown in Fig. 8.1(c).

The area of strip = $L \times dh$
and theoretical velocity of water flowing through strip = $\sqrt{2gh}$

The discharge dQ , through strip is

$$\begin{aligned} dQ &= C_d \times \text{Area of strip} \times \text{Theoretical velocity} \\ &= C_d \times L \times dh \times \sqrt{2gh} \end{aligned} \quad \dots(i)$$

where C_d = Co-efficient of discharge.

The total discharge, Q , for the whole notch or weir is determined by integrating equation (i) between the limits 0 and H .

$$\begin{aligned} \therefore Q &= \int_0^H C_d \cdot L \cdot \sqrt{2gh} \cdot dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2} dh \\ &= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1} \right]_0^H = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H \\ &= \frac{2}{3} C_d \times L \times \sqrt{2g} [H]^{3/2}. \end{aligned} \quad \dots(8.1)$$

- The pressure difference Δp in a pipe of diameter D and length l due to turbulent flow depends on the velocity V , viscosity μ , density ρ and roughness k . Using Buckingham's π theorem, obtain expression for Δp .

Solution. Given :

Δp is a function of D, l, V, μ, ρ, k

$$\therefore \Delta p = f(D, l, V, \mu, \rho, k) \text{ or } f_1(\Delta p, D, l, V, \mu, \rho, k) = 0 \quad \dots(i)$$

\therefore Total number of variables, $n = 7$.

Writing dimensions of each variable,

$$\begin{aligned} \text{Dimension of } \Delta p &= \text{Dimension of pressure} = ML^{-1}T^{-2} \\ D = L, l = L, V &= LT^{-1}, \mu = ML^{-1}T^{-1}, \rho = ML^{-3}, k = L \end{aligned}$$

\therefore Number of fundamental dimensions, $m = 3$

Number of π -terms $= n - m = 7 - 3 = 4$.

Now equation (i) can be grouped in 4 π -terms as

$$f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0 \quad \dots(ii)$$

Each π -term contains $m + 1$ or $3 + 1 = 4$ variables. Out of four variables, three are repeating variables. Choosing D, V, ρ as the repeating variables, we have the four π -terms as

$$\pi_1 = D^{a_1} \cdot V^{b_1} \cdot \rho^{c_1} \cdot \Delta p$$

$$\pi_2 = D^{a_2} \cdot V^{b_2} \cdot \rho^{c_2} \cdot l$$

$$\pi_3 = D^{a_3} \cdot V^{b_3} \cdot \rho^{c_3} \cdot \mu$$

$$\pi_4 = D^{a_4} \cdot V^{b_4} \cdot \rho^{c_4} \cdot k$$

First π -term $\pi_1 = D^{a_1} \cdot V^{b_1} \cdot \rho^{c_1} \cdot \Delta p$

Substituting dimensions on both sides,

$$M^0L^0T^0 = L^{a_1} \cdot (LT^{-1})^{b_1} \cdot (ML^{-3})^{c_1} \cdot ML^{-1}T^{-2}$$

Equating the powers of M, L, T on both sides,

$$\text{Power of } M, \quad 0 = c_1 + 1 \quad \therefore c_1 = -1$$

$$\text{Power of } L, \quad 0 = a_1 + b_1 - 3c_1 - 1, \quad \therefore a_1 = -b_1 + 3c_1 + 1 = 2 - 3 + 1 = 0$$

$$\text{Power of } T, \quad 0 = -b_1 - 2, \quad \therefore b_1 = -2$$

Substituting the values of a_1, b_1 and c_1 in π_1 ,

$$\pi_1 = D^0 \cdot V^{-2} \cdot \rho^{-1} \cdot \Delta p = \frac{\Delta p}{\rho V^2}$$

Second π -term $\pi_2 = D^{a_2} \cdot V^{b_2} \cdot \rho^{c_2} \cdot l$

Substituting dimensions on both sides,

$$M^0L^0T^0 = L^{a_2} \cdot (LT^{-1})^{b_2} \cdot (ML^{-3})^{c_2} \cdot L$$

Equating powers of M, L, T on both sides,

$$\text{Power of } M, \quad 0 = c_2, \quad \therefore c_2 = 0$$

$$\text{Power of } L, \quad 0 = a_2 - b_2 - 3c_2 + 1, \quad \therefore a_2 = b_2 + 3c_2 - 1 = -1$$

$$\text{Power of } T, \quad 0 = -b_2, \quad \therefore b_2 = 0$$

Substituting the values of a_2, b_2, c_2 in π_2 ,

$$\pi_2 = D^{-1} \cdot V^0 \cdot \rho^0 \cdot l = \frac{l}{D}$$

Third π -term $\pi_3 = D^{a_3} \cdot V^{b_3} \cdot \rho^{c_3} \cdot \mu$

Substituting dimensions on both sides,

$$M^0L^0T^0 = L^{a_3} \cdot (LT^{-1})^{b_3} \cdot (ML^{-3})^{c_3} \cdot ML^{-1}T^{-1}$$

Equating the powers of M, L, T on both sides,

$$\text{Power of } M, \quad 0 = c_3 + 1, \quad \therefore c_3 = -1$$

$$\text{Power of } L, \quad 0 = a_3 + b_3 - 3c_3 - 1, \quad \therefore a_3 = -b_3 + 3c_3 + 1 = 1 - 3 + 1 = -1$$

$$\text{Power of } T, \quad 0 = -b_3 - 1, \quad \therefore b_3 = -1$$

Substituting the values of a_3, b_3 and c_3 in π_3 ,

$$\pi_3 = D^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu = \frac{\mu}{DV\rho}$$

Fourth π -term $\pi_4 = D^{a_4} \cdot V^{b_4} \cdot \rho^{c_4} \cdot k$

or $M^0L^0T^0 = L^{a_4} \cdot (LT^{-1})^{b_4} \cdot (ML^{-3})^{c_4} \cdot L$ {Dimension of $k = L$ }

Equating the power of M, L, T on both sides,

$$\text{Power of } M, \quad 0 = c_4, \quad \therefore c_4 = 0$$

$$\text{Power of } L, \quad 0 = a_4 - b_4 - 3c_4 + 1, \quad \therefore a_4 = b_4 + 3c_4 - 1 = -1$$

$$\text{Power of } T, \quad 0 = -b_4, \quad \therefore b_4 = 0$$

Substituting the values of a_4, b_4, c_4 in π_4 ,

$$\pi_4 = D^{-1} \cdot V^0 \cdot \rho^0 \cdot k = \frac{k}{D}$$

Substituting the values of π_1, π_2, π_3 and π_4 in (ii), we get

$$f_1\left(\frac{\Delta p}{\rho V^2}, \frac{l}{D}, \frac{\mu}{DV\rho}, \frac{k}{D}\right) = 0 \quad \text{or} \quad \frac{\Delta p}{\rho V^2} = \phi\left[\frac{l}{D}, \frac{\mu}{DV\rho}, \frac{k}{D}\right]. \text{Ans.}$$

6. Explain Rayleigh method of the dimensional analysis. The time period (t) of a pendulum depends upon the length (L) of the pendulum and acceleration due to gravity (g). Derive an expression for the time period.

► 12.4 METHODS OF DIMENSIONAL ANALYSIS

If the number of variable involved in a physical phenomenon are known, then the relation among the variables can be determined by the following two methods :

1. Rayleigh's method, and
2. Buckingham's π -theorem.

12.4.1 Rayleigh's Method. This method is used for determining the expression for a variable which depends upon maximum three or four variables only. If the number of independent variables becomes more than four, then it is very difficult to find the expression for the dependent variable.

Let X is a variable, which depends on X_1 , X_2 and X_3 variables. Then according to Rayleigh's method, X is function of X_1 , X_2 and X_3 and mathematically it is written as $X = f[X_1, X_2, X_3]$.

This can also be written as $X = KX_1^a \cdot X_2^b \cdot X_3^c$ where K is constant and a , b and c are arbitrary powers.

The values of a , b and c are obtained by comparing the powers of the fundamental dimension on both sides. Thus the expression is obtained for dependent variable.

Problem 12.2 The time period (t) of a pendulum depends upon the length (L) of the pendulum and acceleration due to gravity (g). Derive an expression for the time period.

Solution. Time period t is a function of (i) L and (ii) g

$$\therefore t = KL^a \cdot g^b, \text{ where } K \text{ is a constant} \quad \dots(i)$$

Substituting the dimensions on both sides $T^1 = KL^a \cdot (LT^{-2})^b$

Equating the powers of M , L and T on both sides, we have

$$\text{Power of } T, \quad 1 = -2b \quad \therefore \quad b = -\frac{1}{2}$$

$$\text{Power of } L, \quad 0 = a + b \quad \therefore \quad a = -b = -\left(-\frac{1}{2}\right) = \frac{1}{2}$$

Substituting the values of a and b in equation (i),

$$t = KL^{1/2} \cdot g^{-1/2} = K \sqrt{\frac{L}{g}}$$

The value of K is determined from experiments which is given as

$$K = 2\pi$$

$$\therefore t = 2\pi \sqrt{\frac{L}{g}}. \text{ Ans.}$$

7. The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm², find the intensity of pressure at section 2.

Solution. Given :

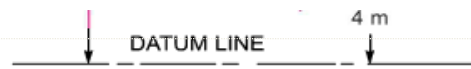


Fig. 6.3

At section 1,

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$$A_1 = \frac{\pi}{4} (.2)^2 = .0314 \text{ m}^2$$

$$p_1 = 39.24 \text{ N/cm}^2 \\ = 39.24 \times 10^4 \text{ N/m}^2$$

$$z_1 = 6.0 \text{ m}$$

At section 2,

$$D_2 = 0.10 \text{ m}$$

$$A_2 = \frac{\pi}{4} (0.1)^2 = .00785 \text{ m}^2$$

$$z_2 = 4 \text{ m}$$

$$p_2 = ?$$

Rate of flow,

$$Q = 35 \text{ lit/s} = \frac{35}{1000} = .035 \text{ m}^3/\text{s}$$

Now

$$Q = A_1 V_1 = A_2 V_2$$

\therefore

$$V_1 = \frac{Q}{A_1} = \frac{.035}{.0314} = 1.114 \text{ m/s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{.035}{.00785} = 4.456 \text{ m/s}$$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\text{or } \frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

$$\text{or } 40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

$$\text{or } 46.063 = \frac{p_2}{9810} + 5.012$$

$$\therefore \frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$$\therefore p_2 = 41.051 \times 9810 \text{ N/m}^2 \\ = \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2. \text{ Ans.}}$$

8. A 30 cm x 15 cm Venturimeter is inserted in a vertical pipe carrying water, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 20 cm. Find the discharge. Take $C_d = 0.98$.

Solution. Given :

Dia. at inlet,

$$d_1 = 30 \text{ cm}$$

∴

$$a_1 = \frac{\pi}{4} (30)^2 = 706.85 \text{ cm}^2$$

Dia. at throat,

$$d_2 = 15 \text{ cm}$$

∴

$$a_2 = \frac{\pi}{4} (15)^2 = 176.7 \text{ cm}^2$$

$$h = x \left[\frac{S_h}{S_o} - 1 \right] = 20 \left[\frac{13.6}{1.0} - 1.0 \right] = 20 \times 12.6 = 252.0 \text{ cm of water}$$

$$C_d = 0.98$$

Discharge,

$$\begin{aligned} Q &= C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \\ &= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 252} \\ &= \frac{86067593.36}{\sqrt{499636.3 - 31222.9}} = \frac{86067593.36}{684.4} \\ &= 125756 \text{ cm}^3/\text{s} = \mathbf{125.756 \text{ lit/s. Ans.}} \end{aligned}$$

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