CBCS SCHEME

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Third Semester B.E. Degree Examination, Dec.2023/Jan.2024
Discrete Mathematical Structure

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define proposition, tautology and contradiction. Determine whether the following compound statement is a tautology or not
$\{(p \vee q) \rightarrow r\} \leftrightarrow \{\sim r \rightarrow \sim (p \vee q)\}$ **(07 Marks)**

b. Using the laws of logic, prove the following :
$[\sim p \wedge (\sim q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$ **(06 Marks)**

c. Find whether the argument is valid.
If a triangle has two equal sides, then it is isosceles.
If a triangle is isosceles, then it has two equal angles.
A certain triangle ABC doesnot have two equal angles.
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$\therefore$ The triangle ABC does not have two equal sides **(07 Marks)**

OR

2 a. Prove that for any 3 proposition p, q, r
$[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)$ is logically equivalent to
$[(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)$ **(07 Marks)**

b. Give (i) a direct proof (ii) an indirect proof and (iii) proof by contradiction for the following statement.
" If n is an odd integer, then n + 9 is an even integer" **(06 Marks)**

c. Establish the validity of the following argument,
$\forall x, (p(x) \vee q(x))$
$\forall x, \sim p(x)$
$\forall x, [\sim q(x) \vee r(x)]$
$\forall x, [s(x) \rightarrow \sim r(x)]$
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$\therefore$ fx, ~s(x) **(07 Marks)**

Module-2

3 a. By mathematical induction prove that,
$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1} \quad \forall n \in z$$ **(07 Marks)**

b. Determine the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$ **(06 Marks)**

c. A certain question paper contains 3 parts A, B, C with 4 questions in Part A, 5 questions in Part B and 6 questions in Part C. It is required to answer 7 questions selecting at least two questions from each part. In how many ways can a student select his seven question for answering? **(07 Marks)**

OR

4 a. For the Fibonacci sequence F_0, F_1, F_2….. Prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$. **(07 Marks)**

b. How many arrangements are there for all letters in the word SOCIOLOGICAL? In how many of there arrangement (i) A & G are adjacent? (ii) All the vowels are adjacent? **(06 Marks)**

c. In how many ways can one distribute eight identical balls into four distinct containers so that,
   (i) No container is left empty.
   (ii) The fourth container gets an odd number of balls. **(07 Marks)**

Module-3

5 a. Let $A = \{1,2,3\}$, $B = \{2,4,5\}$. Determine the following:
   (i) $|A \times B|$
   (ii) Number of relations from A to B
   (iii) Number of relations on A
   (iv) Number of relations from A to B, that contains exactly 5 ordered pairs
   (v) Number of relations on A that contains atleast 7 ordered pairs. **(06 Marks)**

b. Find the least number of ways of choosing three different numbers from 1 to 10, so that all choices have the same sum. **(07 Marks)**

c. Let f, g, h be functions from z to z defined by f(x) = x − 1, g(x) = 3x and $h(x) = \begin{cases} 0, & \text{if x is even} \\ 1, & \text{if x is odd} \end{cases}$. **(07 Marks)**

OR

6 a. Suppose A, B, C $\subseteq Z \times Z$ with $A = \{(x,y) / y = 5x - 1\}$, $B = \{(x,y) / y = 6x\}$, $C = \{(x,y) / 3x - y = -7\}$, find (i) $A \cap B$ (ii) $B \cap C$ (iii) $\overline{\overline{A} \cup \overline{C}}$ **(06 Marks)**

b. Let A = {1, 2, 3, 4, 6} and R be the relations on A defined by aRb iff a is a multiple of b (ii) represent the relation R as a set of ordered pairs (ii) Draw its digraph (iii) write the matrix of R. **(07 Marks)**

c. Draw the Hasse diagram representing the positive divisors of 36 **(07 Marks)**

Module-4

7 a. In how many ways 5 numbers of a's, 4 number of b's and 3 number of c's can be arranged so that all the identical letters are not in a single block? **(06 Marks)**

b. There are n pairs of children's gloves in a box. Each pairs is of a different colour. Suppose the right gloves are distributed at random to n children, and then the left gloves are also distributed to them. Find the probability that (i) no child gets a matching pair. (ii) Every child gets a matching pair (iii) Exactly one child gets a matching pair and (iv) atleast two child gets matching pair. **(07 Marks)**

c. An apple, banana, a mango and an arrange are to be distributed to 4 boys B_1, B_2, B_3, B_4. The boys B_1 and B_2 do not wish to have apple, the boy B_3 does not want banana or mango and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(07 Marks)**

OR

8 a. Find the number of permutations of letters a, b, c, …..z in which name of the patterns spin, game, path or net occurs. **(06 Marks)**

b. Four persons P_1, P_2,P_3,P_4 who arrive late for a dinner party. Find that only one chair at each of 5 tables T_1, T_2, T_3, T_4 and T_5 is vacant. P_1 will not sit at T_1 or T_2, P_2 will not sit at T_2. P_3 will not sit at T_3 or T_4 and P_4 will not sit at T_4 or T_5. Find the number of ways they can occupy the vacant chairs? **(07 Marks)**

c. Obtain the solution of the relation $a_{n+1} - 2a_n = 5$. **(07 Marks)**

Module-5

9 a. Define Isomorphism. Show that the following two graphs are isomorphic. **(06 Marks)**

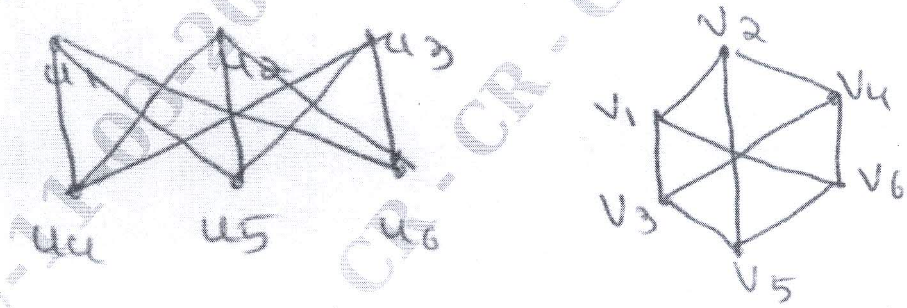

Fig. Q9 (a)

b. Describe Konigsberg Bridge problem. **(07 Marks)**

c. Construct an optimal prefix code for the symbols a, b, c, d, e, f, g, h, i, j that occur with frequencies 78, 16, 30, 35, 125, 31, 20, 50, 80, 3 respectively. **(07 Marks)**

OR

10 a. In every graph, the number of vertices of odd degree is even. **(06 Marks)**

b. A tree with n vertices, has n – 1 edges. **(07 Marks)**

c. Sort the following set of integers using Merge-Sort technique {2, 9, 12, 7, 3, 2, 8, 10, 5} **(07 Marks)**

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