CBCS SCHEME

USN

20MCA14

First Semester MCA Degree Examination, June/July 2023
Mathematical Foundation for Computer Application

Time: 3 hrs. Max. Marks: 100

Note: 1. ***Answer any FIVE full questions, choosing ONE full question from each module.***
2. ***Use of normal distribution tables is permitted.***

Module-1

1 a. If A, B and C are non-empty sets, then prove that :
(i) $A - (B \cap C) = (A - B) \cup (A - C)$
(ii) $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$ **(06 Marks)**

b. How many integers between 1 and 300 (inclusive) are:
(i) Divisible by at least one of 5, 6, 8?
(ii) Divisible by none of 5, 6, 8? **(07 Marks)**

c. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$. **(07 Marks)**

OR

2 a. (i) Define power set of a set. Find the power set of the set $A = \{1, 2, 3\}$.
(ii) Find the sets A and B if $A - B = \{1, 2, 4\}$, $B - A = \{7, 8\}$, $A \cup B = \{1, 2, 4, 5, 7, 8, 9\}$. **(06 Marks)**

b. A survey of 500 television viewers of a sports channel produced the following information 285 watch cricket, 195 watch hockey, 115 watch football, 45 watch cricket and football, 70 watch cricket and hockey, 50 watch hockey and football and 50 do not watch any of the three kinds of games:
(i) How many viewers in the survey watch all three kinds of games?
(ii) How many viewers watch exactly one of the sports? **(07 Marks)**

c. If we select 10-points in the interior of an equilateral triangle of side 1m. Show that there must be at least two points whose distance apart is less than 1/3 m. **(07 Marks)**

Module-2

3 a. Define tautology and contradiction. Prove that, for any propositions p, q, r. The compound proposition $[(p \vee q) \wedge \{(p \to r) \wedge (q \to r)\}] \to r$ is a tautology. **(06 Marks)**

b. Prove the following logical equivalence, without using truth tables;
$(p \to q) \wedge [\sim q \wedge (r \vee \sim q] \Leftrightarrow \sim (q \vee p)$ **(07 Marks)**

c. Give a direct proof of the statement:
(i) "The square of an odd integer is an odd integer"
(ii) For all integers K and ℓ, if K and ℓ are both odd, then $(K + \ell)$ is even and $(K\ell)$ is odd. **(07 Marks)**

OR

4 a. Write down the truth table for converse, inverse and contrapositive. State the converse, inverse and contrapositive of the following conditional, "If a quadrilateral is a parallelogram, then its diagonals bisect each other". **(06 Marks)**

b. Find the validity of the arguments:
I will get grade A in this course or I will not graduate
If I do not graduate, I will join the army
I got graduate
∴ I will not join the army (07 Marks)

c. Let x be a specified number. Write down the negation of the following conditionals:
(i) "If x is an integer, then x is a rational number"
(ii) "If x is not a real number, then it is not a rational number and not an irrational number". (07 Marks)

Module-3

5 a. Let A = {1, 2, 3}, B = {1, 2, 3, 4}. The relations R and S from A to B are represented by the following matrices. Determine the relations $\overline{R}$, $R \cup S$, $R \cap S$ and S^C and also their matrix representations.

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}; \quad M_S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$ (06 Marks)

b. Define partially order set. Draw the Hasse diagram representing the positive divisors of 36. (07 Marks)

c. Let A = {1, 2, 3, 4, 5}, define a relation R on A × A by (x_1y_1) R (x_2y_2) if and only if $x_1 + y_1 = x_2 + y_2$. (07 Marks)

OR

6 a. Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by aRb if and only if a is multiple of b. Represent the relation R as a matrix and draw its digraph. (06 Marks)

b. Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}, define the relation R by (x, y) ∈ R if and only if (x − y) is multiple of 5. Verify that R is an equivalence relation. Find the partition of A induced by R. (07 Marks)

c. Let A = {1, 2, 3, 4} and R, S are relations on A defined by R = {(1, 2), (1, 3), (2, 4), (4, 4)}, S = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 3), (2, 4)}, find RoS, SoR, R^2, S^2. Write down their matrices. (07 Marks)

Module-4

7 a. The probability density function P(X) of a variate X is given by the following table:

X	0	1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13k

For what value of K, does this represent a valid probability distribution? Also find $P(X < 4)$, $P(X \geq 5)$ and $P(3 < X \leq 6)$. (06 Marks)

b. In a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain at least 3 defective parts. (07 Marks)

c. If x is an exponential variate with mean 4, evaluate the followings:
(i) $P(0 < x < 1)$
(ii) $P(x > 2)$
(iii) $P(-\infty < x < 10)$ (07 Marks)

OR

8 a. A random variable x has the density function

$$P(x) = \begin{cases} Kx^2, & -3 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

Evaluate K and find (i) $P(1 \le x \le 2)$ (ii) $P(x \le 2)$ (iii) $P(x > 1)$ **(06 Marks)**

b. The number of telephone lines busy at an instant of time is a binomial varaite with $p = 0.2$, if an instant of time, 10-lines are chosen at random, what is the probability that :
(i) 5 lines are busy (ii) At most 2-lines are busy (iii) All lines are busy **(07 Marks)**

c. The weekly wages of workers in a company are normally distributed with mean of Rs.700 and standard deviation of Rs.50. Find the probability that the weekly wage of a randomly chosen worker is: (i) Between Rs.650 and Rs.750 (ii) More than Rs.750
Given $\phi(1) = 0.3413$. **(07 Marks)**

Module-5

9 a. Define the following with an example for each :
(i) Complete graph (ii) Regular graph (iii) Bipartite graph **(06 Marks)**

b. Determine whether the graphs shown in Fig.Q9(b) are isomorphic:

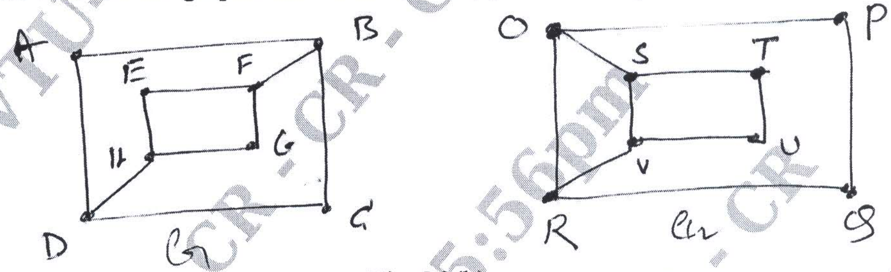

Fig.Q9(b) **(07 Marks)**

c. Explain Konisberg bridge problem. **(07 Marks)**

OR

10 a. Define the following with an example each:
(i) Euler circuit
(ii) Hamilton cycle
(iii) Hamilton path **(06 Marks)**

b. Show that complete graph K_5 (namely, the Kuratowski's first graph) is a non-linear graph. **(07 Marks)**

c. Find the chromatic numbers of the following graphs shown in Fig.Q10(c):

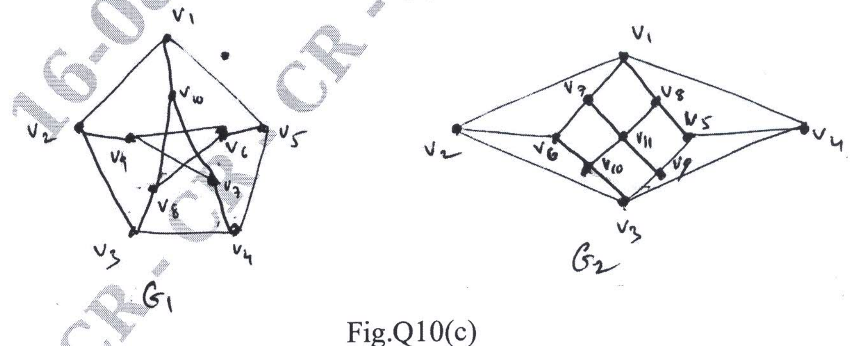

Fig.Q10(c) **(07 Marks)**

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