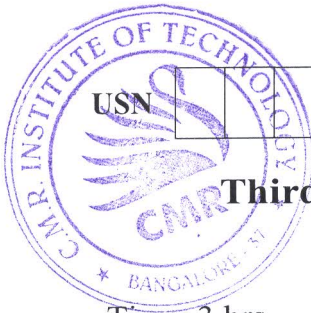


CBCS SCHEME

15MATDIP31



Third Semester B.E. Degree Examination, Dec.2023/Jan.2024 Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Show that

$$\frac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^4} = \cos 8\theta + i\sin 8\theta \quad (05 \text{ Marks})$$

b. Prove that $[\vec{b} \times \vec{c}, \vec{c} \times \vec{a}, \vec{a} \times \vec{b}] = [\vec{a}, \vec{b}, \vec{c}]^2$ (05 Marks)

c. Prove that $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$ (06 Marks)

OR

2 a. Find the real part of $\frac{1}{1 + \cos\theta + i\sin\theta}$ (05 Marks)

b. If $\vec{a} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$, find (i) $\vec{a} \cdot \vec{b}$ (ii) Angle between \vec{a} & \vec{b} . (05 Marks)

c. Show that $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[a, b, c]$ (06 Marks)

Module-2

3 a. If $y = \tan^{-1}x$, prove that $(1 + x^2)y_2 + 2xy_1 = 0$ and hence show that $(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$ (06 Marks)

b. Find the angle between the radius vector and the tangent for the curve $r^m = a^m(\cos m\theta + \sin m\theta)$ (05 Marks)

c. Find the pedal equation of $r^n = a^n \cos n\theta$ (05 Marks)

OR

4 a. If $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$ (06 Marks)

b. If $u = f(x - y, y - z, z - x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ (05 Marks)

c. Find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ where $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$ (05 Marks)

Module-3

5 a. Evaluate the integral $\int_0^\pi x \sin^2 x \cos^4 x dx$ using Reduction formula. (05 Marks)

b. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy dy dx$ (05 Marks)

c. Evaluate $\int_{-1}^1 \int_0^2 \int_{x-2}^{x+2} (x + y + z) dy dx dz$ (06 Marks)

OR

- 6 a. Evaluate $\int_0^{\pi} \sin^4 x \, dx$, using Reduction formula. (05 Marks)
- b. Evaluate $\int_0^1 x^{3/2}(1-x)^{3/2} \, dx$, using Reduction formula. (06 Marks)
- c. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) \, dy \, dx$, using Reduction formula. (05 Marks)

Module-4

- 7 a. A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2 \cos 3t$, $z = 2 \sin 3t$ where t is the time. Find the velocity and acceleration at any time t and also their magnitudes at $t = 0$. (05 Marks)
- b. Find $\text{div } \vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ (06 Marks)
- c. Find the value of the constant 'a' such that the vector field $\vec{F} = (axy - z^3)\mathbf{i} + (a - 2)x^2\mathbf{j} + (1 - a)xz^2\mathbf{k}$ is irrotational. (05 Marks)

OR

- 8 a. If $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ represents the parametric equation of the curve. Find the angle between the tangents at $t = 1$ and $t = 2$. (05 Marks)
- b. Find the angle between the normals to the surface at the points $(4, 1, 2)$ and $(3, 3, -3)$. (05 Marks)
- c. Show that $\vec{F} = \frac{x\mathbf{i} + y\mathbf{j}}{x^2 + y^2}$ is both solenoidal and irrotational. (06 Marks)

Module-5

- 9 a. Solve: $x^2y \, dx - (x^3 + y^3)dy = 0$ (06 Marks)
- b. Solve: $(2x + y + 1)dx + (x + 2y + 1)dy = 0$ (05 Marks)
- c. Show that $\frac{dy}{dx} - \frac{2y}{x} = x + x^2$ (05 Marks)

OR

- 10 a. Solve: $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$ (05 Marks)
- b. Solve: $\frac{dy}{dx} + y \cot x = \cos x$ (05 Marks)
- c. Solve: $\frac{dy}{dx} + \frac{y}{x} = y^2x$ (06 Marks)

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