USN

15MATDIP31

Third Semester B.E. Degree Examination, Dec.2023/Jan.2024
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Show that
$$\frac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^4} = \cos 8\theta + i\sin 8\theta$$ (05 Marks)

b. Prove that $[\vec{b}\times\vec{c},\ \vec{c}\times\vec{a},\ \vec{a}\times\vec{b}] = [\vec{a}\ \ \vec{b}\ \ \vec{c}]^2$ (05 Marks)

c. Prove that $\hat{i}\times(\vec{a}\times\hat{i}) + \hat{j}\times(\vec{a}\times\hat{j}) + \hat{k}\times(\vec{a}\times\hat{k}) = 2\vec{a}$ (06 Marks)

OR

2 a. Find the real part of $\dfrac{1}{1+\cos\theta + i\sin\theta}$ (05 Marks)

b. If $\vec{a} = 3i - 2j + 4k$ and $\vec{b} = i + j - 2k$, find (i) $\vec{a}.\vec{b}$ (ii) Angle between $\vec{a}$ & $\vec{b}$. (05 Marks)

c. Show that $[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}] = 2[a, b, c]$ (06 Marks)

Module-2

3 a. If $y = \tan^{-1}x$, prove that $(1 + x^2)y_2 + 2xy_1 = 0$ and hence show that
$(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$ (06 Marks)

b. Find the angle between the radius vector and the tangent for the curve
$r^m = a^m(\cos m\theta + \sin m\theta)$ (05 Marks)

c. Find the pedal equation of $r^n = a^n \cos n\theta$ (05 Marks)

OR

4 a. If $u = \log\left(\dfrac{x^4 + y^4}{x + y}\right)$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3$ (06 Marks)

b. If $u = f(x - y, y - z, z - x)$, show that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$ (05 Marks)

c. Find $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$ where $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$ (05 Marks)

Module-3

5 a. Evaluate the integral $\displaystyle\int_0^{\pi} x\sin^2 x\cos^4 x\,dx$ using Reduction formula. (05 Marks)

b. Evaluate $\displaystyle\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$ (05 Marks)

c. Evaluate $\displaystyle\int_{-1}^{1}\int_0^{2}\int_{x-2}^{x+2} (x + y + z)\,dy\,dx\,dz$ (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

6 a. Evaluate $\int_0^{\pi} \sin^4 x \, dx$, using Reduction formula. **(05 Marks)**

b. Evaluate $\int_0^1 x^{3/2}(1-x)^{3/2} \, dx$, using Reduction formula. **(06 Marks)**

c. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) \, dy \, dx$, using Reduction formula. **(05 Marks)**

Module-4

7 a. A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$ where t is the time. Find the velocity and acceleration at any time t and also their magnitudes at $t = 0$. **(05 Marks)**

b. Find $\operatorname{div}\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ **(06 Marks)**

c. Find the value of the constant 'a' such that the vector field $\vec{F} = (axy - z^3)i + (a-2)x^2 j + (1-a)xz^2 k$ is irrotational. **(05 Marks)**

OR

8 a. If $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ represents the parametric equation of the curve. Find the angle between the tangents at $t = 1$ and $t = 2$. **(05 Marks)**

b. Find the angle between the normals to the surface at the points (4, 1, 2) and (3, 3, –3). **(05 Marks)**

c. Show that $\vec{F} = \dfrac{xi + yj}{x^2 + y^2}$ is both solenoidal and irrotational. **(06 Marks)**

Module-5

9 a. Solve : $x^2 y \, dx - (x^3 + y^3) dy = 0$ **(06 Marks)**

b. Solve : $(2x + y + 1)dx + (x + 2y + 1)dy = 0$ **(05 Marks)**

c. Show that $\dfrac{dy}{dx} - \dfrac{2y}{x} = x + x^2$ **(05 Marks)**

OR

10 a. Solve : $(y^3 - 3x^2 y)dx - (x^3 - 3xy^2)dy = 0$ **(05 Marks)**

b. Solve : $\dfrac{dy}{dx} + y \cot x = \cos x$ **(05 Marks)**

c. Solve : $\dfrac{dy}{dx} + \dfrac{y}{x} = y^2 x$ **(06 Marks)**

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