CBCS SCHEME

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BMATS201

Second Semester B.E./B.Tech. Degree Examination, Dec.2023/Jan.2024

Mathematics – II for CSE Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	**M**	**L**	**C**
Q.1	a.	Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dydxdz$.	7	L2	CO1
	b.	Evaluate $\int_{0}^{\infty}\int_{0}^{\infty}e^{-(x^2+y^2)}\,dxdy$ by changing into polar coordinates.	7	L3	CO1
	c.	Show that $\beta(m,n)=\dfrac{\gamma(m)\gamma(n)}{\gamma(m+n)}$	6	L2	CO1
		OR			
Q.2	a.	Evaluate $\int_{0}^{1}\int_{y}^{\sqrt{y}}(x^2y+xy^2)dxdy$ by changing the order of integration.	7	L3	CO1
	b.	Show that $\int_{0}^{\pi/2}\dfrac{d\theta}{\sqrt{\sin\theta}}\times\int_{0}^{\pi/2}\sqrt{\sin\theta}\,d\theta=\pi$	7	L2	CO1
	c.	Using mathematical tools, write the code to find the area of an ellipse by double integration $A=4\int_{0}^{a}\int_{0}^{\frac{b}{a}\sqrt{a^2-x^2}}dydx$, taking $a=4, b=6$.	6	L3	CO5
		Module – 2			
Q.3	a.	Find the directional derivative of $\phi=4xz^3-3x^2y^2z$ at (2, -1, 2) along vector $2i-3j+6k$.	7	L2	CO2
	b.	Show that the vector $\vec{F}=\dfrac{xi+yi}{x^2+y^2}$ is both solenoidal and irrotational.	7	L2	CO2
	c.	Prove that the spherical coordinate system is orthogonal.	6	L3	CO2
		OR			
Q.4	a.	Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z^2+y^2-x=3$ at (2, -1, 2).	7	L2	CO2
	b.	Express the vector $\vec{A}=z\hat{i}-2x\hat{j}+y\hat{k}$ in cylindrical coordinates.	7	L2	CO2
	c.	Using mathematical tools, write the code to find the curl of $\vec{F}=x^2yz\hat{i}+y^2zx\hat{j}+z^2xy\hat{k}$.	6	L3	CO5

		Module – 3			
Q.5	**a.**	Prove that the subset $W = \{(x, y, z) : ax + by + cz = 0; x, y, z \in R\}$ of the vector space R^3 is a subspace of R^3.	7	L2	CO3
	b.	Determine the following vectors are linearly independent or not, $x_1 = (2, 2, 1)$, $x_2 = (1, 3, 7)$ and $x_3 = (1, 2, 2)$ in R^3.	7	L2	CO3
	c.	Show that the function $T : R^2 \rightarrow R^3$ given by $T(x, y) = (x + y, x - y, y)$ is a linear transformation.	6	L2	CO3
		OR			
Q.6	**a.**	Determine whether the vectors $v_1 = (1, 2, 3)$, $v_2 = (3, 1, 7)$ and $v_3 = (2, 5, 8)$ are linearly dependent or linearly independent.	7	L2	CO3
	b.	Verify the Rank-Nullity theorem for the linear transformation $T : R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.	7	L2	CO3
	c.	Consider the vectors $u = (1, 2, 4)$, $v = (2, -3, 5)$, $w = (4, 2, -3)$ in R^3. Find: i) $\langle u.v \rangle$ ii) $\langle u.w \rangle$ iii) $\langle v.w \rangle$ iv) $\langle (u + v).w \rangle$	6	L2	CO3
		Module – 4			
Q.7	**a.**	Find an approximate value of the root of the equation $x^3 - x^2 - 1 = 0$, using the Regula-Falsi method upto four decimal places of accurancy, where root lies between 1.4 and 1.5.	7	L2	CO4
	b.	Using Newton's divided difference formula evaluate f(4) from the following: <table><tr><td>x</td><td>0</td><td>2</td><td>3</td><td>6</td></tr><tr><td>f(x)</td><td>-4</td><td>2</td><td>14</td><td>158</td></tr></table>	7	L2	CO4
	c.	Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ by using Trapezoidal rule by taking 7 ordinates.	6	L3	CO4
		OR			
Q.8	**a.**	Find an approximate root of the equation $x \log_{10} x - 1.2 = 0$ corrected to five decimal places where root lies near 2.5 by Newton-Raphson method.	7	L2	CO4
	b.	The area A of a circle of diameter d is given for the following values. Calculate the area of a circle of diameter 82 by using Newton's forward interpolation formula. <table><tr><td>d</td><td>80</td><td>85</td><td>90</td><td>95</td><td>100</td></tr><tr><td>A</td><td>5026</td><td>5674</td><td>6362</td><td>7088</td><td>7854</td></tr></table>	7	L2	CO4
	c.	Use Simpson's 1/3rd rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking seven ordinates.	6	L2	CO4

Module – 5

Q.9	a.	Find by Taylor's series method the value of y at x = 0.1 to five places of decimals from $\frac{dy}{dx} = x^2y - 1$ with an initial condition y(0) = 1.	7	L2	CO4
	b.	Using the Runge-Kutta method of fourth order solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) = 1 at x = 0.2 taking h = 0.2.	7	L2	CO4
	c.	Given that $\frac{dy}{dx} = x^2(1+y)$ and y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548 and y(1.3) = 1.979. Compute y at x = 1.4 by applying Milne's method.	6	L2	CO4

OR

Q.10	a.	Using modified Euler's method, solve $\frac{dy}{dx} = 3x + \frac{y}{2}$ at x = 0.1 corrected to four decimal places by taking h = 0.1, with initial condition y(0) = 1.	7	L2	CO4
	b.	Given that $\frac{dy}{dx} = x - y^2$ and y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762. Compute y(0.8) by Milne's method.	7	L2	CO4
	c.	Using mathematical tools, write the code to find the solution of $\frac{dy}{dx} = 1 + \frac{y}{x}$ at y(2) taking h = 0.2. Given that y(1) = 2 by Runge-Kutta method of 4th order.	6	L3	CO5

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