CBCS SCHEME

USN

15MAT11

First Semester B.E. Degree Examination, Dec.2023/Jan.2024
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. **(06 Marks)**

b. Prove that the following pairs of curves intersect orthogonally $r = a\sec^2\left(\frac{\theta}{2}\right)$; $r = a\,\text{cosec}^2\left(\frac{\theta}{2}\right)$. **(05 Marks)**

c. Show that for the curve $r = a(1+\cos\theta)$, $\frac{\rho^2}{r}$ is const. **(05 Marks)**

OR

2 a. Find the n^{th} derivative of $\frac{x^2}{(x+2)(2x+3)}$. **(06 Marks)**

b. Find the pedal equation for the curve $r = a\sin^3\left(\frac{\theta}{3}\right)$. **(05 Marks)**

c. For the ellipse $x = a\cos t$; $y = b\sin t$, find $\frac{ds}{dt}$. **(05 Marks)**

Module-2

3 a. Expand $\log(1+e^x)$ upto the term containing x^4 using MaClaurin's series. **(06 Marks)**

b. Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$. **(05 Marks)**

c. If $u = (x^2+y^2+z^2)^{-\frac{1}{2}}$, show that $\Sigma u_{xx} = 0$ **(05 Marks)**

OR

4 a. If $u = \sin^{-1}\left(\frac{x^2y^2}{x+y}\right)$ then show that $xu_x + yu_y = 3\tan u$. **(06 Marks)**

b. Expand sin x inpowers of $\left(x - \frac{\pi}{2}\right)$ upto 4^{th} degree terms on the Taylor's series. **(05 Marks)**

c. If $u = z - x$, $v = y - z$, $w = x + y + z$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. **(05 Marks)**

Module-3

5 a. Find the angle between the tangents to the curve $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3k$ at $t = \pm 1$. **(06 Marks)**

b. Find the directional derivative of $\phi = 2xy + z^2$ at the point (1, –1, 3) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. **(05 Marks)**

c. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, −1, 2). **(05 Marks)**

OR

6 a. Find the divergence and curl of the vector $\vec{F} = (xyz + y^2z)\hat{i} + (3x^2 + y^2z)\hat{j} + (xz^2 - zy^2)\hat{k}$ at (1, −1, 1). **(06 Marks)**

b. Show that $\vec{F} = 3x^2y\hat{i} + (x^3 - 2yz^2)\hat{j} + (3z^2 - 2y^2z)\hat{k}$ is irrotational and hence find the scalar potential ϕ. **(05 Marks)**

c. Prove that $\text{div}(\phi\vec{F}) = \phi(\text{div}\vec{F}) + \nabla\phi.\vec{F}$. **(05 Marks)**

Module-4

7 a. Obtain the reduction formulae for $\int_0^{\pi/2} \cos^n x dx$. **(06 Marks)**

b. Evaluate $\int_0^4 x^3\sqrt{4x - x^2}dx$. **(05 Marks)**

c. Solve $y[xy\sin(xy) + \cos(xy)]dx + [xy\sin(xy) - \cos(xy)]xdyx$. **(05 Marks)**

OR

8 a. Evaluate $\int_0^{2a} x^{\frac{7}{2}}(2a - x)^{-\frac{1}{2}}dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} = \frac{1}{x^2y^3 + xy}$. **(05 Marks)**

c. Find the orthogonal trajectories of the family of curves $\frac{2a}{r} = 1 - \cos\theta$. **(05 Marks)**

Module-5

9 a. Apply Gauss-Jordan method to solve $2x + 5y + 7z = 52$; $2x + y - z = 0$; $x + y + z = 9$. **(06 Marks)**

b. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$ by elementary transformation. **(05 Marks)**

c. Find the dominant eigen value and the corresponding eigen vector of the matrix by power method $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by as $[1, 0.8, -0.8]^T$. **(05 Marks)**

OR

10 a. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12yz + 4zx - 8xy$ to canonical form. **(06 Marks)**

b. Diagonalize the matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$ and hence find A^4. **(05 Marks)**

c. Show that the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$; $y_3 = x_1 - 2x_3$ is Regular and hence find the inverse transformation. **(05 Marks)**

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