Imp QUESTION MODULE-05 ①

HEAT EXCHANGERS

1) CLASSIFICATION OF HEAT EXCHANGERS

Heat exchangers are classified ~~int~~ as follows:-

i. According to the nature of heat exchanging process.
   a) Direct contact heat exchangers
   b) Regenerators
   c) Recuperates.

ii. According to direction of flow of fluids
   a) Parallel flow heat exchangers
   b) Counter flow heat exchangers
   c) Cross flow heat exchangers.

iii. According to design and construction
   a) Concentric tube heat exchangers.
   b) Shell and tube exchanger.
   c) compact heat exchangers.

iv. According to the physical state of fluids.
   a) Condenser.
   b) Evaporator.

i) ~~Dir~~

a) Direct contact heat exchangers.

In this type of heat exchangers, heat transfer between the hot and cold fluids. is achieved by their complete physical mixing in one another.

Ex:- Cooling towers, Jet condensers.

b) Regenerators

In a regenerators type heat exchanger, the hot and cold fluids pass alternatively through a space containing solid particles called matrix. The heat stored in the matrix is transferred to the cold fluid by allowing it to pass over the heated matrix. Thus, fluids flow through the same space but alternatively and do not get mixed.

Ex:- Pre heaters for steam power plant, Blast furnaces.

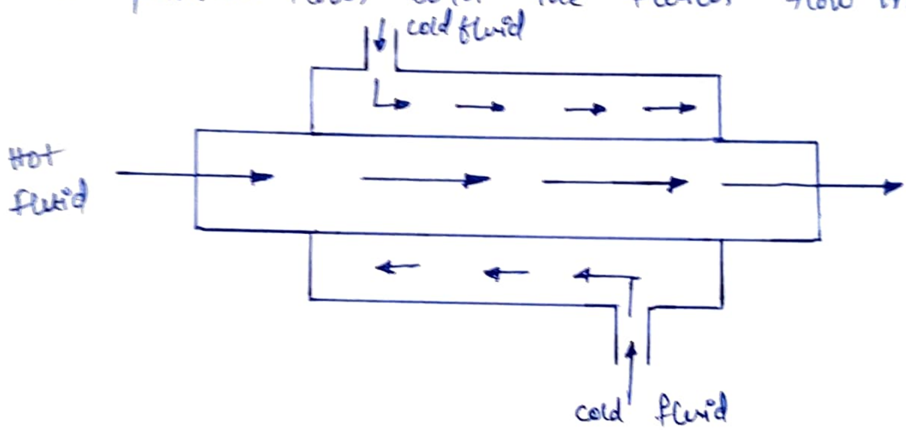
c> Recuperator:

In this the hot and cold fluids flow simultaneously on either side of a separating wall. The heat is transferred by a combination of convection to and from the wall and conduction through the wall. The wall can include extended surface such as fins or other heat transfer enhancement devices.

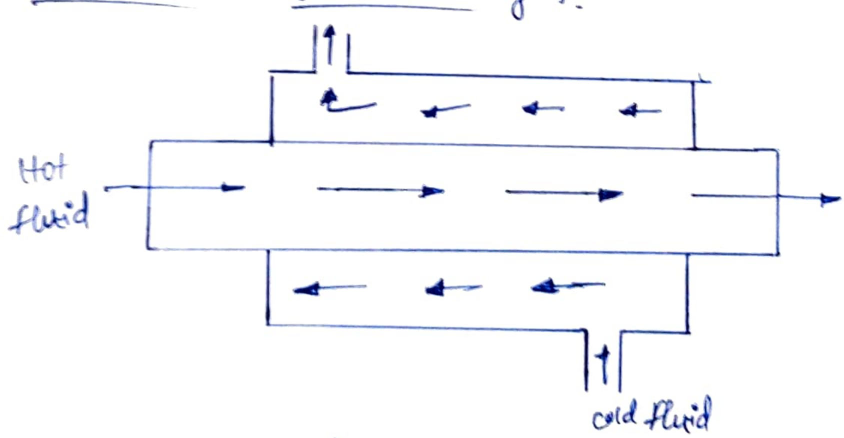
ii>

a> Parallel flow Heat Exchangers:

In parallel flow, both the fluids flow in same direction.



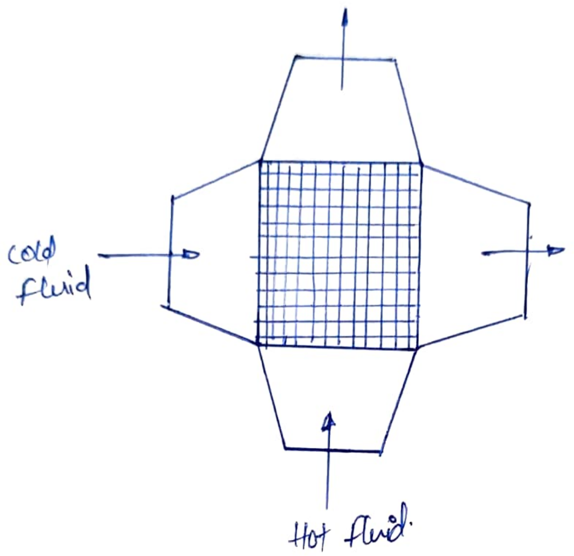
b> counter flow Heat Exchangers:



In counter flow the two fluids flow in opposite direction.

c) Cross flow Heat Exchangers:

In cross flow heat exchangers the heat and cold fluids flow at right angles to each other.



iii)

a) Concentric tube Heat Exchangers:

In this, two concentric tubes are used. Each carries one of the fluids. The direction of flow may be parallel or counter flow.

b) Shell and tube Heat Exchangers:

In this, one of the fluids flow through a bundle of tubes enclosed by a shell and the outer fluid is forced through the shell and flows over the tubes.

c) Compact Heat Exchangers

They have a very large heat transfer area per unit volume of the heat exchanger.

Applications in which compact heat exchangers are required include (i) an automobile heater core in which engine coolant is circulated through tubes and the passenger compartment air is blown over the finned exterior surface of the tubes.

iii) Refrigerator condensers.

iv)
a) Condenser.

In this, condensing fluid remains at constant temperature through out the heat exchanger. The temperature of the colder fluid gradually increases from inlet to outlet.

b) Evaporator:

In this cold fluid remains at constant temperature, while temperature of hot fluid gradually decreases from inlet to outlet.

2) FOULING

The phenomenon of rust formation and deposition of fluid impurities on the tube surface of heat exchanger is called fouling.

The surface deposits increase thermal resistance with a corresponding drop in the performance of heat exchanger. The effect of scale on heat flow is considered by specifying an equivalent scale heat transfer coefficient.

Thermal resistance due to fouling

$R_{fi} = \frac{1}{A_i h_{fi}}$ → inside the pipe

$R_{fo} = \frac{1}{A_o h_{fo}}$ → outside the tube

$\left.\begin{matrix} h_{fi} = h_{si} \\ h_{fo} = h_{so} \end{matrix}\right\}$

h_{fi} and h_{fo} are the heat transfer co-efficients between scale and fluid,

Now the overall heat transfer coefficient becomes,

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{1}{h_{fi}} + \frac{r_i}{k}\log_e\frac{r_o}{r_i} + \frac{r_i}{r_o}\frac{1}{h_{fo}} + \frac{r_i}{r_o}\frac{1}{h_o}$$ and

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{1}{h_{fo}} + \frac{r_o}{k}\log_e\frac{r_o}{r_i} + \frac{r_i}{r_o}\frac{1}{h_{fi}} + \frac{r_o}{r_i}\frac{1}{h_i}$$

Fouling factor

The reciprocal of scale heat transfer co-efficient is called as fouling factor.

i.e., Fouling factor $R_f = \frac{1}{h_f}$

i.e, $R_{fo} = \frac{1}{h_{fo}}$ and $R_{fi} = \frac{1}{h_{fi}}$ for outer and inner sides of the pipe.

fouling factors are determined experimentally by testing the heat exchangers in both the clean and dirty conditions.

i.e., $R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}}$.

3) Logarithmic Mean Temperature difference Method (LMTD)

The Thermal analysis of heat Exchangers depend on variables like inlet and outlet fluid temperature, overall heat transfer coefficient, total surface area of heat exchanger and the heat transfer rate.

The temperature difference b/w hot and cold fluids is $\Delta T = T_h - T_c$. Since ΔT is varying with position along the length of the heat exchanger, the heat transfer becomes

$Q = UA\Delta T_m$

where, ΔT_m = Mean Temperature difference across the heat exchanger.

For heat exchangers this mean temperature difference will be in the form of a logarithmic relation. Hence it is referred as Logarithmic Mean temperature Difference (LMTD).

Assumptions are considered for the analysis of heat exchanger under LMTD.

a) The overall heat transfer coefficient is uniform through out the heat exchanger.

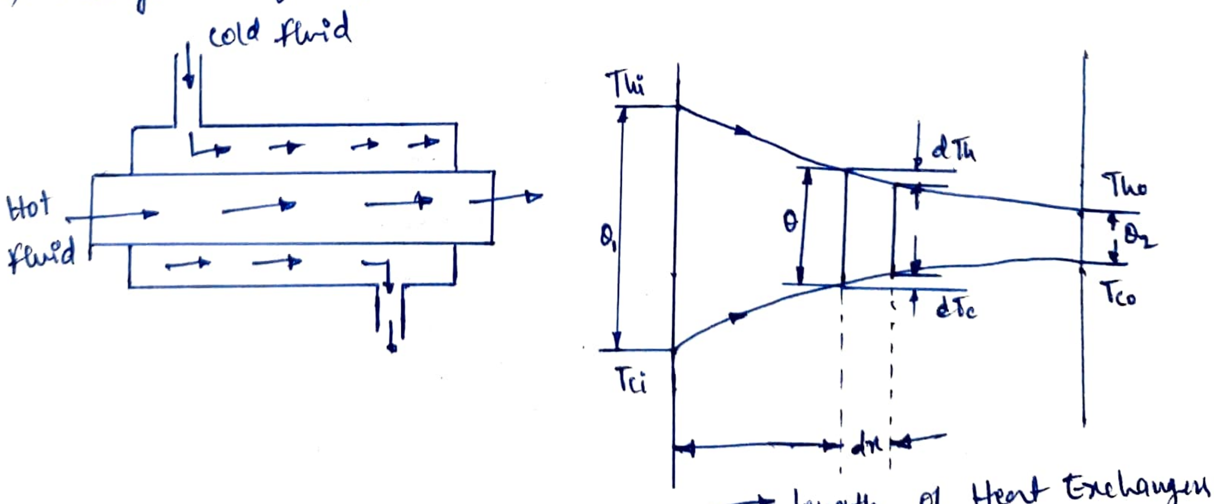
b) The Specific heats and mass flow rates of both the fluids are constant.

c) The potential and kinetic energy changes are negligible.

d) The heat exchange takes only between the two fluids.

e) The temperature of both the fluids are constant over a given cross-section.

4) Analysis of parallel flow heat exchanger using LMTD.



let T_{hi} = Inlet temperature of hot fluid

T_{ho} = outlet temperature of hot fluid

T_{ci} = Inlet temperature of cold fluid
T_{co} = outlet temperature of cold fluid
C_{ph} & C_{pc} are specific heats of hot & cold fluids
m_h & m_c are mass flow rate of hot & cold fluids.

Also let $\theta_1 = T_{hi} - T_{ci}$
$\theta_2 = T_{ho} - T_{co}$

Total heat transfer between the two fluids

$$Q = m_h C_{ph} (T_{hi} - T_{ho}) = m_c C_{pc} (T_{co} - T_{ci})$$

(or) $$Q = C_h (T_{hi} - T_{ho}) = C_c (T_{co} - T_{ci})$$

where $C_h = m_h C_{ph}$ & $C_c = m_c C_{pc}$ are called heat capacity rates of hot & cold fluids.

Consider a small element of thickness 'dx' as shown in figure.

For hot fluid

$$dQ = -m_h C_{ph}\, dT_h = -C_h dT_h$$

(or) $$dT_h = \frac{-dQ}{C_h}$$

for cold fluid,

$$dQ = m_h C_{pc}\, dT_c = C_c dT_c$$

(or) $$dT_c = \frac{dQ}{C_c}$$

But for the element

$$\theta = T_h - T_c$$

$$d\theta = dT_h - dT_c$$

$$= \cancel{dT_h}$$

$$= \frac{-dQ}{C_h} - \frac{dQ}{C_c}$$

$$d\theta = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$d\theta = -U\,dA\,\theta\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \qquad \therefore [dQ = U dA.\theta]$$

$$\frac{d\theta}{\theta} = -U dA\left[1/C_h + 1/C_c\right]$$

Integrating along the heat exchanger length between inlet and outlet of heat exchanger.

i.e., $$\int_1^2 \frac{d\theta}{\theta} = -U\left[\frac{1}{C_h} + \frac{1}{C_c}\right]\int_0^A dA$$

$$\log_e \frac{\theta_2}{\theta_1} = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$= -UA\left[\frac{1/Q}{T_{hi} - T_{ho}} + \frac{1/Q}{T_{co} - T_{ci}}\right]$$

$$= \frac{-UA}{Q}\left[T_{hi} - T_{ho} + T_{co} - T_{ci}\right]$$

$$= \frac{-UA}{Q}\left[(T_{hi} - T_{ci}) - (T_{ho} - T_{co})\right]$$

$$\log_e \frac{\theta_2}{\theta_1} = \frac{-UA}{Q}\left[\theta_1 - \theta_2\right]$$

$$-\log_e \frac{\theta_2}{\theta_1} = \frac{UA}{Q}\left[\theta_1 - \theta_2\right]$$

$$\log_e \left(\frac{\theta_2}{\theta_1}\right)^{-1} = \frac{UA}{Q}\left[\theta_1 - \theta_2\right]$$

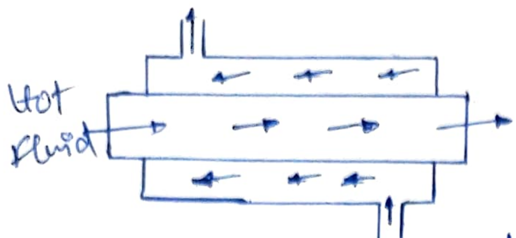
$$\log_e \frac{\theta_1}{\theta_2} = \frac{UA}{Q}(\theta_1 - \theta_2)$$

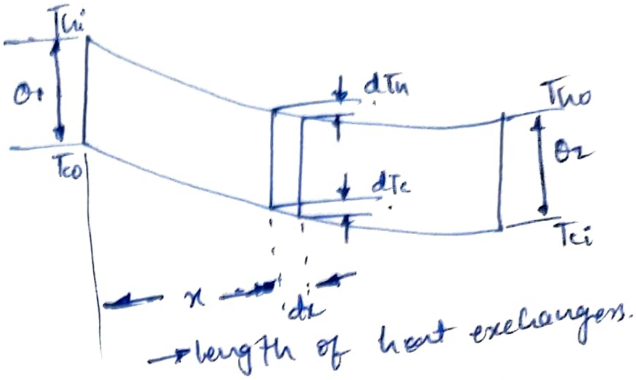
$$Q = UA \frac{\theta_1 - \theta_2}{\log_e \frac{\theta_1}{\theta_2}}$$

$$\boxed{Q = UA\,\theta_m}$$

where $\theta_m = \dfrac{\theta_1 - \theta_2}{\log_e \frac{\theta_1}{\theta_2}}$ is called logarithmic mean temp. difference.

(5) Analysis of Counter flow Heat Exchanger using LMTD,





let T_{hi} & T_{ho} are temp/. of hot fluid at inlet & outlet

T_{ci} & T_{co} are temp/. of cold fluid at inlet & outlet

m_h & m_c are mass flow rates of hot & cold fluids.

C_{ph} & C_{pc} are specific heats of hot & cold fluids.

let heat transfer b/w two fluids as,

$$Q = m_h C_{ph} (T_{hi} - T_{ho}) = m_c C_{pc} (T_{co} - T_{ci})$$

$$Q = C_h (T_{hi} - T_{ho}) = C_c (T_{co} - T_{ci})$$

also $\theta_1 = T_{hi} - T_{co}$

$\theta_2 = T_{ho} - T_{ci}$

for a small elemental area dA, heat transfer is

$$dQ = -m_h C_{ph} dT_h = -C_h dT_h \quad \&$$

$$dQ = -m_c C_{pc} dT_c = -C_c dT_c$$

(or) $dT_h = \dfrac{-dQ}{C_h}$ & $dT_c = \dfrac{-dQ}{C_c}$

from the diagram at a distance of x

$\theta = T_h - T_c$

(or) $d\theta = dT_h - dT_c$

$$= \frac{-dQ}{C_h} - \left(-\frac{dQ}{C_c}\right) = -dQ\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

$$d\theta = -U dA \theta \left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

$$\frac{d\theta}{\theta} = -U\left(\frac{1}{C_h} - \frac{1}{C_c}\right) dA$$

Integrating b/w inlet and outlet conditions

$$\int_1^2 \frac{d\theta}{\theta} = -U\left[\frac{1}{C_h} - \frac{1}{C_c}\right]\int_0^A dA$$

$$\log_e \frac{\theta_2}{\theta_1} = -UA\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$= -UA\left[\frac{1}{\frac{Q}{T_{hi}-T_{ho}}} - \frac{1}{\frac{Q}{T_{co}-T_{ci}}}\right]$$

$$= \frac{-UA}{Q}\left[(T_{hi}-T_{ho}) - (T_{co}-T_{ci})\right] = \frac{-UA}{Q}\left[(T_{hi}-T_{co}) - (T_{ho}-T_{ci})\right]$$

$$\log_e \frac{\theta_2}{\theta_1} = \frac{-UA}{Q}\left[\theta_1 - \theta_2\right] \qquad \left[\because \theta_1 = T_{hi}-T_{co},\ \theta_2 = T_{ho}-T_{ci}\right]$$

$$\log_e \frac{\theta_1}{\theta_2} = \frac{UA}{Q}(\theta_1-\theta_2)$$

$$\boxed{Q = UA\frac{\theta_1-\theta_2}{\log_e \frac{\theta_1}{\theta_2}} = UA\theta_m}$$

6> a> Capacity Rates

Capacity rate is the product of mass flow rate and specific heat of fluid at constant pressure.

for cold fluid, capacity rate. $C_c = m_c C_{pc}$

hot fluid, capacity rate $C_h = m_h C_{ph}$

b> Capacity Ratio

It is defined as the ratio of minimum to maximum capacity rates.

Capacity rate of hot fluid $= C_h = m_h C_{ph}$

— " — of cold fluid $= C_c = m_c C_{pc}$

$\therefore$ capacity ratio. $C = \frac{C_c}{C_h}$. if $C_h > C_c$

(or) capacity ratio. $C = \frac{C_h}{C_c}$. if $C_h > C_c$.

(E)

(7) Analysis of Heat Exchangers by NTU - Effectiveness method.

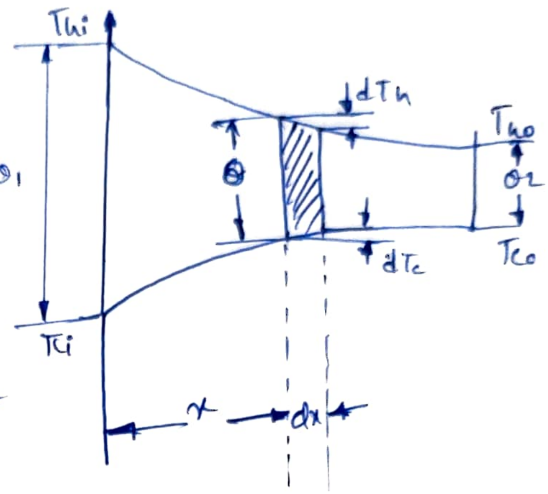
i) Parallel flow Heat exchanger.

We know that,

$$d\theta = -dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$= U\,dA\,\theta\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\frac{d\theta}{\theta} = -U\left[\frac{1}{C_h} + \frac{1}{C_c}\right]dA$$



Integrating b/w inlet & outlet conditions, we get

$$\log_e \frac{\theta_2}{\theta_1} = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\frac{\theta_2}{\theta_1} = e^{-UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]}$$

$$\frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}} = e^{-UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]} \longrightarrow (1)$$

but Effectiveness $\epsilon = \dfrac{C_h(T_{hi} - T_{ho})}{C_{min}(T_{hi} - T_{ci})}$ (or) $T_{ho} = T_{hi} - \epsilon\dfrac{C_{min}}{C_h}(T_{hi} - T_{ci}) \rightarrow (2)$

Also $\epsilon = \dfrac{C_c(T_{co} - T_{ci})}{C_{min}(T_{hi} - T_{ci})}$ (or) $T_{co} = T_{ci} + \epsilon\dfrac{C_{min}}{C_c}(T_{hi} - T_{ci}) \rightarrow (3)$

equation (2) - (3) gives

$$T_{ho} - T_{co} = T_{hi} - \epsilon\frac{C_{min}}{C_h}(T_{hi} - T_{ci}) - \left[T_{ci} - \epsilon\frac{C_{min}}{C_c}(T_{hi} - T_{ci})\right]$$

$$= (T_{hi} - T_{ci}) - \epsilon C_{min}\left[\frac{1}{C_h} + \frac{1}{C_c}\right](T_{hi} - T_{ci})$$

$$= (T_{hi} - T_{ci})\left[1 - \epsilon C_{min}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\right]$$

$$\frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}} = 1 - \epsilon C_{min}\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \longrightarrow (4)$$

equating ① & ④

$$1 - \epsilon C_{min}\left(\frac{1}{C_h} + \frac{1}{C_c}\right) = e^{-UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)}$$

$$\epsilon C_{min}\left[\frac{1}{C_h} + \frac{1}{C_c}\right] = 1 - e^{-UA\left(1/C_h + 1/C_c\right)}$$

$$\epsilon = \frac{1 - e^{-UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)}}{C_{min}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)}$$

$$\epsilon = \frac{1 - e^{-UA\left(\frac{C_c}{C_h} + 1\right)}}{\frac{C_{min}}{C_c}\left[\frac{C_c}{C_h} + 1\right]}$$

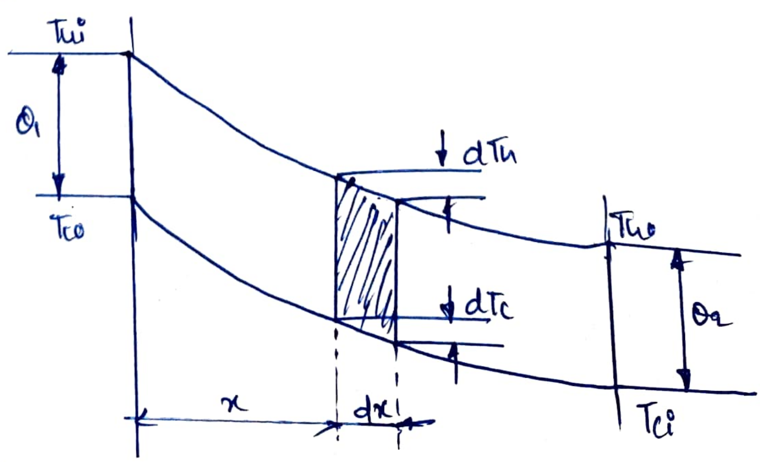
Assume $C_c = C_{min}$ and $C_h = C_{max}$, $C = \frac{C_{min}}{C_{max}}$,

$$NTU = \frac{UA}{C_{min}}$$

$$\therefore \epsilon = \frac{1 - e^{-\frac{UA}{C_{min}}\left(\frac{C_{min}}{C_{max}} + 1\right)}}{\frac{C_{min}}{C_{min}}\left[\frac{C_{min}}{C_{max}} + 1\right]}$$

$$\boxed{\therefore \epsilon = \frac{1 - e^{-NTU(C+1)}}{C+1}}$$

ii) Counter Flow Heat Exchanger



WKT,

$$d\theta = -dQ\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

$$= dQ\left(\frac{1}{C_c} - \frac{1}{C_h}\right)$$

$$d\theta = U\,dA\,\theta\left(\frac{1}{C_c} - \frac{1}{C_h}\right)$$

$$\frac{d\theta}{\theta} = U\left(\frac{1}{C_c} - \frac{1}{C_h}\right)dA$$

Integrating b/w inlet and outlet conditions,

$$\log_e \frac{\theta_2}{\theta_1} = U\left[\frac{1}{C_c} - \frac{1}{C_h}\right]A$$

$$\frac{\theta_2}{\theta_1} = e^{UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)}$$

$$\frac{T_{ho} - T_{ci}}{T_{hi} - T_{co}} = e^{UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)}$$

$$\frac{T_{hi} - T_{co}}{T_{ho} - T_{ci}} = e^{-UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)} \longrightarrow (1)$$

But Effectiveness $\epsilon = \dfrac{C_h(T_{hi} - T_{ho})}{C_{min}(T_{hi} - T_{ci})}$ (or)

$$T_{ho} = T_{hi} - \epsilon \frac{C_{min}}{C_h}(T_{hi} - T_{ci})$$

Also $\epsilon = \dfrac{C_c(T_{co} - T_{ci})}{C_{min}(T_{hi} - T_{ci})}$ (or) $T_{co} = T_{ci} + \epsilon \dfrac{C_{min}}{C_c}(T_{hi} - T_{ci})$

Substituting T_{ho} & T_{co} relation in eqn (1)

$$\frac{T_{hi} - T_{ci} - \epsilon \frac{C_{min}}{C_c}(T_{hi} - T_{ci})}{T_{hi} - \epsilon \frac{C_{min}}{C_h}(T_{hi} - T_{ci}) - T_{ci}} = e^{-UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)}$$

$$\frac{T_{hi} - T_{ci} - \epsilon \frac{C_{min}}{C_c}(T_{hi} - T_{ci})}{(T_{hi} - T_{ci}) - \epsilon \frac{C_{min}}{C_h}(T_{hi} - T_{ci})} = e^{-UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)}$$

$$\frac{1 - \epsilon \frac{C_{min}}{C_c}}{1 - \epsilon \frac{C_{min}}{C_h}} = e^{-UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)}$$

$$1 - \epsilon \frac{C_{min}}{C_c} = e^{-UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)} - \epsilon \cdot \frac{C_{min}}{C_h} e^{-UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)}$$

$$\epsilon \times \left[\frac{C_{min}}{C_h} e^{-UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)} - \frac{C_{min}}{C_c}\right] = e^{-UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)} - 1$$

$$\epsilon = \frac{e^{-UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)} - 1}{\frac{C_{min}}{C_h} e^{-UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)} - \frac{C_{min}}{C_c}}$$

$$= \frac{e^{\frac{-UA}{C_c}\left[1 - \frac{C_c}{C_h}\right)} - 1}{\frac{C_{min}}{C_h} e^{\frac{-UA}{C_c}\left(1 - \frac{C_c}{C_h}\right)} - \frac{C_{min}}{C_c}}$$

But $NTU = \frac{UA}{C_{min}}$ and $c = \frac{C_{min}}{C_{max}}$. & Assume $C_h = C_{max}$, $C_c = C_{min}$

$$\therefore \epsilon = \frac{e^{-NTU(1-c)} - 1}{C e^{-NTU(1-c)} - \frac{C_{min}}{C_{min}}}$$

$$\epsilon = \frac{e^{-NTU(1-c)} - 1}{C e^{-NTU(1-c)} - 1}$$

(or) $$\boxed{\epsilon = \frac{1 - e^{-NTU(1-c)}}{1 - C e^{-NTU(1-c)}}}$$