CBCS SCHEME

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22MCA11

First Semester MCA Degree Examination, June/July 2023
Mathematical Foundation for Computer Applications

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	**a.**	Define (i) Set (ii) Power set (iii) Subset , with an example each	6	L3	CO1
	b.	For any sets A and B, prove that (i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$	7	L3	CO1
	c.	Find all the eigen values and eigen vector corresponding to the largest eigen value of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$	7	L3	CO1
		OR			
Q.2	**a.**	State pigeonhole principle. ABC is an equilateral triangle whose sides are of length 1 m. If we select 10 points inside the triangle, prove that atleast 2 of these points are such that the distance between them is less than 1/3 m.	6	L3	CO1
	b.	In a survey of 260 students the following data were obtained 64 has taken maths course, 94 taken C.S. 58 had taken Business course, 24 taken maths and business, 26 has taken maths and C.S., 22 had taken C.S. and Business course, 14 had taken all the three courses. Find the number of students who had taken, (i) Only the C.S. (ii) None of the course	7	L3	CO1
	c.	Lest $f: R \to R$ defined by $f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \le 0 \end{cases}$ Determine (i) f(0), f(t), f(5/3), f(-5/3) (ii) $f^{-1}(0)$, $f^{-1}(t)$, $f^{-1}(-1)$, $f^{-1}(3)$, $f^{-1}(-3)$, $f^{-1}(-6)$	7	L3	CO1
		Module – 2			
Q.3	**a.**	Write the converse, inverse and contrapositive of the conditional statement. "If the home team wins then it is raining".	6	L2	CO3
	b.	Define Tautology and prove that $[(p \vee q)] \wedge [(p \to q) \wedge (q \to r)] \to r$, is a tautology.	7	L2	CO3
	c.	Give a direct proof, an indirect proof and a proof by contradiction for the following statement "If n is an odd integer then n+9 is an even integer".	7	L2	CO3
		OR			
Q.4	**a.**	Prove the following is valid argument: $\neg p \to q$ $q \to r$ $\neg r$ ――――― $\therefore p$	6	L2	CO3
	b.	Using laws of logic prove the following : i) $[p \vee q] \vee [(\neg p \wedge \neg q \wedge r)] \equiv (p \vee q \vee r)$ ii) $(p \to q) \wedge [\neg q \wedge (r \wedge \neg q)] \equiv \neg(p \vee q)$	7	L2	CO3

Q. No.		Question	Marks	Level	CO
	c.	Test the validity of the argument $\forall x, [p(x) \rightarrow q(x)]$ $\forall x, [q(x) \rightarrow r(x)]$ $\exists x, \neg r(x)$ ——————— $\therefore \exists x, \neg p(x)$	7	L2	CO3
		Module – 3			
Q.5	a.	If $A = \{1, 3, 5\}$, $B = \{2, 3\}$, $C = \{4, 6\}$, find the following : (i) $(A\times B)\cup(B\times C)$ (ii) $(A\cup B)\times C$ (iii) $(A\times B)\cap(B\times A)$	6	L2	CO5
	b.	Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 3), (3, 3), (4,4)\}$ be a relation on A. Verify that R is an equivalence relation.	7	L2	CO5
	c.	Let R be a relation on the set $A = \{1, 2, 3, 4\}$ defined by xRy if and only if "x divides y". Prove that (A, R) is a poset. Draw its Hasse diagram.	7	L2	CO5
		OR			
Q.6	a.	Let $A = \{1, 2, 3, 4\}$ and let R be the relation on A defined by xRy iff "x divided y" written x/y. i) Write down R as a set of ordered point ii) Draw the digraph of R iii) Determine the in-degree and out-degree of the vertices in the digraph.	6	L2	CO5
	b.	Let $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4\}$. The relation R and S from A to B are represented by the following matrices $M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ $M_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ Determine the relations $\overline{R}$, $R\cup S$, $R\cap S$ and their matrix representations.	7	L2	CO5
	c.	Draw the Hasse diagram representation of the partial ordering $\{(a, b) \mid a \text{ divides } b\}$ on $\{1, 2, 3, 4, 6, 12\}$	7	L2	CO5
		Module – 4			
Q.7	a.	Find the value of k such that the following distribution represents a finite probability distribution. Hence find its mean, variance. Also find $p(x < 1)$, $p(x > 0)$, $p(-2 < x \leq 3)$ x: -3, -2, -1, 0, 1, 2, 3 p(x): k, 2k, 3k, 4k, 3k, 2k, k	6	L3	CO2
	b.	If 2% of fuses manufactured by a firm are defective, find the probability that a box containing 200 fuses has i) At least one defective fuse ii) Three or more defective fuses iii) Exactly two defective fuses.	7	L3	CO2
	c.	The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted the disease. What is the probability that i) At least 10 survive ii) From 3 to 8 survive iii) Exactly 5 survive	7	L3	CO2
		OR			
Q.8	a.	A random variable x has the following probability density function $f(x) = \begin{cases} kx^2, & -3 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$ Evaluate k and find (i) $p(1 \leq x \leq 2)$ (ii) $p(x \leq 2)$ (iii) $p(x > 1)$	6	L3	CO2

			Marks	Level	CO
	b.	The market of 1000 students in an examination follows normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) More than 75 (iii) between 65 and 75. Given that $\phi(1) = 0.3413$	7	L3	CO2
	c.	A random variable X has the following probability density function for various values of x : (see table below) For what value of k does this represents a valid probability distribution? Find $p(x < 4)$, $p(x \geq 5)$, $p(3 \leq x \leq 6)$.	7	L3	CO2

x :	0	1	2	3	4	5	6
p(x) :	k	3k	5k	7k	9k	11k	13k

Module – 5

			Marks	Level	CO
Q.9	a.	Define the following with an example : i) Complete graph ii) Bipartite graph iii) Complement graph	6	L2	CO4
	b.	Show that the following graphs are Isomorphic. [Refer Fig.Q9(b)(i), Fig.Q9(b)(ii)]	7	L2	CO4
	c.	Explain the Konigsberg bridge problem.	7	L2	CO4

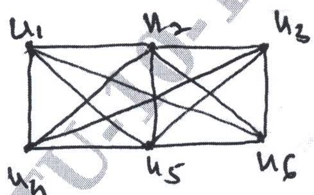

Fig.Q9(b)(i)

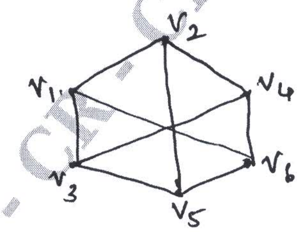

Fig.Q9(b)(ii)

OR

			Marks	Level	CO
Q.10	a.	Define the terms : (i) Regular graph (ii) Planar graphs (iii) Hamilton path with suitable example for each.	6	L2	CO4
	b.	Show that the bipartite graph $K_{2,2}$ and $K_{2,3}$ are planar graphs. [Refer Fig.Q10b(i) and Fig.Q10b(ii)]	7	L2	CO4
	c.	Find the chromatic polynomial and chromatic number for the cycle C_4 of length 4. [Refer Fig.Q10(c)]	7	L2	CO4

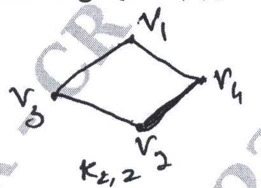

Fig.Q10(b)(i)

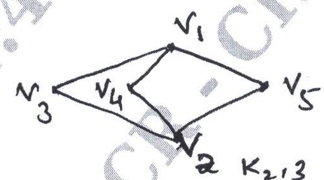

Fig.Q10(b)(ii)

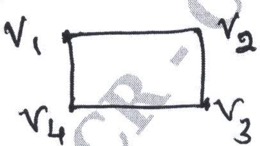

Fig.Q10(c)

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