CBCS SCHEME

USN

17CS/IS36

Third Semester B.E. Degree Examination, Dec.2023/Jan.2024
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define the following with their truth table:
i) Conjunction ii) Disjunction iii) Conditional iv) Biconditional. **(08 Marks)**

b. Define Tautology. Prove that for any propositions p, q, r the compound proposition.
$[(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p - r)$ is a tautology. **(06 Marks)**

c. Define logical equivalence. Show that the compound propositions $p \wedge (\neg q \vee r)$ and $p \vee (q \wedge \neg r)$ are not logically equivalent. **(06 Marks)**

OR

2 a. Define the following with example:
i) Open statement ii) Quantifier iii) Universal quantifier iv) Existential quantifier. **(08 Marks)**

b. Prove the following by using truth table:
i) $[p \wedge (p \rightarrow q) \wedge r] \Rightarrow [(p \vee q) \rightarrow r]$
ii) $\{[p \vee (q \vee r)] \wedge \neg q\} \Rightarrow p \vee q$. **(06 Marks)**

c. Test the validity of the following argument:
If I study, I will not fail in the examination.
If I do not watch TV in the evenings, I will study.
I failed in the Examination.
$\therefore$ I must have watched TV in the evenings. **(06 Marks)**

Module-2

3 a. Find an explicit definition of the following sequences defined recursively by
i) $a_1 = 4, a_n = a_{n-1} + n$ for $n \geq 2$
ii) $a_1 = 7, a_n = 2a_{n-1} + 1$ for $n \geq 2$. **(08 Marks)**

b. Prove that every positive integer $n \geq 24$ can be written as a sun of $5'^s$ and/or $7'^s$. **(06 Marks)**

c. State mathematical induction principle. Prove by mathematical induction that, for all positive integers $n \geq 1$,
$1 + 2 + 3 + 4 + \ldots\ldots + n = \frac{1}{2} n (n + 1)$. **(06 Marks)**

OR

4 a. A woman has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations:
i) There is no restriction on the choice.
ii) Two particular persons will not attend separately. **(06 Marks)**

b. Find the coefficient of
i) x^9y^3 in the expansion of $(2x - 3y)^{12}$.
ii) $x^2y^2z^3$ in the expansion of $(3x\text{-}2y\text{-}4z)^7$. **(08 Marks)**

c. Prove the following identities
i) $c(n, r\text{-}1) + c(n, r) = c(n + 1, r)$
ii) $c(m, 2) + c(n, 2) = c(m + n, 2) - mn$. **(06 Marks)**

Module-3

5 a. Define the following with example:
   i) Cartesian product of sets A and B
   ii) One-one function
   iii) Onto function
   iv) Composition of two functions. **(08 Marks)**

b. State pigeonhole principle. Find the least number of ways of choosing three different number from 1 to 10 so that all choices have the same sum. **(06 Marks)**

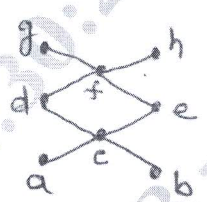
c. Define invertible function. Let A = {1, 2, 3, 4, 5, 6} and B = {6, 7, 8, 9, 10}. If a function f: $A \to B$ is defined by f = {(1, 7), (2, 7), (3, 8), (4, 6), (5, 9), (6, 9)}. Determine $f^{-1}(6)$ and $f^{-1}(9)$. If $B_1 = \{7, 8\}$ and $B_2 = \{8, 9, 10\}$, find $f^{-1}(B_1)$ and $f^{-1}(B_2)$. **(06 Marks)**

OR

6 a. Define the following with example:
   i) Reflexive relation
   ii) Symmetric relation
   iii) Antisymmetric relation
   iv) Transitive relation. **(08 Marks)**

b. Let A = {1, 2, 3, 4} and let R be the relation on A defined by xRy if and only if "x divides y" written x/y.
   i) Write down R as a set of order pairs.
   ii) Draw the diagraph of R.
   iii) Find M(R). **(06 Marks)**

c. Consider the Hasse diagram of a poset (A, R) given below:
   If B = {c, d, e}, find
   i) All upper bounds of B
   ii) All lower bounds of B
   iii) LUB of B
   iv) GLB of B

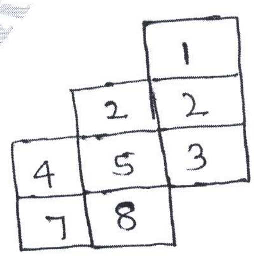


(06 Marks)

Module-4

7 a. Find the number of permutations of the English letters which contain i) Exactly two ii) at least two iii) Exactly three iv) at least three, of the patterns CAR, DOG, PUN and BYTE. **(08 Marks)**

b. By using the expansion formula obtain the rook polynomial for the board C shown below.



(06 Marks)

c. Prove that, for any positive integer η,

$$n! = \sum_{k=0}^{n} \binom{n}{k} d_k$$

(06 Marks)

OR

8 a. Solve the recurrence relation $a_n = a_{n-1} + a_{n-2}$ given $a_0 = 1$, $a_2 = 3$. **(08 Marks)**

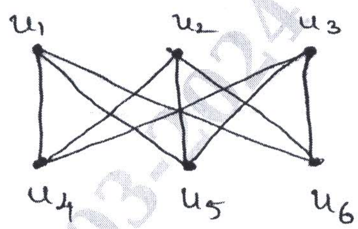
b. What is Derangement? Give the formula for 'du' and hence evaluate d_5, d_6, d_7, d_8. **(06 Marks)**

c. A bank pays 6% interest on every deposit compounding monthly. A person deposits 2000 Rs. in a bank. How much amount will be getting after 60-months? **(06 Marks)**

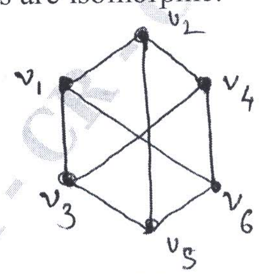
Module-5

9 a. Define the following with example:
i) Multigraph ii) General graph iii) Complete graph iv) Bipartite graph. **(08 Marks)**

b. Define isomorphism. Show that the following two graphs are isomorphic:



G1



G2

(06 Marks)

c. Write a note on Konigsberg bridge problem. Define Euler's circuit and Euler's trail. **(06 Marks)**

OR

10 a. Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code. **(08 Marks)**

b. Define tree. Prove that a tree with 'n' vertices has n – 1 edges. **(06 Marks)**

c. Define merge sort. Using merge sort method sort the following list.
7, 3, 8, 4, 5, 10, 6, 2, 9. **(06 Marks)**

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