21EC33

Third Semester B.E. Degree Examination, Dec.2023/Jan.2024
Basic Signal Processing

Time: 3 hrs.

Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1   a.   Write the complete solution $x = x_p + x_n$. to

$$A = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 8 & 10 \end{bmatrix}.$$

(10 Marks)

   b.   Define the four fundamental vector spaces and find the Dimension and basis for four fundamental subspaces for :

$$A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 4 & 8 & 12 \\ 3 & 6 & 7 & 13 \end{bmatrix}.$$

(10 Marks)

OR

2   a.   Illustrate the transformation of the plane that comes from four matrices and list the transformations T(x) that are not possible with Ax.   **(10 Marks)**

   b.   Compute $A^T A$ and their eigen values and unit eigen vectors for V and u. Then check $AV = u\Sigma$.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}.$$

(10 Marks)

Module-2

3   a.   What is an orthogonal matrix. Apply the gram Schmidt process to

$$a = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad c = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ and write the result in the form of A = QR.   **(10 Marks)**

   b.   Find the projection of b onto the column space of A.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}.$$

(10 Marks)

OR

4 a. Find eigen values and eigen vectors for the matrix. A can the matrix be diagonalized.

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

(10 Marks)

   b. i) What is a positive definite matrix? Mention the methods of testing positive definiteness.
(04 Marks)

   ii) Decide for or against the positive definiteness of

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}.$$

(06 Marks)

Module-3

5 a. Define a signal. List the elementary signals. Differentiate between even and odd signals, energy and power signals.
(10 Marks)

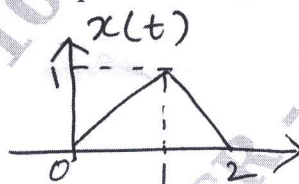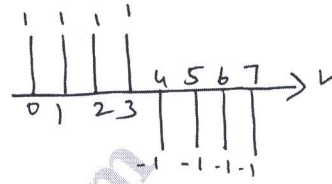   b. Sketch the even the odd part of the signals shown in Fig.Q5(b)(i) and 5(b)(ii).

Fig.Q5(b)(i)   Fig,Q5(b)(ii)   (08 Marks)

   c. Determine whether the following signal is periodic or not. If periodic find the fundamental period $x(n) = \cos\left(\dfrac{n\pi}{5}\right) \sin\left(\dfrac{n\pi}{3}\right)$.
(02 Marks)

OR

6 a. Determine whether the following systems are memoryless, causal time invariant, linear and stable. i) $y(n) = nx(n)$ ii) $y(t) = x(t/2)$.
(08 Marks)

   b. For the signal x(t) and y(t) shown in Fig.6(a) sketch the following signals :
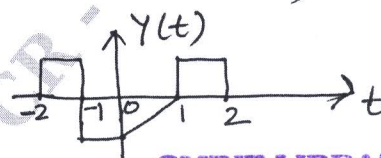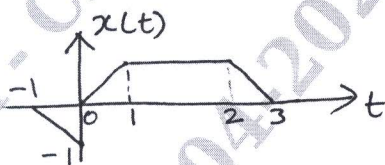   i) $x(t+1)y(t-2)$ ii) $x(t)y(t-1)$

Fig.Q6(b)   (08 Marks)

   c. Sketch the waveform of the signal : $x(t) = u(t+1) - 2u(t) + u(t-1)$.
(04 Marks)

Module-4

7 a. Compute the following convolution :

   i) $y(t) = e^{-2}u(t-2) * \{u(t-2) - u(t-12)\}$

   ii) $y(n) = \alpha^n \{u(n) - u(n-6)\} * 2\{u(n) - 4(n-15)\}$.
(14 Marks)

   b. Prove the following :

   i) $x(t) * \delta(t-t_0) = x(t-t_0)$

   ii) $x(n) * u(n) = \sum\limits_{k=-\infty}^{n} X(k)$.
(06 Marks)

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OR

8 a. Evaluate the step response for LTI system represented by the following impulse response :
   i) h(t) = u(t + 1) – u(t –1)
   ii) h(n) =(1/2)^{n}u(n). **(08 Marks)**

 b. Determine whether the following system defined by their impulse response are causes, memoryless and stable.
   i) $h(t) = e^{-2t}u(t-1)$
   ii) $h(n) = 2u(n) - 2u(n-5)$. **(08 Marks)**

 c. A system consists of several subsystems connected as shown in Fig.Q8(c). Find the operator H relating x(t) to y(t) for the following subsystem operators.
   H1 : $Y_1(t) = X_1(t)X_1(t-1)$
   H2 : $Y_2(t) = |X_2(t)|$
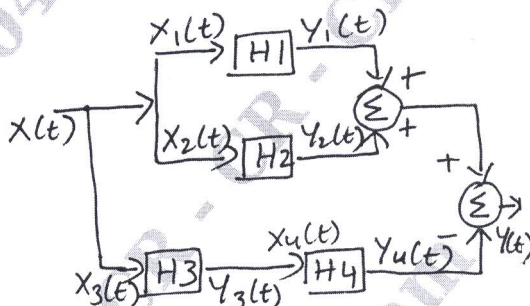   H3 : $Y_3(t) = 1 + 2X_3(t)$
   H4 : $Y_4(t) = \cos(X_4(t))$.

Fig.Q8(c) **(04 Marks)**

Module-5

9 a. Describe the properties of region of convergence and sketch the ROL of two sided, right sided and lift sided sequence. **(08 Marks)**

 b. Find Z – tranform of the following and specify its ROC
$$x(n) = \sin\left(\frac{\pi}{4}n - \frac{\pi}{2}\right)u(n-2)$$
$$x(n) = \left(\frac{2}{3}\right)^n u(n) * 2^n u(-n-3).$$ **(08 Marks)**

 c. Find inverse Z-transform if $X(z) = \dfrac{\left(\frac{1}{4}\right)z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$. **(04 Marks)**

OR

10 a. Describe the transfer function and the impulse response for the causal LTI system described by the differential equation :
$$y(n) - \frac{1}{4}y(n-1) - \frac{3}{8}y(n-2) = -x(n) + 2x(n-1).$$ **(10 Marks)**

 b. Determine the impulse response of the following transfer function if :
   i) The system is causal
   ii) The system is stable
   iii) The system is stable and causal at the same time : $H(z) = \dfrac{3z^{+2} - z}{(z-2)\left(z+\frac{1}{2}\right)}$. **(10 Marks)**

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