Max. Marks: 80

Time: 3 hrs

Second Semester B.E. Degree Examination, Dec.2023/Jan.2024 **Engineering Mathematics - II**

Note: Answer any FIVE full questions, choosing ONE full question from each module.

1 a. Solve
$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-x} + \sin 2x$$
 by inverse differential operator method. (06 Marks)

b. Solve
$$y'' + y' + y = x^2 + x + 1$$
 by inversed differential operator method. (05 Marks)

c. Solve
$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x}$$
 by the method of variation of parameters. (05 Marks)

2 a. Solve
$$y'' + 2y' + y = x \sin x$$
 by inverse differential operator method. (06 Marks)

b. Solve
$$(D^2 - 4D + 3)y = 2xe^{3x}$$
 by inverse differential operator method.

c. Solve
$$(D^2 - 3D + 2)y = x^2 + e^x$$
 by the method of undetermined co-efficient.

3 a. Solve
$$(2x+1)^2 y'' - 6(2x+1)y' + 16y = 8(2x+1)^2$$

b. Solve
$$x^2p^4 + 2xp - y = 0$$
 by solving for y

c. Solve
$$y = 2px + y^2p^3$$

(05 Marks)

4 a. Solve
$$x^2y'' + xy' + 9y = 3x^2 \sin[3\log x]$$
.

b. Solve
$$xy\left(\frac{dy}{dx}\right)^2 - (x^2 + y^2)\frac{dy}{dx} + xy = 0$$
.

c. Solve
$$xp^2 - py + kp + a = 0$$
 by reducing into Clairaut's form. Hence find singular solution. (05 Marks)

Module-3

- a. Form the partial differential equation by eliminating the arbitrary function, (05 Marks) $z = \phi(x + ay) + \psi(x - ay).$
 - Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos y$, given that $\frac{\partial z}{\partial y} = -2 \cos y$ when x = 0 and z = 0 when $y = n\pi$. (05 Marks)
 - Derive one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$. (06 Marks)

- Obtain the partial differential equation, $\phi(x+y+z, x^2+y^2+z^2)=0$. (05 Marks)
 - b. Solve $\frac{\partial^2 z}{\partial y^2} = z$ given that z = 0 and $\frac{\partial z}{\partial y} = \sin x$ when y = 0. (05 Marks)
 - Find the solution of neat equation, $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables. (06 Marks)

- a. Evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2) dz dy dx$. (05 Marks)
 - b. Evaluate $\int_{0}^{1} \int_{x}^{\sqrt{x}} xy \, dy dx$ by changing the order of integration. (05 Marks)
 - c. Evaluate $\int_{-\sqrt{2-x}}^{2} dx$ by using Beta and Gamma functions. (06 Marks)

- a. Evaluate $\iint xy(x+y)dydx$ taken over the region between $y=x^2$ and y=x. (05 Marks)
 - Find the area of the circle $x^2 + y^2 = a^2$ by double integration. (05 Marks)
 - c. Show that $\int_{0}^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin \theta}} \times \int_{0}^{\frac{\pi}{2}} \sqrt{\sin \theta} d\theta = \pi$ (06 Marks)

- a. Find $L[te^{-2t} \sin 4t]$. (05 Marks)
 - b. Given that $f(t) = \begin{cases} a, & 0 \le t \le a \\ -a, & a < t \le 2a \end{cases}$, where f(t+2a) = f(t). Show that $L\{f(t)\} = \frac{a}{5} \tanh\left(\frac{as}{2}\right)$. (05 Marks)
 - c. Solve $y'' + 4y' + 4y = e^t$, given that y(0) = y'(0) = 0 by using Laplace transform method. (06 Marks)

OR

- a. Find $L^{-1} \left[\frac{s+1}{s^2+6s+9} \right]$ (05 Marks)
 - b. Find $L^{-1} \left| \frac{1}{s(s^2 + a^2)} \right|$ by using convolution theorem. (05 Marks)
 - Express $f(t) = \begin{cases} \cos t, & 0 < t \le \pi \\ 1, & \pi < t \le 2\pi \end{cases}$ in term of unit step function and hence find its Laplace $t > 2\pi$

(06 Marks) transform.