CBCS SCHEME

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15MATDIP41

Fourth Semester B.E. Degree Examination, Dec.2023/Jan.2024
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the rank of a matrix by reducing to echelon form:

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$$ **(05 Marks)**

b. Solve the system of equations by Gauss elimination method,
$x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$ **(05 Marks)**

c. Using Cayley-Hamilton find the inverse of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. **(06 Marks)**

OR

2 a. Find the rank of a matrix by using row elementary transformations:

$$A = \begin{bmatrix} 1 & 1 & -1 & 3 \\ 2 & -2 & 6 & 8 \\ 3 & 5 & -7 & 8 \end{bmatrix}$$ **(05 Marks)**

b. Show that the system is consistant and hence solve:
$x + 2y - z = 3$, $3x + y + 2z = 1$, $2x - 2y + 3z = 2$ **(05 Marks)**

c. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$. **(06 Marks)**

Module-2

3 a. Solve $\dfrac{d^3y}{dx^3} - 6\dfrac{d^2y}{dx^2} + 11\dfrac{dy}{dx} - 6y = 0$. **(05 Marks)**

b. Solve $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 3e^x$. **(05 Marks)**

c. Solve by the method of variation of parameters $y'' + a^2y = \sec ax$. **(06 Marks)**

OR

4 a. Solve $\dfrac{d^2x}{dt^2} - 6\dfrac{dx}{dt} + 9x = 5e^{-2t}$. **(05 Marks)**

b. Solve $\dfrac{d^2y}{dx^2} + 4y = \sin x$. **(05 Marks)**

c. Solve by the method of undetermined coefficients $y'' + 3y' + 2y = 12x^2$. **(06 Marks)**

Module-3

5 a. Find the Laplace transform of $\sin 5t \cos 2t$. **(05 Marks)**

b. Prove that $L[\cos at] = \frac{s}{s^2 + a^2}$. **(05 Marks)**

c. Given $f(t) = \begin{cases} E & 0 < t < a/2 \\ -E & a/2 < t < a \end{cases}$ where $f(t + a) = f(t)$, show that $L\{f(t)\} = \frac{E}{s} \tan h\left(\frac{as}{4}\right)$. **(06 Marks)**

OR

6 a. Find the Laplace transform of $\cos t \cos 2t \cos 3t$. **(05 Marks)**

b. Find the Laplace transform of $te^t \sin t$. **(05 Marks)**

c. Express the function $f(t) = \begin{cases} t & 0 < t < 4 \\ 5 & t > 4 \end{cases}$ interms of Heaviside unit step function and hence find its Laplace transform. **(06 Marks)**

Module-4

7 a. Evaluate $L^{-1}\left\{\frac{2s^2 + 5s - 4}{s(s-1)(s+2)}\right\}$. **(08 Marks)**

b. Employ Laplace transform to solve equation $y'' + 5y' + 6y = 5e^{2t}$, $y(0) = 2$, $y'(0) = 1$. **(08 Marks)**

OR

8 a. Find the inverse Laplace transform of $\log\left(\frac{s-a}{s-b}\right)$. **(08 Marks)**

b. Using Laplace transform, solve $y'' - 2y' + y = e^{2t}$ given that $y(0) = 0$, $y'(0) = 1$. **(08 Marks)**

Module-5

9 a. A bag contains 7 white, 6 red and 5 black balls. Two balls are drawn at random. Find the probability that they will both be white. **(05 Marks)**

b. A problem in mathematics is given to three students A, B, C. Whose changes of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the probability that the problem is solved? **(05 Marks)**

c. In a bolt factory, machines A, B and C manufactures respectively 25%, 35% and 40% of the total of their output 5, 4 and 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B? **(06 Marks)**

OR

10 a. Prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. **(05 Marks)**

b. A box contain three white balls and two red balls. If two balls are drawn in succession, find the probability that the first removed ball is white and the second is red. **(05 Marks)**

c. In a certain college, 4% men students and 1% of women students are taller than 1.8 m. Furthermore 60% of students are women. If a student is selected at random and is found taller than 1.8 m. What is the probability that the student is woman? **(06 Marks)**

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