

CBCS SCHEME

18MAT31



Third Semester B.E. Degree Examination, Dec.2023/Jan.2024 Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the Laplace transform of i) $e^t \sin 3t$ ii) $\frac{\cos at - \cos bt}{t}$. (07 Marks)
- b. The square wave function $f(t)$ with period "a" is defined by

$$f(t) = \begin{cases} E & , 0 \leq t < a/2 \\ -E & , a/2 \leq t < a \end{cases}$$
 Show that $L\{f(t)\} = (E/s) \tan h (as/4)$. (06 Marks)
- c. Using Laplace transform method to solve $y'' - 3y' + 2y = e^{3t}$, $y(0) = 1$ and $y'(0) = 0$. (07 Marks)

OR

- 2 a. Find i) $L^{-1}\left\{\frac{s+2}{s^2-4s+13}\right\}$ ii) $L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\}$ (07 Marks)
- b. Find the Laplace transform of $f(t) = \begin{cases} t-1 & , 1 < t < 2 \\ 3-t & , 2 < t < 3 \end{cases}$ by using unit - step function. (06 Marks)
- c. Find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$, using Convolution theorem. (07 Marks)

Module-2

- 3 a. Find a Fourier series to represent $(x - x^2)$ from $x = -\pi$ to $x = \pi$. (07 Marks)
- b. Find the half - range cosine series for the function $f(x) = (x-1)^2$ in the interval $0 < x < 1$. (06 Marks)
- c. The following table gives the variations of periodic current over a period :

t(sec):	0	T/6	T/3	T/2	2T/3	5T/6	T
A(amp) :	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic. (07 Marks)

OR

- 4 a. Obtain Fourier series for the function

$$f(x) = \begin{cases} \pi x & , 0 \leq x \leq 1 \\ \pi(2-x) & , 1 \leq x \leq 2 \end{cases}$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$. (07 Marks)

- b. Find the half range Fourier sine series of $f(x) = x(\pi - x)$, $0 \leq x \leq \pi$. (06 Marks)

- c. Obtain Fourier series for the function $f(x)$, given

$$\text{by } f(x) = \begin{cases} 1 + \frac{2x}{\pi} & , -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi} & , 0 \leq x \leq \pi \end{cases}$$

(07 Marks)

Module-3

- 5 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2 & , |x| \leq 1 \\ 0 & , |x| > 1 \end{cases}$$

Hence evaluate $\int_0^{\infty} \left(\frac{\sin x - x \cos x}{x^3} \right) \cos(x/2) dx$.

(07 Marks)

- b. Find the Fourier sin transform of e^{-ax} , $a > 0$.

(06 Marks)

- c. Solve $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ with $u_0 = u_1 = 0$ by using Z - transforms.

(07 Marks)

OR

- 6 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1 & , |x| < 1 \\ 0 & , |x| > 1 \end{cases} \quad \text{Hence evaluate } \int_0^{\infty} \frac{\sin x}{x} dx.$$

(07 Marks)

- b. Find the Z - transform of $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$.

(06 Marks)

- c. Find the Inverse Z - transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$.

(07 Marks)

Module-4

- 7 a. Employ Taylor's method to obtain approximate value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$.

(07 Marks)

- b. Solve the differential equation $\frac{dy}{dx} = xy^2$ under the initial condition $y(0) = 1$ by using modified Euler's method at the point $x = 0.05$ ($h = 0.05$).

(07 Marks)

- c. Apply Milne's predictor - corrector formulae to compute $y(1.4)$ correct to four decimal

places. Given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and following the data $y(1) = 2$, $y(1.1) = 2.2156$,
 $y(1.2) = 2.4649$, $y(1.3) = 2.7514$.

(06 Marks)

OR

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- 8 a. Using fourth order Runge Kutta method, compute $y(0.2)$. Given that $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$ (take $h = 0.2$).

(07 Marks)

- b. Using modified Euler's method to find $y(0.2)$ correct to four decimals by solving the equation $\frac{dy}{dx} = x - y^2$, $y(0) = 1$ by taking $h = 0.1$. (Perform 2 iterations in each step).

(07 Marks)

- c. Given $\frac{dy}{dx} = x^2(1+y)$ and $y(1) = 1$, $y(1.1) = 1.2330$, $y(1.2) = 1.5480$, $y(1.3) = 1.9790$.

Evaluate $y(1.4)$ by Adams - Bashforth method.

(06 Marks)

Module-5

- 9 a. Using fourth order Runge – Kutta method solve $y'' = x(y')^2 - y^2$ for $x = 0.2$ correct to four decimal places. Initial conditions are $x = 0$, $y = 1$ and $y' = 0$. (07 Marks)
- b. Solve the variational problem $\delta \int_0^1 \{x + y + (y')^2\} dx = 0$ under the conditions $y(0) = 1$ and $y(1) = 2$. (07 Marks)
- c. With usual notation prove that $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$. (06 Marks)

OR

- 10 a. Apply Milne's method to compute $y(0.4)$ given $y'' + xy' + y = 0$, $y(0) = 1$, $y(0.1) = 0.995$, $y(0.2) = 0.9802$, $y(0.3) = 0.956$ and $y'(0) = 0$, $y'(0.1) = -0.0995$, $y'(0.2) = -0.196$, $y'(0.3) = -0.2863$. (07 Marks)
- b. Solve the variational problem $\delta \int_0^{\pi/2} \{y^2 - (y')^2\} dx = 0$, $y(0) = 0$, $y(\pi/2) = 2$. (07 Marks)
- c. Prove that the geodesics on a plane are straight lines. (06 Marks)
