CBCS SCHEME

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18MAT41

Fourth Semester B.E. Degree Examination, Dec.2023/Jan.2024
Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. State and prove Cauchy – Riemann equations in Cartesian form. **(07 Marks)**

b. Find the analytic function $f(z) = u + iv$, given that $u - v = e^x[\cos y - \sin y]$. **(07 Marks)**

c. If y(z) is an analytic function, then show that :

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2 .$$ **(06 Marks)**

OR

2 a. Determine the analytic function f(z), where imaginary part is $\left(\gamma - \frac{K^2}{\gamma}\right)\sin\theta$, $r \neq 0$. Hence find the real part 07 f(z). **(07 Marks)**

b. Find the analytic function f(z), whose real part is $u = \log\sqrt{x^2 + y^2}$. **(07 Marks)**

c. Show that $f(z) = z^u$ is analytic and hence find its derivative. **(06 Marks)**

Module-2

3 a. Discuss the transformation $w = z^2$. **(07 Marks)**

b. State and prove Cauchy's integral theorem. **(07 Marks)**

c. Evaluate : $\int_0^{(2+i)} (\bar{z})^2 dz$, along the real axis up to 2 and then vertically to 2 + i. **(06 Marks)**

OR

4 a. Evaluate : $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where c is the circle $|2| = 3$. **(07 Marks)**

b. Find the bilinear transformation that maps the points $z = 1, i, -1$ onto $w = 0, 1, \infty$. **(07 Marks)**

c. Evaluate : $\int_{(1-i)}^{(2+i)} (2x + iy + 1)\, dz$ along the straight line joining the points (1, –1) and (2, 1). **(06 Marks)**

Module-3

5 a. A coin is tossed twice. If x represents the number of heads turning up, find the probability distribution of x. also find its mean and variance. **(07 Marks)**

b. If 2% of the fuses manufactured by a firm are defective. Find the probability that a box containing 200 fuses contains : i) no defective fuses ii) 3 or more defective fuses. **(07 Marks)**

c. In a normal distribution, 31% of the items are below 45 and 8% of the items are above 64. Find the mean and standard deviation of the distribution. Given that : A(1.4) = 0.42 and A(0.5) = 0.1915. **(06 Marks)**

OR

6 a. Find the constant K such that

$$f(x) = \begin{cases} Kx^2; & -3 \le x \le 3 \\ o; & \text{otherwise} \end{cases}$$

is a probability density function. Also find :

i) $P(1 \le x \le 2)$

ii) $P(x \le 2)$

iii) $P(x > 1)$. **(07 Marks)**

b. When a coin is tossed 4 items, find the probability of getting

i) exactly one head

ii) at most 3 heads

iii) at least 2 heads. **(07 Marks)**

c. If x is an exponential variate with mean 5. Evaluate :

i) $P(0 < x <)$

ii) $P(-\infty < x < 10)$

iii) $P(x \le 0)$ or $(x \ge 1)$. **(06 Marks)**

Module-4

7 a. Find the coefficient of correlation and the lines of regression for the following data :

x	1	2	3	4	5
y	2	5	3	8	7

(07 Marks)

b. Fit a curve of the form $y = ax^b$ for the data :

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

(07 Marks)

c. If the equations of regression lines of two variables x and y are $x = 19.13 - 0.879$ and $y = 11.64 - 0.5x$. Find the correlation coefficient and the means of x and y. **(06 Marks)**

OR

8 a. Compute the rank correlation coefficient for the following data :

x	68	64	75	50	64	80	75	40	55	64
y	62	58	68	45	81	60	68	48	50	70

(07 Marks)

b. Fit a parabola $y = a + bx + cx^2$ by the method of least squares to the following data :

x	1	2	3	4	5	6	7
y	2.3	5.2	9.7	16.5	29.4	35.5	54.4

(07 Marks)

c. Compute the mean values of x and y and the coefficient correlation for the regression lines $2x + 3y + 1 = 0$ and $x + 6y - 4 = 0$. **(06 Marks)**

Module-5

9 a. The joint probability distribution of two random variables x and y is defined by the function $P(x, y) = \frac{1}{27}(2x + y)$, where x and y assume the values 0, 1, 2. Find the marginal distributions of x and y. Also compute E(x) and E(y). **(07 Marks)**

b. Fit a Poisson distribution for the following data and test the goodness of fit. Given that $\Psi^2_{0.05} = 9.49$ for degrees of freedom 4. **(07 Marks)**

c. Write short notes on :
i) Null hypothesis
ii) Type – I and Type – II
iii) Level of significance. **(06 Marks)**

OR

10 a. Joint probability distribution of two random variables is given by the following data :

x \ y	–3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find :
i) Marginal distributions of x and y
ii) Cov(x, y)
iii) P(x, y). **(07 Marks)**

b. The following are the I·Q's of a randomly chosen sample of 10 boys.
70, 120, 110, 101, 88, 83, 95, 98, 107, 100
Does this data support the hypothesis that the population mean of I·Q's is 100 at 5% level of significance? Given $t_{0.05} = 2.26$. **(07 Marks)**

c. A sample of 900 items is found to have the mean 3.4. Can it be reasonably regarded as a truly random sample from a large population with mean 3.25 and standard deviation 1.61 at 5% level of significance? Given $Z_{0.05} = 1.96$ (Two Tailed Test). **(06 Marks)**

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