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Third Semester B.E. Degree Examination, Dec.2023/Jan.2024 Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Define dot product between two vectors \vec{a} and \vec{b} . Find the sine of the angle between $\vec{a} = 4\vec{i} - \vec{j} + 3\vec{k}$ and $\vec{b} = -2\vec{i} + \vec{j} - 2\vec{k}$. (08 Marks)
- b. Express $\frac{3+4i}{3-4i}$ in the form of $x+iy$ and hence find its modulus and amplitude. (06 Marks)
- c. Find the real part of $\frac{1}{1+\cos\theta+i\sin\theta}$. (06 Marks)

OR

- 2 a. Prove that $\left[\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta} \right]^4 = \cos 8\theta + i\sin 8\theta$ (08 Marks)
- b. If $\vec{A} = 2\vec{i} - 2\vec{j} + \vec{k}$ and $\vec{B} = 2\vec{i} - 2\vec{j} + \vec{k}$. Show that $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$ are orthogonal. (06 Marks)
- c. Find the value of λ so that the vectors $\vec{A} = 3\vec{i} + 5\vec{j} - 3\vec{k}$, $\vec{B} = \vec{i} + \lambda\vec{j} + 2\vec{k}$ and $\vec{C} = 2\vec{i} - 2\vec{j} + \vec{k}$ are co-planar. (06 Marks)

Module-2

- 3 a. If $y = \tan^{-1} x$, prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. (08 Marks)
- b. Find the angle between the curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$. (06 Marks)
- c. If $u = \tan^{-1} \left[\frac{x^3 + y^3}{x - y} \right]$. Show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$. (06 Marks)

OR

- 4 a. Obtain the Maclaurin's series expansion of $\tan x$ upto the term containing x^5 . (08 Marks)
- b. Find the Pedal equation to the curve $r^m = a^m \cos m\theta$. (06 Marks)
- c. If $u = f(x - y, y - z, z - x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (06 Marks)

Module-3

- 5 a. Obtain a reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x dx$ ($n > 0$). (08 Marks)
- b. Evaluate $\int_0^a x \sqrt{ax - x^2} dx$. (06 Marks)
- c. Evaluate $\iint_R xy dx dy$ where R is the I quadrant of the circle $x^2 + y^2 = a^2$, $x \geq 0$, $y \geq 0$. (06 Marks)

OR

- 6 a. Obtain a reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x dx$ ($n > 0$). (08 Marks)
- b. Using reduction formula evaluate $\int_0^1 x^5 (1-x^2)^{\frac{5}{2}} dx$. (06 Marks)
- c. Evaluate $\int_0^1 \int_0^2 \int_0^2 x^2 yz dx dy dz$. (06 Marks)

Module-4

- 7 a. A particle moves along the curve $x = (1-t^3)$, $y = (1+t^2)$ and $z = (2t-5)$. Determine the components of velocity and acceleration at $t = 1$ in the direction $2i + j + 2k$. (08 Marks)
- b. Find the directional derivative of $\phi = x^2 yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2i - j - 2k$. (06 Marks)
- c. If $\vec{F} = (x + y + 1)i + j - (x + y)k$, show that $\vec{F} \cdot \text{curl} \vec{F} = 0$ (06 Marks)

OR

- 8 a. Find $\text{div} \vec{F}$ and $\text{curl} \vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ (08 Marks)
- b. Show that $\vec{F} = \frac{xi + yj}{x^2 + y^2}$ is solenoidal. (06 Marks)
- c. Find the values of the constants a, b, c such that $\vec{F} = (x + y + az)i + (6x + 2y - z)j + (x + cy + 2z)k$ is irrotational. (06 Marks)

Module-5

- 9 a. Solve: $(1 + y^2)dx = (\tan^{-1} y - x) dy$. (08 Marks)
- b. Solve: $(y^3 - 3x^2 y)dx - (x^3 - 3xy^2)dy = 0$. (06 Marks)
- c. Solve: $\left[x \tan\left(\frac{y}{x}\right) - y \sec^2\left(\frac{y}{x}\right) \right] dx + x \sec^2\left(\frac{y}{x}\right) dy = 0$. (06 Marks)

OR

- 10 a. Solve: $\frac{dy}{dx} + \frac{y}{x} = y^2 x$. (08 Marks)
- b. Solve: $y(x + y)dx + (x + 2y - 1)dy = 0$. (06 Marks)
- c. Solve: $[y^2 e^{xy^2} + 4x^3]dx + [2xye^{xy^2} - 3y^2]dy = 0$ (06 Marks)
