CBCS SCHEME

USN

21MAT31

Third Semester B.E. Degree Examination, Dec.2023/Jan.2024
Transform Calculus Fourier Series & Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the Laplace transform of, (i) $e^{-3t}\sin 5t.\cos 3t$ (ii) $\dfrac{e^{at}-e^{bt}}{t}$. **(06 Marks)**

b. If a periodic function of period 'a' is defined by $f(t)=\begin{cases} E, & \text{for } 0<t<\dfrac{a}{2} \\ -E, & \text{for } \dfrac{a}{2}<t<a \end{cases}$ then show that $L\{f(t)\}=\dfrac{E}{S}\tanh\left(\dfrac{as}{4}\right)$. **(07 Marks)**

c. Using convolution theorem find the inverse Laplace transform of $\dfrac{s}{(s+2)(s^2+9)}$. **(07 Marks)**

OR

2 a. Express the function $f(t)=\begin{cases} \cos t & \text{for } 0<t<\pi \\ \cos 2t & \text{for } \pi<t<2\pi \\ \cos 3t & t>2\pi \end{cases}$ interms of unit step function and hence find its Laplace transform. **(07 Marks)**

b. Find the inverse laplace transform of $\dfrac{2s^2-6s+5}{s^3-6s^2+11s-6}$. **(06 Marks)**

c. Solve the differential equation $\dfrac{d^2y}{dt^2}+4\dfrac{dy}{dt}+4y=e^{-t}$ with $y(0)=y'(0)=0$ by using Laplace transform. **(07 Marks)**

Module-2

3 a. Find a Fourier series to represent $f(x)=|x|$ in $-\pi\le x\le\pi$. **(06 Marks)**

b. Obtain the half-range cosine series for $f(x)=x\sin x$ in $(0,\pi)$ and hence show that $\dfrac{\pi-2}{4}=\dfrac{1}{1.3}-\dfrac{1}{3.5}+\dfrac{1}{5.7}-\ldots\ldots\infty$ **(07 Marks)**

c. Express y as a Fourier series up to second harmonics for the following data :

x:	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y:	1	1.4	1.9	1.7	1.5	1.2	1.0

(07 Marks)

OR

4 a. Obtain the Fourier series expansion for the function, $f(x) = 2x - x^2$ in (0, 2). **(06 Marks)**

b. Find the half range sine series for the function, $f(x) = \begin{cases} \frac{1}{4} - x & \text{for } 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \text{for } \frac{1}{2} < x < 1 \end{cases}$ **(07 Marks)**

c. The following table gives the variation of periodic current over period :

t sec :	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
A (amp) :	1.98	1.30	1.05	1.30	−0.88	−0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic. **(07 Marks)**

Module-3

5 a. Find the Fourier transform of the function $f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$. Hence evaluate

$$\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) dx.$$ **(06 Marks)**

b. Find the Fourier sine and cosine transform of $f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 2 - x & \text{if } 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$. **(07 Marks)**

c. Find the z-transform of $\cosh\left(n\frac{\pi}{2} + \theta\right)$. **(07 Marks)**

OR

6 a. Find the Fourier sine transform of $f(x) = e^{-ax}$, a>0. **(06 Marks)**

b. Find the inverse z transform of $\frac{18z^2}{(2z-1)(4z+1)}$. **(07 Marks)**

c. Solve the difference equation $u_{n+2} + 6u_{n+1} + 9u_n = z^n$ with $u_0 = u_1 = 0$ using z-transform. **(07 Marks)**

Module-4

7 a. Classify the following partial differential equations :

(i) $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$.

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + (1 - y^2)\frac{\partial^2 u}{\partial y^2} = 0$, $-\infty < x < \infty$, $-1 < y < 1$.

(iii) $(1 + x^2)\frac{\partial^2 u}{\partial x^2} + (5 + 2x^2)\frac{\partial^2 u}{\partial x \partial t} + (4 + x^2)\frac{\partial^2 u}{\partial t^2} = 0$.

(iv) $(x+1)\frac{\partial^2 u}{\partial x^2} - 2(x+2)\frac{\partial^2 u}{\partial x \partial y} + (x+3)\frac{\partial^2 u}{\partial y^2} = 0$. **(10 Marks)**

b. Evaluate the values at the mesh points for the equation $u_{tt} = 16u_{xx}$ taking h = 1 upto t = 1.25. The boundary conditions are u(0, t) = u(5, t) = 0 and the initial conditions are $u(x, 0) = x^2(5 - x)$ and $u_t(x, 0) = 0$. **(10 Marks)**

OR

8 a. Using Schmidt two-level formula to solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ under the conditions,

(i) $u(0, t) = u(1, t) = 0 \quad t \geq 0$

(ii) $u(x, 0) = \sin \pi x, \quad 0 < x < 1$ by taking $h = \frac{1}{4}$ and $\alpha = \frac{1}{6}$ co. **(10 Marks)**

b. Solve the two-dimensional Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ at the interior mesh points of the square region and the values of u at the mesh points on the foundary are shown in Fig.Q8 (b).

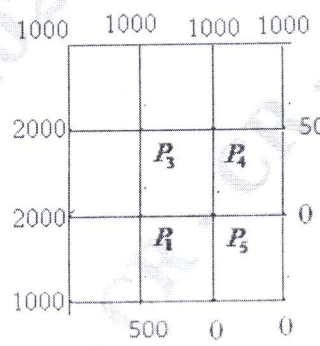

Fig. Q8 (b) **(10 Marks)**

Module-5

9 a. Using Runge-Kutta method of 4^{th} order to solve the differential equation $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 4y = 0$ with $y(0) = 0.2$ and $y'(0) = 0.5$ for $x = 0.1$. Correct to four decimal places. **(07 Marks)**

b. State and prove Euler's equation. **(07 Marks)**

c. Find the extremal of the functional $I = \int_0^{\frac{\pi}{2}} (y^2 - y'^2 - 2y\sin x)dx$ under the end conditions $y(0) = 0, \ y\left(\frac{\pi}{2}\right) = 0$ **(06 Marks)**

OR

10 a. Apply Milne's method to compute y(0.3). Given that $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$, $y'(0) = 0$, $y'(0.2) = 0.1996$, $y'(0.4) = 0.3937$, $y'(0.6) = 0.5689$ **(07 Marks)**

b. Prove that the shortest distance between two points in a plane is a straight line. **(07 Marks)**

c. Find the extremal of the functional $I = \int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x)dx$ **(06 Marks)**

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