Internal Assessment Test III – July 2023

Sub:	Finite Element Methods					**Code:**	18ME61
Date:	04/ 07 / 2023	Duration: 90 mins	Max Marks: 50	**Sem:**	VI	**Branch:**	MECH

Note: Answer all questions.

		Marks	OBE CO	OBE RBT
1	Briefly explain the different types of boundary conditions in heat transfer analysis	**06**	CO4	L2
2	A composite wall is as shown in figure 1 consists of three materials. The outer temperature T_o is 20^oC. Convective heat transfer takes place on the inner surface of the wall with $T_\infty = 800^oC$. The convective heat transfer coefficient is 25 $W/m^2\ ^oC$. Determine the temperature distribution in the wall.	**14**	CO4	L2

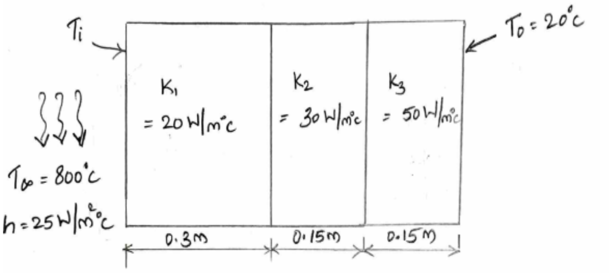

Figure 1

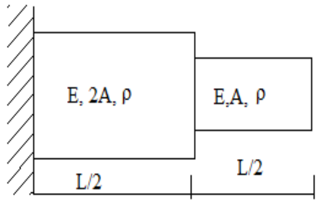

Figure 2

3	Derive consistent mass matrix for truss element.	**10**	CO5	L3
4	Evaluate Eigen values & Eigen vector for stepped bar shown in figure 2 when it is subjected to axial vibration, with fixed free end condition as shown. Draw mode shapes	**20**	CO5	L3

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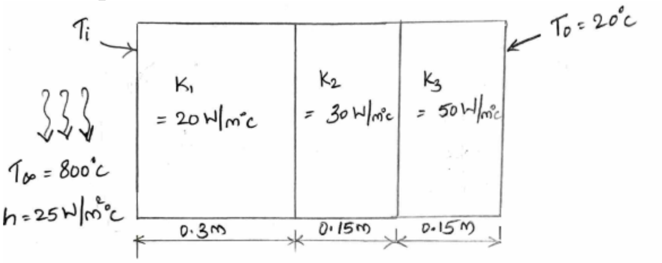


Figure 1

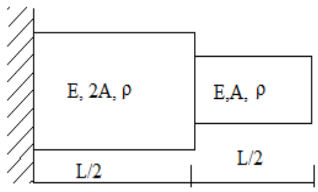


Figure 2

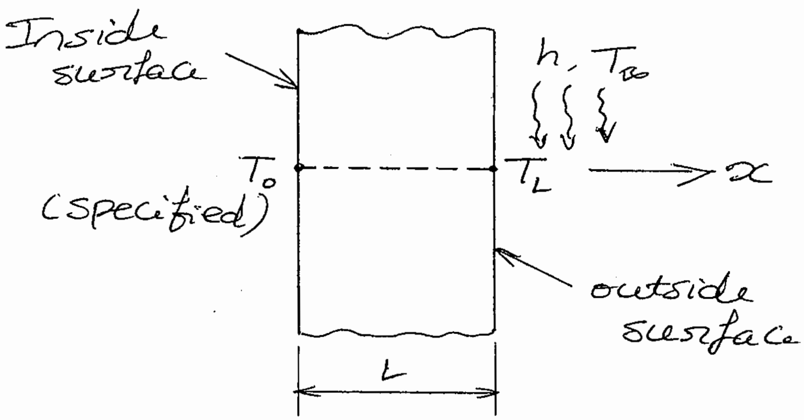
3	Derive consistent mass matrix for truss element.	**10**	CO5	L3
4	Evaluate Eigen values & Eigen vector for stepped bar shown in figure 2 when it is subjected to axial vibration, with fixed free end condition as shown. Draw mode shapes	**20**	CO5	L3

Types of boundary conditions:-

The boundary conditions are mainly of three kinds.

1) Specified temperature boundary condition
2) Specified heat flux (Insulated) boundary cond?
3) Convection boundary condition.

For example consider the wall of a tank as shown in the figure below. The tank contains a hot liquid at a temperature 'T_o' with an airstream of temperature T_∞ passed on the outside maintaining a wall temperature of 'T_L' at the boundary.

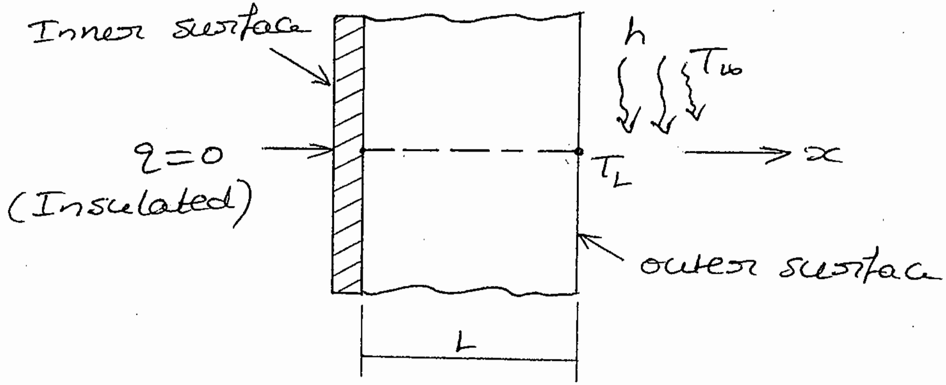


The boundary condition for this problem is

At $x=0$, $T=T_o$

At $x=L$, $q=h(T_L-T_\infty)$

As another example, consider a wall as shown in the figure below, where the inside surface is insulated & the outside is a convective surface.

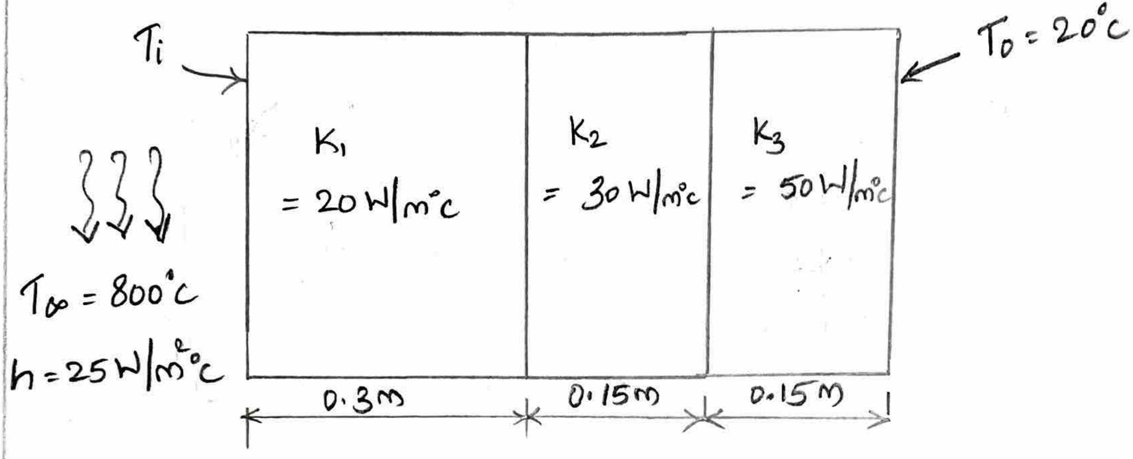


The boundary condition for this problem is

At $x=0$, heat flux $q=0$

At $x=L$ heat flux $q=h(T_L-T_\infty)$

2 A Composite wall is as shown in figure Consists of three materials. The outer temperature T_0 is $20°c$. Convective heat transfer takes place on inner Surface of the wall with $T_\infty = 800°c$. The Convective heat transfer Coefficient is $25\ W/m^2°c$. Determine the temperature distribution in the wall.

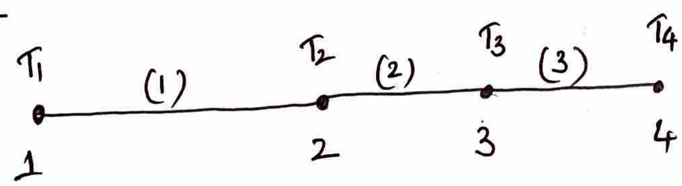


Sol

$K_1 = 20\ W/m°c$; $K_2 = 30\ W/m°c$; $K_3 = 50\ W/m°c$

$h = 25\ W/m^2°c$; $T_\infty = 800°c$

F.E. Model



Element Stiffness matrix

$$[K] = \frac{AK}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

For element 1

$$[K_1] = \frac{1 \times 20}{0.3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 66.67 & -66.67 \\ -66.67 & 66.67 \end{bmatrix} \end{array}\begin{array}{c} 1 \\ 2 \end{array}$$

For element 2

$$K_2 = \frac{1(30)}{0.15}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{array}{ccc} 2 & 3 & \\ \left[\begin{matrix} 200 & -200 \\ -200 & 200 \end{matrix}\right] & & \begin{matrix} 2 \\ 3 \end{matrix} \end{array}$$

For element 3

$$K_3 = \frac{1(50)}{0.15}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{array}{ccc} 3 & 4 & \\ \left[\begin{matrix} 333.33 & -333.33 \\ -333.33 & 333.33 \end{matrix}\right] & & \begin{matrix} 3 \\ 4 \end{matrix} \end{array}$$

Global stiffness

$$[K] = \begin{array}{cc} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} & \\ \begin{bmatrix} 66.67 & -66.67 & 0 & 0 \\ -66.67 & 266.67 & -200 & 0 \\ 0 & -200 & 533.33 & -333.33 \\ 0 & 0 & -333.33 & 333.33 \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \end{array}$$

At node 1 ; Convection is taking place

$$[K_h] = hA\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = 25(1)\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{array}{cc} \begin{matrix} 1 & 2 \end{matrix} & \\ \begin{bmatrix} 25 & 0 \\ 0 & 0 \end{bmatrix} & \begin{matrix} 1 \\ 2 \end{matrix} \end{array}$$

Global stiffness

$$[K] = [K] + [K_h]$$

$$[K] = \begin{bmatrix} 91.67 & -66.67 & 0 & 0 \\ -66.67 & 266.67 & -200 & 0 \\ 0 & -200 & 533.33 & -333.33 \\ 0 & 0 & -333.33 & 333.33 \end{bmatrix}$$

Force Vector

$$[F_h] = A h T_\infty \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= 1(25)(800)\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 20{,}000 \\ 0 \end{bmatrix}$$

Equilibrium eqn $[K][T] = [F]$

$$\begin{bmatrix} 91.67 & -66.67 & 0 & 0 \\ -66.67 & 266.67 & -200 & 0 \\ 0 & -200 & 533.33 & -333.33 \\ 0 & 0 & -333.33 & 333.33 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 20000 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Since Temp at node 4 ; $T_4 = 20°C$

$$-200T_2 + 533.33T_3 - 333.33T_4 = 0$$

$$-200(T_2) + 533.33T_3 - 333.33(20) = 0$$

$$-200T_2 + 533.33T_3 = 6666.67$$

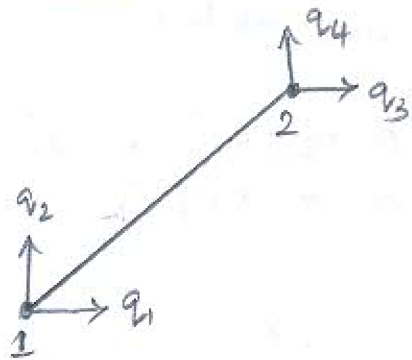
$$\therefore \begin{bmatrix} 91.67 & -66.67 & 0 \\ -66.67 & 266.67 & -200 \\ 0 & -200 & 533.33 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 20000 \\ 0 \\ 6666.67 \end{bmatrix}$$

$T_1 = 304.76°c$; $T_2 = 119.05°c$; $T_3 = 57.14°c$

Nodal temp.

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 304.76 \\ 119.05 \\ 57.14 \end{bmatrix} °c$$

3 **For truss element**



Kinetic Energy for a System

$$= \frac{1}{2} \times \text{mass} \times (\text{velocity})^2$$

$$= \frac{1}{2} \times \text{density} \times \text{volume} \times (\text{velocity})^2 \qquad 4$$

$$= \frac{1}{2} \int_V \rho . dv . \dot{u}^T . \dot{u}$$

$$= \frac{1}{2} \int_V \rho . dv . [N\dot{q}]^T N \dot{q}$$

$$= \frac{1}{2} \int_V \rho . \dot{q}^T N^T N \dot{q} \, dv$$

$$= \frac{1}{2} \int_{le} \rho \dot{q}^T N^T N \dot{q} A \, dx$$

$$= \frac{1}{2} \int_{-1}^{+1} \rho \dot{q}^T N^T N \dot{q} A \frac{le}{2} d\xi$$

$$= \frac{1}{2} \dot{q}^T \left[\frac{\rho A le}{2} \int_{-1}^{+1} N^T N d\xi \right] \dot{q}$$

$$= \frac{1}{2} \dot{q}^T m^e \dot{q}$$

where $m^e \rightarrow$ mass matrix

$$N = \begin{bmatrix} N_1 & 0 & N_2 & 0 \\ 0 & N_1 & 0 & N_2 \end{bmatrix} ; \quad N^T = \begin{bmatrix} N_1 & 0 \\ 0 & N_1 \\ N_2 & 0 \\ 0 & N_2 \end{bmatrix}$$

$$m^e = \frac{\rho A le}{2} \int_{-1}^{+1} N^T N . d\xi$$

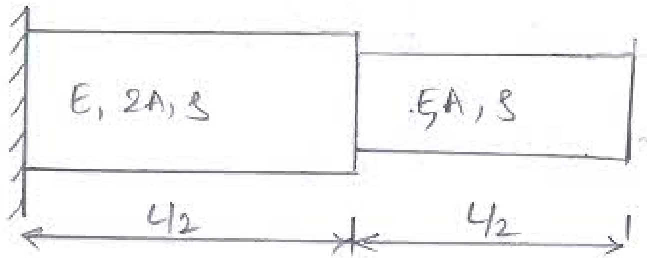
$$M^e = \frac{\rho A l_e}{2} \int_{-1}^{+1} \begin{bmatrix} N_1 & 0 \\ 0 & N_1 \\ N_2 & 0 \\ 0 & N_2 \end{bmatrix} \begin{bmatrix} N_1 & 0 & N_2 & 0 \\ 0 & N_1 & 0 & N_2 \end{bmatrix} d\xi$$

$$= \frac{\rho A l_e}{2} \int_{-1}^{+1} \begin{bmatrix} N_1^2 & 0 & N_1N_2 & 0 \\ 0 & N_1^2 & 0 & N_1N_2 \\ N_1N_2 & 0 & N_2^2 & 0 \\ 0 & N_1N_2 & 0 & N_2^2 \end{bmatrix}$$

$$= \frac{\rho A l_e}{2} \begin{bmatrix} 2/3 & 0 & 1/3 & 0 \\ 0 & 2/3 & 0 & 1/3 \\ 1/3 & 0 & 2/3 & 0 \\ 0 & 1/3 & 0 & 2/3 \end{bmatrix}$$

$$= \frac{\rho A l_e}{6} \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

4 Find eigen values & eigen vectors for stepped bar when it is subjected to axial vibration, with fixed free end condition as shown. Draw mode shape.



Sol <u>F.E. Model</u>

<u>Elemental stiffness matrix</u>

$$K = \frac{EA}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K_1 = \frac{4EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{EA}{L}\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \end{array}\begin{array}{c} 1 \\ 2 \end{array}$$

$$K_2 = \frac{2EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{EA}{L}\begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \end{array}\begin{array}{c} 2 \\ 3 \end{array}$$

<u>Global Stiffness</u>

$$K = \frac{EA}{L}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 4 & -4 & 0 \\ -4 & 6 & -2 \\ 0 & -2 & 2 \end{bmatrix} \end{array}\begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

Elemental Mass matrix

$$m^e = \frac{\rho A l_e}{6}\begin{bmatrix} 2 & +1 \\ +1 & 2 \end{bmatrix}$$

$$m^{e_1} = \frac{\rho \cdot 2A \cdot L}{2 \cdot 6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \frac{\rho AL}{12}\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \end{array}\begin{array}{c} 1 \\ 2 \end{array}$$

$$m^{e_2} = \frac{\rho . AL}{12}\begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \end{array}\begin{array}{c} 2 \\ 3 \end{array}$$

Global Mass matrix

$$m^e = \frac{\rho AL}{12}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 4 & 2 & 0 \\ 2 & 6 & 1 \\ 0 & 1 & 2 \end{bmatrix} \end{array}\begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

Eqn of motion

$$|K - \lambda M| = 0$$

$$\frac{EA}{L}\begin{bmatrix} 4 & -4 & 0 \\ -4 & 6 & -2 \\ 0 & -2 & 2 \end{bmatrix} - \lambda\begin{bmatrix} 4 & 2 & 0 \\ 2 & 6 & 1 \\ 0 & 1 & 2 \end{bmatrix}\frac{\rho AL}{12} = 0$$

$$\left|\begin{bmatrix} 4 & -4 & 0 \\ -4 & 6 & -2 \\ 0 & -2 & 2 \end{bmatrix} - \frac{\rho L^2}{12E}\lambda\begin{bmatrix} 4 & 2 & 0 \\ 2 & 6 & 1 \\ 0 & 1 & 2 \end{bmatrix}\right| = 0$$

$$\left| \begin{bmatrix} 6 & -2 \\ -2 & 2 \end{bmatrix} - b \begin{bmatrix} 6 & 1 \\ 1 & 2 \end{bmatrix} \right| = 0 \qquad \because b = \frac{\rho L^2}{12E} \cdot \lambda$$

$$\begin{vmatrix} 6-6b & -(2+b) \\ -(2+b) & 2-2b \end{vmatrix} = 0$$

$$(6-6b)(2-2b) - (2+b)^2 = 0$$

$$12 - 12b - 12b + 12b^2 - 4 - b^2 - 4b = 0$$

$$8 - 28b + 11b^2 = 0$$

$b_1 = 0.3279$ $\qquad$ $b_2 = 2.217$

$$\frac{\rho L^2}{12E} \cdot \lambda = 0.3279$$

$$\lambda_1 = \frac{3.9348\,E}{\rho L^2} = \omega_1^2$$

$$\omega_1 = \frac{1.98}{L}\sqrt{\frac{E}{\rho}} \quad r/s$$

$$\frac{\rho L^2}{12E} \cdot \lambda = 2.217$$

$$\lambda_2 = \frac{26.604\,E}{\rho L^2} = \omega_2$$

$$\omega_2 = \frac{5.16}{L}\sqrt{\frac{E}{\rho}} \quad r/s$$

For first mode shape

$$|K - \lambda M|[x] = 0$$

$$\left(\frac{EA}{L}\begin{bmatrix}6 & -2\\ -2 & 2\end{bmatrix} - \left(3.9348\,\frac{E}{8L^2}\right)\cdot\frac{8AL}{12}\begin{bmatrix}6 & 1\\ 1 & 2\end{bmatrix}\right)\begin{bmatrix}x_2^{(1)}\\ x_3^{(1)}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\left(\begin{bmatrix}6 & -2\\ -2 & 2\end{bmatrix} - \begin{bmatrix}1.98 & 0.33\\ 0.33 & 0.66\end{bmatrix}\right)\begin{bmatrix}x_2^{(1)}\\ x_3^{(1)}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

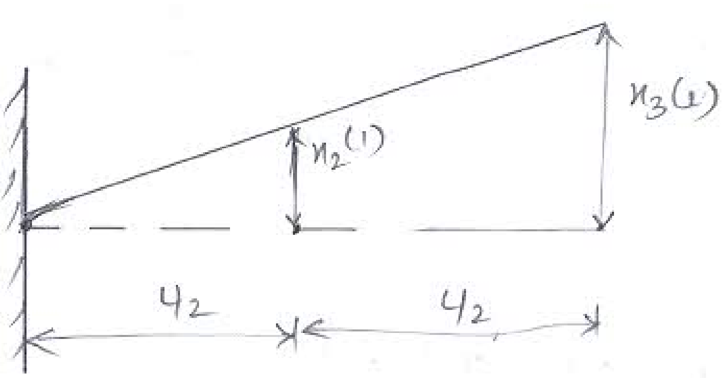
$$\begin{bmatrix}4.02 & -2.33\\ -2.33 & 1.34\end{bmatrix}\begin{bmatrix}x_2^{(1)}\\ x_3^{(1)}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

$$4.02\,x_2^{(1)} - 2.33\,x_3^{(1)} = 0$$

$$x_3^{(1)} = 1.72\,x_2^{(1)}$$

Eigen vector or 1st mode shape

$$\left[x^{(1)}\right] = \begin{bmatrix}x_2^{(1)}\\ x_3^{(1)}\end{bmatrix} = \begin{bmatrix}x_2^{(1)}\\ 1.72\,x_2^{(1)}\end{bmatrix} = x_2^{(1)}\begin{bmatrix}1\\ 1.72\end{bmatrix}$$



For Second mode shape

put $\omega_2^2 = \lambda_2$ in $\left| K - \lambda M \right| [x] = 0$.

$$\left(\frac{EA}{\ell} \begin{bmatrix} 6 & -2 \\ -2 & 2 \end{bmatrix} - \left[\frac{26.604\, E}{8\ell^2} \right] \begin{bmatrix} 6 & 1 \\ 1 & 2 \end{bmatrix} \frac{8A\ell}{12} \right) \begin{bmatrix} x_2^{(2)} \\ x_3^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} 6 & -2 \\ -2 & 2 \end{bmatrix} - 2.217 \begin{bmatrix} 6 & 1 \\ 1 & 2 \end{bmatrix} \right) \begin{bmatrix} x_2^{(2)} \\ x_3^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} 6 & -2 \\ -2 & 2 \end{bmatrix} - \begin{bmatrix} 13.302 & 2.217 \\ 2.217 & 4.434 \end{bmatrix} \right) \begin{bmatrix} x_2^{(2)} \\ x_3^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 - 13.302 & -2 - 2.217 \\ -2 - 2.217 & 2 - 4.434 \end{bmatrix} \begin{bmatrix} x_2^{(2)} \\ x_3^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -7.302 & -4.217 \\ -4.217 & -2.434 \end{bmatrix} \begin{bmatrix} x_2^{(2)} \\ x_3^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-7.302\, x_2^{(2)} - 4.217\, x_3^{(2)} = 0$$

$$x_3^{(2)} = -1.72\, x_2^{(2)}$$

$$\left[x^{(2)}\right] = \begin{bmatrix} x_2^{(2)} \\ x_3^{(2)} \end{bmatrix} = \begin{bmatrix} x_2^{(2)} \\ -1.72\,x_3^{(2)} \end{bmatrix}$$

$$= x_2^{(2)} \begin{bmatrix} 1 \\ -1.72 \end{bmatrix}$$ //