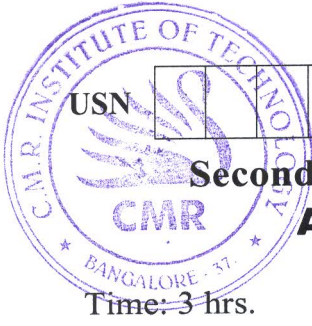


# CBCS SCHEME

18MAT21



## Second Semester B.E. Degree Examination, Dec.2023/Jan.2024 Advanced Calculus and Numerical Methods

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

### Module-1

- 1 a. Find grade 'φ' when φ is given by  $\phi = 3x^2y - y^3z^2$  at the point (1, -2, -1). (06 Marks)
- b. A vector field is given by  $\vec{A} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$ . Show that the field is irrotational. (07 Marks)
- c. Find the angle between the surfaces  $x^2 + y^2 + z^2 = 9$  and  $z = x^2 + y^2 - 3$  at (2, -1, 2). (07 Marks)

OR

- 2 a. Verify Green's theorem in the plane for  $\oint_C (xy + y^2)dx + x^2dy$ , where C is the closed curve bounded by  $y = x$  and  $y = x^2$ . (06 Marks)
- b. Evaluate by Stokes theorem  $\oint_C yzdx + zxdy + xydz$ , where C is the curve  $x^2 + y^2 = 1, z = y^2$ . (07 Marks)
- c. Using the divergence theorem, evaluate  $\iiint_S \vec{F} \cdot \hat{n} ds$ , where  $\vec{F} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$  and S is the surface of the sphere  $x^2 + y^2 + z^2 = a^2$ . (07 Marks)

### Module-2

- 3 a. Solve  $\frac{d^3y}{dx^3} + y = 0$ . (06 Marks)
- b. Solve  $y'' - 4y' + 13y = \cos 2x$ . (07 Marks)
- c. Solve  $\frac{d^2y}{dx^2} + y = \tan x$  by the method of variation or parameters. (07 Marks)

OR

- 4 a. Solve  $x^2y'' - xy' - xy' + 2y = x$  by Cauchy method. (06 Marks)
- b. Solve  $(2x + 1)^2 y'' - 2(2x + 1)y' - 12y = 6x$  by Legendre's method. (07 Marks)
- c. A particle moves along the x - axis according to the law  $\frac{d^2x}{dt^2} + 6 \frac{dx}{dt} + 25x = 0$ . If the particle is started at  $x = 0$  with an initial velocity of 12 ft/sec to the left, determine nets. (07 Marks)

### Module-3

- 5 a. Form partial differential equation by eliminating the arbitrary constants 'a' & 'b'.  $z = ax^2 + by^2$ . (06 Marks)
- b. Form partial differential equation by eliminating the arbitrary function 'f'.  $z = x^n \cdot f\left(\frac{y}{x}\right)$  (07 Marks)
- c. Solve  $\frac{\partial^3 z}{\partial x^2 \partial y} = \sin(2x + 3y)$ . (07 Marks)

OR

- 6 a. Solve  $\frac{\partial^3 z}{\partial x^2} + 4z = 0$ . Given that when  $x = 0$ ,  $z = e^{2y}$  and  $\frac{\partial z}{\partial x} = 2$ . (06 Marks)
- b. Solve  $p \cot x + q \cot y = \cot z$ . (07 Marks)
- c. Find solution of one – dimensional heat equation : (07 Marks)
- $$\frac{\partial u}{\partial t} = c^2 \cdot \frac{\partial^2 y}{\partial x^2}.$$

**Module-4**

- 7 a. Discuss the convergence of  $\sum_{n=1}^{\infty} \frac{(n+1)^n x^n}{n^{n+1}}$ . (06 Marks)
- b. Test for convergence of the series  $\frac{1^2}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \frac{4^2}{2^4} + \dots$ . (07 Marks)
- c. Test the positive series  $+1 + 2 + 3 + \dots + n$ . (07 Marks)

OR

- 8 a. Solve Bessel's differential equation leading to  $J_n(x)$ . (06 Marks)
- b. Express the polynomial  $f(x) = 4x^3 - 2x^2 - 3x + 8$  in terms of Legendre polynomials. (07 Marks)
- c. Using Rodrigues's formula, obtain expressions for  $P_0(x)$ ,  $P_1(x)$ ,  $P_2(x)$ ,  $P_3(x)$ . (07 Marks)

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**Module-5**

- 9 a. Using Newton's forward interpolation formula, find  $y$  at  $x = 8$  from the following table :

x :	0	5	10	15	20	25
y :	7	11	14	18	24	32

(06 Marks)

- b. Using Newton's divided difference formula, evaluate  $f(8)$  and  $f(15)$ , given

x :	4	5	7	10	11	13
f(x) :	48	100	294	900	1210	2028

(07 Marks)

- c. Find a real root of the equation  $f(x) = x^3 - 2x - 5 = 0$  by Regula Falsi method correct to three decimal places. (07 Marks)

OR

- 10 a. Evaluate  $\int_0^6 \frac{dx}{1+x^2}$  by using Weddle's Rule. (06 Marks)
- b. Evaluate  $\int_2^8 \log_{10} x \, dx$  taking 6 subintervals correct to four decimal places by Simpson's  $\left(\frac{3}{8}\right)^{\text{th}}$  rule. (07 Marks)
- c. Use Newton – Raphson method to find a real root of the equation  $x e^x - 2 = 0$  correct to three decimal places. (07 Marks)

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